B.Tech.

Fifth Semester Examination

Applied Numerical Technique & Computing (ME-311-F)

Note : Solve any five questions.

Q. 1. (a) Derive general error formula. If $N = \frac{4x^2y^3}{z^4}$ and $\Delta x = \Delta y = \Delta z = 0.001$ compute maximum relative error in N when $x = 1, y = 2, z = 3$.

Ans. Let $y = f(x_1, x_2 \ldots x_n)$ be a function of n variables $x_1, x_2 \ldots x_n$

Suppose δy be the error in y due to errors $\delta x, \delta x_2 \ldots \delta x_n$ in $x_1, x_2, \ldots x_n$ respect. Then

$$y + \delta y = f(x + \delta x_1, x + \delta x_2, \ldots x + \delta x_n) \quad \text{.... (i)}$$

Expanding by Taylor's Series

$$y + \delta y = f(x_1, x_2, \ldots x_n) + \left(\frac{\partial f_o}{\partial x_1}\delta x_1 + \ldots + \frac{\partial f}{\partial x_n}\delta x_n\right)$$

$$+ \frac{1}{\sqrt{2}}\left[\frac{\partial}{\partial x_1}\delta x_1 + \frac{\partial}{\partial x_2}\delta x_2 + \ldots \frac{\partial}{\partial x_n}\delta x_n\right]^2 f + \ldots$$

terms involving higher powers..... (ii)

$$y + \delta y = f(x_1, x_2, \ldots x_n) + \left(\frac{\partial f_1}{\partial x_1}\delta x_1 + \ldots + \frac{\partial f}{\partial x_n}\delta x_n\right) \text{approx}$$

$$\delta y = \frac{\partial f}{\partial x_1}\delta x_1 + \frac{\partial f}{\partial x_2}\delta x_2 + \ldots \frac{\partial f}{\partial x_n}\delta x_n \quad \text{.... (iii)}$$

Error in y calculated by equation (iii) $\quad N = \frac{4x^2y^3}{z^4}$

$$\Delta x = \Delta y = \Delta z = 0.001, x = 1, y = 2, z = 3$$

$$\frac{\partial N}{\partial x} = \frac{8xy^3}{z^4}, \frac{\partial N}{\partial y} = \frac{12x^2y^2}{z^4}, \frac{\partial N}{\partial z} = -\frac{16x^2y^3}{z^5}$$

$$\Delta N = \frac{\partial N}{\partial x}\delta x + \frac{\partial N}{\partial y}\delta y + \frac{\partial N}{\partial z}\delta z$$

$$(\Delta N)_{max} = \left|\frac{\partial N}{\partial x}\Delta x\right| + \left|\frac{\partial N}{\partial y}\Delta y\right| + \left|\frac{\partial N}{\partial z}\Delta z\right|$$

$$= \left|\frac{8xy^3}{z^4} \times .001\right| + \left|\frac{12x^2y^2}{z^4} \times .001\right| + \left|-\frac{16x^2y^3}{z^5} \times .001\right|$$

$$=\left|\frac{8\times 8}{81}\times .001\right|+\left|\frac{12\times 4}{81}\times .001\right|+\left|-\frac{16\times 8}{243}\times .001\right|$$

$$= 0019$$

Q. 1. (b) Find first three non-vanishing terms in the Taylor's expansion of $(x) = \sin 2x$ about $x = \frac{T_1}{4}$. Use the polynomial to find $\sin\left(\frac{T_1}{3}\right)$. Obtain a bound of error in using this approximation.

