Section–D

8. (a) Form the partial differential equation by eliminating