Ans.

$$f(n) = \sin 2x \qquad \text{about } x = T_1/4$$

$$f'(n) = 2\cos 2x \qquad f'(T_1/4) = 2\cos(2T_1/4)$$

$$f''(n) = -2^2 \sin 2x \qquad f''(T_1/4) = -2^2 \sin(2T_1/4)$$

$$f'''(n) = -2^3 \cos 2x \qquad f'''(T_1/4) = -2^3 \cos(T_1/4)$$

$$\sin 2x = f(n) = f(a) + (n-a)f'(a) + \frac{(n-a)^2}{21} f''(a) e \ldots\ldots$$

$$= \sin\left(\frac{T_1}{2}\right) + \left(x - \frac{T_1}{4}\right) 2\cos\left(\frac{T_1}{2}\right) + \frac{(x - T_1/4)^2}{2!} 2^2 \sin + \ldots\ldots\ldots$$

Now replace $x = T_1/6$

$$\sin(T_1/3) = \sin(T_1/2) + (-T_1/2)\cos(T_1/12) + \frac{(-T_1/12)}{2} \cdot 2^2 \sin + \ldots\ldots\ldots$$

Q. 2. (a) Fit the experimental curve $y = ae^{bx}$ to the following data :

x	2	4	6	8
y	25	38	56	84

Ans.

$$y = ae^{bx}$$

$$\log y = \log a + bx \log e$$

$$\log y = \log a + bx$$

$$y = A + bx$$

$(T_1/2)$ The normal equations are $y = \log y \qquad A = 206\Delta$

$$\sum y = nA + b\sum x$$

$$\sum xy = A\sum y + b\sum x^2$$

x	y	$\log y = y$	x^2	xy
2	25	1.3979	4	2.7958
4	38	1.5798	16	6.3192
6	56	1.7482	36	10.4892
8	84	1.9243	64	15.3944
20		6.6502	120	34.4986

$$\left.\begin{array}{r}6.6502 = 4A + 20b \\ 34.9986 = 20A + 120b\end{array}\right]$$

$$A = 1.2256 \qquad b = 0.08739$$

$$1.2256 = \log a \qquad a = 16.81$$

$$y = 16.81e^{0.8739x} \quad \textbf{Ans.}$$

Q. 2. (b) Find Newton's polynomial satisfying the following data. Hence find $f(2.5)$

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	19

Ans.

x	$f(n)$	$\Delta f(n)$	$\Delta^2 f(n)$	$\Delta^3 f(n)$	$\Delta^4 f(n)$	$\Delta^5 f(n)$
0	1					
		13				
1	14		−6			
		1		1		
2	15		−2		0	
		−5		1		0
4	5		2		0	
		1		1		
5	6		6			
		13				
6	8					

By Newton's divided Interpolation

$$f(n) = f(x_0) + (x - x_0)\Delta f(n_0) + (x - x_0)(x - x_1) f(n_0)$$
$$+ (x - x_0)(x - x_1)(x - x_1)\Delta^3 f_1(n_0) + \ldots\ldots\ldots\ldots$$
$$= 1 + (x - 0)13 + (x - 0)(x - 1)(-6) + (x - 0)(4x - 1)(x - 2)(1)$$
$$f(n) = x^3 - 9x^2 + 21x + 1$$
$$f(2.5) = (2.5)^3 - 9(2.5)^2 + 21(2.5) + 1$$
$$= 15.625 - 156.25 + 52.5 + 1$$
$$= 12.875$$

Q. 3. (a) Find the first and second derivative of the function tabulated below $x = 1.10$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	1.296	2.432	4

Ans.

x	$f(n)$	$\Delta f(n)$	$\Delta^2 f(n)$	$\Delta^3 f(n)$	$\Delta^4 f(n)$	$\Delta^5 f(n)$
1.0	0					
		0.1280				
1.2	0.128		0.298			
		0.4160		0.018		
1.4	0.544		0.316		0.06	
		0.7520		0.078		−0.1
1.6	1.296		0.394		−0.04	
		1.1360		0.038		
1.8	2.432		0.432			
		1.5680				
2.0	4					

Since we are to find $f'(1.1)$ & $f''(1.1)$ so we use Newton forward formula

$$y = f(n_0)+\mu\,\Delta f(n_0)+\frac{\mu(\mu-1)}{2}\Delta^2 f(n_0)+\frac{\mu(\mu-1)(\mu-2)}{3!}\Delta^3 f(n_0) \quad \text{.... (i)}$$

$$\mu=\frac{x-a}{h} \quad \text{..... (ii)}$$

$$\frac{dy}{d\mu}=\Delta f(n_0)+\frac{2\mu-1}{2}\Delta^2 y_0+\left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0+\left(\frac{2\mu^3-9\mu^2+11\mu-3}{12}\right)\Delta^4 y_0+\dots \quad \text{..... (iii)}$$

Differentiating equation (ii)

$$\frac{dx}{dn}=\frac{1}{h}$$

$$\frac{dy}{dn}=\frac{dy}{du}\frac{du}{dx}$$

$$\frac{dy}{dn}=\frac{1}{h}\left[\Delta f(n_0+\left(\frac{2\mu-1}{2}\right)\Delta^2 y_0+\left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0+\left(\frac{2\mu^3-9\mu^2+11u-3}{12}\right)\Delta^4 y_0\right]$$

$$x=1.1,\quad \mu=\frac{1.1-1.00}{0.2}=\frac{1}{2}$$

$$\frac{dy}{dn}=\text{but } x=1.1$$

$$f'(1.1)=\frac{1}{0.2}\left[0.1280+\left\{\frac{2(\tfrac12-1)}{2}\right\}0.298+\left\{\frac{3(\tfrac12)^2-6(\tfrac12)+2}{6}\right\}0.018\right.$$

$$\left.\left\{\frac{2(\tfrac12)^3-9(\tfrac12)^2+11(\tfrac12)-3}{12}\right\}(0.6)+\left\{\frac{5(\tfrac12)^4-40(\tfrac12)^3+105(\tfrac12)^2-100(\tfrac12)+24}{120}\right\}(0.1)\right]$$

$= 0.66724$

$$f''(x) = \frac{d}{du}\left(\frac{dy}{du}\right)\frac{du}{dn}$$

$$= \frac{l}{h^2}\left(\Delta^2 f(n_0) + (\mu - 1)\Delta^3 f(n_0) + \left(\frac{6u^2 - 18\mu + 11}{12}\right)\Delta^4(y_0) + \left(\frac{2u^3 - 12u^2 + 21u - 10}{12}\right)\Delta^5(y_0) \ldots\right.$$

At 1.1

$f''(1.1) = 8.13125$

Q. 3. (b) A river is 80 ft. wide. The depth *d* ft. at a distance *x* ft. from the bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
d	0	4.5	7.3	9.2	12.4	15.1	14	8	3.2

Find approximately the area of cross-section by :

(i) Trapezoidal Rule,

(ii) Simpson's $\frac{1}{3}$ rd rule

Ans. (i) By Trapezoidal Rule :

$$A = \int_0^{80} y d_n = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots\ldots\ldots y_{n-1}]$$

$$= \frac{10}{2}[(0+3.2) + 2(4.5+7.3+9.2+12.4+15.1+14+8)]$$

$$= 5[3.2 + 2[70.5] = 5[3.2 + 141]$$

$$= 5[144.2]$$

$$= 721 \text{ sq. ft.}$$

(ii) By Simpson's $\frac{1}{3}$ Rule :

$$A = \int_0^{80} y d_n = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + y_{n-1}) + 2(y_2 + y_4 + \ldots\ldots y_{n-2})]$$

$$= \frac{10}{3}[(0+3.2) + 4(4.5+9.2+15.1+8) + 2(7.3+12.4+14)]$$

$$= \frac{10}{3}[3.2 + 4(36.8) + 2(33.7)]$$

$$\frac{2178}{3} = \frac{10}{3}[3.2 + 147.2 + 67.4]$$

$$\frac{2178}{3} = 726 \text{ sq. ft.}$$

Q. 4. (a) Find a root of the equation $\cos x = 3x - 1$ correct to three decimal places by Secant method.

Ans. By Secant Method

$$f(n) = \cos 3x - \cos x - 1$$
$$f(0) = -2 \qquad (-\text{ve}) = f(a)$$
$$f(1) = 1.4597 \qquad (+\text{ve}) = f(b)$$
$$a = 0, b = 1$$
$$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{0(1.4597) - 1(-2)}{1.4597 + .2} = \frac{2}{3.4597}$$
$$x_1 = 0.5791$$
$$f(n_1) = 3(0.5781) - \cos(0.5781) - 1$$
$$= -0.2656$$
$$a = 1, b = 0.5781 \quad a\begin{bmatrix} 0.5781 \\ 0.6430 \end{bmatrix} \quad b \begin{bmatrix} 0.6667 \\ 0.6439 \end{bmatrix}$$
$$x_2 = \frac{1(-0.26561) - 0.5781(1.4597)}{-0.2656 - 1.4597}$$
$$= 0.6430$$
$$f(n_2) = 3(0.6430) - \cos(0.6430) - 1$$
$$= -0.0709$$
$$a = 0.5781 \qquad b = 0.6430$$
$$x_3 = \frac{0.5781(-0.0709) - 0.6430(-0.2656)}{-0.0709 + 0.2656}$$
$$x_3 = 0.6672$$
$$f(x_3) = 1.6771$$
$$x_4 = \frac{0.6430(1.6771) - (0.6667)(-0.0709)}{1.6771 + 0.0709}$$
$$x_4 = 0.6439$$

Root is 0.6439 **Ans.**

Q. 4. (b) Use Newton Raphson method to find a root of the equation $x^3 + x^2 - 100 = 0$ correct to three decimal places.

Ans. $f(n) = x^3 - x^2 - 100 = 0$

Newton Raphson Method

$$f(0) = -100 = -\text{ve}$$
$$f(5) = 0$$
$$f(6) = 80$$
$$f(4) = 64 - 16 - 100 = 52$$
$$f'(n) = 3x^2 - 2x$$

Take
$$x_0 = 4.96$$
$$x_1 = x_0 - \frac{f(n_0)}{f'(x_0)} = 4.96 - \frac{(-2.5777)}{63.8848}$$
$$= 4.96 + 0.04035$$
$$x_1 = 5.00035$$
$$n_2 = 5.0003 - \frac{f(5.0003)}{f'(5.0003)}$$
$$= 5.0003 - \frac{0.01950}{65.0084}$$
$$= 5.0003 - 0.00029$$
$$= 5.00001$$

Root is 5.00001 **Ans.**

Q. 5. (a) Apply UV factorization method to solve the equations :

$$3x+2y+7z=4; \quad 2x+3y+z=5; \quad 3x+4y+z=7$$

Ans.
$$3x+2y+7z=4$$
$$2x+3y+z=5$$
$$3x+4y+z=7$$

The equations can be written in the form

$$\begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_3 & 2 \end{bmatrix}\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

Where
$$LU = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \qquad \text{..... (i)}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21}u_{11} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ l_{31}u_{11} & l_{13}u_{12}+l_{33}u_{22} & l_{31}u_{13}+l_{32}u_{23}+u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix} \qquad \text{.... (ii)}$$

Equating
$$u_{11} = 3, u_{12} = 2, u_{13} = 7$$
$$l_{21}u_{11} = 2 \Rightarrow l_{21} = 2/3$$
$$l_{31}u_{11} = 3 \Rightarrow l_{31} = 1$$
$$l_{21}u_{12} + u_{22} = 3 \Rightarrow u_{22} = 5/3$$

$$l_{21}u_{13} + u_{23} = 1 \Rightarrow u_{23} = -11/3$$

$$l_{31}u_{12} + l_{32}u_{22} = 4 \Rightarrow l_{32} = 6/5$$

Similarly $$u_{33} = 8/5$$

Putting in equation (i)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & 8/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix} \quad \text{..... (iii)}$$

$$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad \text{..... (iv)}$$

From equation (iii) $$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$$

$\Rightarrow$ $$u = 4, v = 7/3, w = 1/5$$

From equation (iv) $$\begin{bmatrix} 3 & 2 & 7 \\ 0 & 5/3 & -11/3 \\ 0 & 0 & -8/5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7/3 \\ 1/5 \end{bmatrix}$$

$\Rightarrow$ $$x = 7/8, y = 9/8, \& z = -1/8$$

Q. 5. (b) Solve the following equations by Gauss Siedal method :

$$\mathbf{10x - 2y - 2z = 6; x - 10y + 2z = -7; -x - y + 10z = 8}$$

Ans. $10x - 2y - 2z = 6,\ x - 10y + 2z = -7, -x - y + 10z = 8$

$$x = \frac{1}{10}(2y + 2z + 6)$$

$$y = \frac{1}{10}(x + 2z + 7)$$

$$z = \frac{1}{10}(x + y + 8)$$

First approximation $$y = 0, z = 0 \Rightarrow x_1 = 6/10 = .6$$

Put $$x = .6$$

$$y_1 = \frac{1}{10}(.6 + 7) = .76$$

$$x_1 = .6,\ y_1 = .76 \Rightarrow z_1 = .936$$

Second approximation $$y = .76, z = .936$$

$\Rightarrow$ $$x_2 = \frac{1}{10}(2 \times .76 + 2(.936) + 6)$$

$$= .9392$$

$$y = \frac{1}{10}(.9392 + 2 \times .936 + 7)$$

$$= 1.8264$$

$$z_2 = \frac{1}{10}(.9392 + 1.8264 + 3)$$

$$= 1.07624$$

Third approximation
$$x_3 = \frac{1}{10}(2 \times 1.8264 + 2 \times 1.07624 + 6)$$

$$= 1380528$$

$$y_3 = \frac{1}{10}(1.3805 + 2 \times 1.0762 + 7)$$

$$= 1.0533$$

$$z_3 = \frac{1}{10}(1.3805 + 1.0533 + 8)$$

$$= 1.0434$$

Fourth approximation
$$x_4 = \frac{1}{10}[2 \times 1.0533 + 2 \times 1.0434 + 6]$$

$$= 1.0193$$

$$y_4 = \frac{1}{10}[1.0193 + 2 \times 1.0434 + 7]$$

$$= 1.0106$$

$$z_4 = \frac{1}{10}[1.0193 + 1.0106 + 8]$$

$$= 1.00299$$

$$z = y = z = 1$$

Q. 6. (a) Find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Ans.
$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Characteristic equation $|A - \lambda I| = 0$

$$\begin{bmatrix} 1-\lambda & 6 & 1 \\ 1 & 2-\lambda & 0 \\ 0 & 0 & 3-\lambda \end{bmatrix} = 0$$

$$(1-\lambda)(2-\lambda)(3-\lambda)-6(3-\lambda)=0$$

$$(3-\lambda)[2-\lambda-2\lambda+\lambda^2-6]=0$$

$$\lambda=3,4,-1$$

Maximum value $=4$

$$\begin{bmatrix}1-4 & 6 & 1\\ 1 & 2-4 & 0\\ 0 & 0 & 3-4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3 & 6 & 1\\ 1 & -2 & 0\\ 0 & 0 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$x_3=0$$

$$-3x_1+6x_2=0 \Rightarrow -x_1+2x_2=0$$

$$x_1-2x_2=0 \Rightarrow x_1-2x_2=0$$

Let $x_2=k$

$$x_1=2k$$

Eigen vector corresponds to $\lambda=4$ is $k\begin{bmatrix}2\\1\\0\end{bmatrix}$.

Q. 6. (b) Find eigen values and eigen vectors of the matrix A by Jacobi's method $A=\begin{bmatrix}5 & 0 & 1\\ 0 & -2 & 0\\ 1 & 0 & 5\end{bmatrix}$.

Ans.

$$A=\begin{bmatrix}5 & 0 & 1\\ 0 & -2 & 0\\ 1 & 0 & 5\end{bmatrix}$$

$$|A-\lambda I|=0 \Rightarrow \begin{bmatrix}5-\lambda & 0 & 1\\ 0 & -2-\lambda & 0\\ 1 & 0 & 5-\lambda\end{bmatrix}=0$$

$$-(5-\lambda)^2(2+\lambda)+(\lambda+2)=0$$

$$(\lambda+2)[1-25-\lambda^2+10\lambda]=0$$

$$(\lambda+2)[\lambda^2-10\lambda+24]=0$$

$$(\lambda+2)(\lambda^2-6\lambda-4\lambda+24)=0$$

$$(\lambda+2)(\lambda-6)(\lambda-4)=0$$

$$\lambda=-2,4,6$$

When $\lambda = -2$

$$\begin{bmatrix} 7 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$7x_1 + x_3 = 0$$

$$x_1 + 7x_3 = 0 \Rightarrow x_1 = -7x_3$$

$$x_2 = k_1$$

$$-49x_3 + x_3 = 0$$

$$x_3 = 0$$

$$x_1 = 0$$

Eigen vector $k_1 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

When $\lambda = 4$ $\quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & -6 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$x_2 = 0$$

$$x_1 + x_3 = 0$$

$$x_1 = -x_3$$

$$x_3 = k_2$$

$$x_1 = -k_2$$

Eigen vector corresponds $\quad x_2 = k_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

When $\lambda = 6$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -8 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 = x_3 = k_3$$

$$x_2 = 0$$

$$x_3 = k_3 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Q. 7. (a) Given $\frac{dy}{dx}=x+y^2$, $y(0)=1$ Use modified Euler's method to find $y(0.1)$, $y(0.2)$ & $y(0.3)$.

Ans.

$$\frac{dy}{dn}=x+y^2 \quad y(0)=1$$

$$f(n,y)=x+y^2$$

$$y_{n+1}=y_n+h\,f(x_n,y_n)$$

$$y_{n+1}=y_n+\frac{h}{2}[f(n_n y_n)+f(n_{n+1},y_{n+1}]$$

$$y(0.1)=1+0.1(0+1^2)$$

$$=1+0.1(1)=1.1$$

Modified formula

$$x_n+1=0.1$$

$$y_n+1=1.1$$

$$y_{(0.1)}=1+0.05[(0+1^2)+(0.1+(1.1)^2)]$$

$$=1+0.5[1+0.1+1.21]$$

$$=1+0.05[2.31]$$

$$=1.1155$$

Value of $y_{(0.2)}$

$$y_{n+1}=y_n+(0.1)(x_n+y_n^2)$$

$$y_{(0.2)}=y_{(0.1)}+(0.1)[0.1+y_{(0.1)}^2]$$

$$=1.1155+(0.1)[0.1+(1.1155)^2]$$

$$=1.1155+(0.1)[1.3443]$$

$$=1.1155+0.1344$$

$$=1.2499$$

Modified formula

$$y_{(0.2)}=y_n+(0.05)[(x_n+y_n^2)+(x_{n+1}^2+y_{n+1}^2)]$$

$$=1.2499+0.05[(0.1)+(1.1155)^2]+(0.2+1.2499^2)$$

$$=1.2499+(0.05)[1.3443+0.2+1.56225]$$

$$=1.2499+0.05(3.1066)$$

$$=1.2499+0.1553$$

$$=1.40523$$

$$y_{(0.3)}=1.40523+(0.1)[0.2+1.9746]$$

$$=1.6227$$

Modified formula

$$y_{(0.3)}=y_{(0.2)}+0.05\,[x_{(0.2)}+y_{0.2}{}^2+(x_{0.3}+y_{0.3}{}^2)]$$

$$=1.6227+0.05\,[0.2+1.9747+0.3+2.6332]$$

$$y_{(0.3)} = 1.8781$$

Q. 7. (b) Using Runge Kutta method of order 4, solve $y'' = y + xy'$, $y(0) = 1$, $y'(0) = 0$, to find $y(0.2)$ any $y'(0.2)$.

Ans. $y'' = y + xy', \ y(0) = 1, \ y'(0) = 0$

$$y(0.2), \ y'(0.2) = 2.$$

Substitute $\dfrac{dy}{dx} = z = f(x, y, z)$

$$\frac{d^2 y}{dn^2} = xz + y = g\ (n, y, z)$$

Initial condition $x_0 = 0, y_0 = 1, z_0 = 0$

$$h = 0.2$$

$$k_1 = hf(x_0, y_0, z_0) = hz_0 = 0.2 \times 0 = 0$$

$$m_1 = hg\ (x_0, y_0, z_0) = h\ (x_0 z_0 + y_0)$$

$$= 0.2\,[0 \times 0 + 1] = 0.2$$

$$k_2 = hf(x_0 + h/2, y_0 + \frac{k_1}{2}, z_0 + m/2)$$

$$= 0.2\,[z_0 + m/2] = 0.2\left[0 + \frac{0.2}{2}\right]$$

$$= 0.02$$

$$m_2 = hg\,[x_0 + h/2, y_0 + k_1/2, z_0 + m_1/2]$$

$$= 0.2\,[(x_0 + h/2)\,(z_0\ m/2) + (y_0 + k_1/2)]$$

$$= 0.2\left[\left(\frac{0.2}{2}\right)\left(\frac{0.2}{2}\right) + \left(1 + \frac{0}{2}\right)\right]$$

$$02[(0.1)\,(0.1) + 1] = (1.01)\,0.2$$

$$= .202$$

$$k_3 = hf\ (x_0\ h/2, \ y_0 + k_2/2, \ z_0 + m_2/2)$$

$$= h\ (z_0 + m_2/2) = 0.2\left[0 + \frac{.202}{2}\right]$$

$$0.2 + .101 = .0202$$

$$m_3 = hg\ (x_0 + h/2, \ y_0 + k/2, \ z_0 + m_2/2)$$

$$= h\,[(x_0 + h/2)\,(y_0 + m_2/2) + (y_0 + k_2/2)]$$

$$= h\left[\left(0 + \frac{0.2}{2}\right)\left(0 + \frac{0.202}{2}\right) + \left(1 + \frac{0.02}{2}\right)\right]$$

$$= 0.2\,[(0.1)\,(0.101) + 1.01]$$

$$= 0.2[.0101 + 1.01] = 0.2[1.0201]$$

$$m_3 = 0.20402$$

$$k_4 = 0.2(z_0 + m_3)$$

$$= 0.2[0 + 0.20402] = .040804$$

$$m_4 = (0.2)[(0+0.2)(0+0.20402) + 1 + 0.0202]$$

$$= 0.2[0.040804 + 1.0202]$$

$$= 0.2[0.061004]$$

$$= 0.122008$$

$$y(0.2) = y(0) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$= 1 + \frac{1}{6}[0 + 2(0.02) + 2(0.0202) + 0.040804]$$

$$= 1 + \frac{1}{6}[004 + 0.0404 + 0.040804]$$

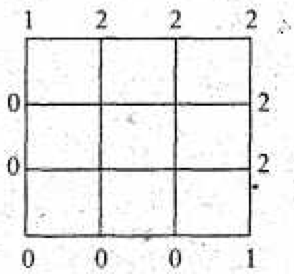
$$= 1.0202$$

$$z(02) = z(0) + \frac{1}{6}(m_1 + 2m_2 + 2m_3 + m_4)$$

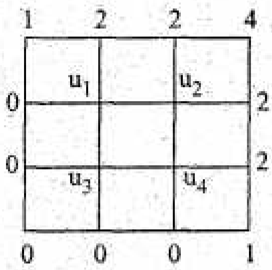
$$= 0 + \frac{1}{6}[0.2 + 2(.202) + 210.20402) + 0.122008]$$

$$= 0189008$$ **Ans.**

Q. 8. (a) Solve $U_{xx} + U_{xy} = 0$ for square mesh with boundary values in the figure below :



Ans.



Initially let $u_4 \sum 0$

Diagonal 5-pt formal $$u_{ij} = \frac{1}{4}[u_{i-1,j+1} + u_{i+1,j-1} + u_{i-1,j-1} + u_{i+1,j+1}]$$

Standard 5 pt formula $$u_{ij} = \frac{1}{4}[u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}]$$

$$u_{i,j}^{(n+1)} = \frac{1}{4}[u_{i-1,j}^{(n+1)} + u_{i+1,j}^{(n)} + u_{i,j+1}^{(n+1)} + u_{i,j-1}^{(n)}]$$

By Diagonal formula $$\mu_1 = \frac{1}{4}[1 + u_4 + 2 + 0]$$

$$= \frac{1}{4}[1 + 0 + 2 + 0] = .75$$

$$u_2 = \frac{1}{4}[u_1 + 2 + 2 + u_4]$$

$$= \frac{1}{4}[.75 + 2 + 2 + 0] = 1.187$$

$$u_3 = \frac{1}{4}[0 + u_1 + u_4 + 0] = .187$$

$$u_4 = \frac{1}{4}[u_2 + u_3 + 2 + 0] = 843$$

Now, $$u_1^{(n+1)} = \frac{1}{4}[0 + u_2^{(n)} + 2 + u_3^{(n)}]$$

$$u_2^{(n+1)} = \frac{1}{4}[u_1^{(n+1)} + 2 + 2 + u_4^{(n)})]$$

$$u_3^{(n+1)} = \frac{1}{4}[0 + u_1^{(n+1)} + + u_4^{(n)} + 0)]$$

$$u_4^{(n+1)} = \frac{1}{4}[2 + 0 + u_2^{(n+1)} + u_3^{(n+1)}]$$

But $n = 0$ $\quad u_1^{(1)} = .843, u_2^{(1)} = 1.421, u_3^{(1)} = .421, u_4^{(1)} = .961$

$n = 1$ $\quad u_1^{(2)} = .961, u_2^{(2)} = 1.481, u_3^{(2)} = .481, u_4^{(2)} = .991$

$n = 2$ $\quad u_1^{(3)} = .991, u_2^{(3)} = 1.496, u_3^{(3)} = .496, u_4^{(3)} = .998$

$n = 3$ $\quad u_1^{(4)} = .998, u_2^{(4)} = 1.499, u_3^{(4)} = .499, u_4^{(4)} = .999$

$n = 4$ $\quad u_1^{(5)} = .999, u_2^{(5)} = 1.49995, u_3^{(5)} = .4995, u_4^{(5)} = .9997$

Hence
$$u_1 = .999$$
$$u_2 = 1.4995$$
$$u_3 = .4995$$
$$u_4 = .9997$$

Q. 8. (b) Write computer programme for Taylor series method of solving ordinary differential equation initial value problems.

Ans. /* Method to solve first order initial value problem */

Program to solve an IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$

```
# include <stdio.h>
# include <math.h>
floatf() ;
main()
{
        float x0, y0, h, xf, x, y;
        int i, iter ;
        FILE * fp ;
        fp = fopen ("result", "w") ;
        printf(" Input initial point xo, initial value y0\n ) ;
        Printf(" step size h and final value xP \ n") ;
        Scanf("%f %f %f %f ", &x0  &y0 &h , &xP) ;
        fprintf(fp. "Initial point x0 = %f, initial", x0) ;
        fprintf(fp,"value y0 = %f  \ n:, y0) ;
        fprintf(fp, "step size = %f \ n", h ) ;
        fprintf(fp,"Final value = %f \ n", xf) ;
        iter = (xf - x0 ) / n + 1;
        for (i = 1; ic = iter ; i + +)
        {       y = y0 + h * f(x0, y0 );
                x = x0 + h
        if (x < xf)
        {       x0 = x ;
                y0 = y ;
}
fprintf (fp, "At x = % 6.4 f, y = % 12.6 e\n", x, y) ;
fprintf ("\ n Please see File 'result' for results\n\n") ;
fclose(fp);
return 0;
}
```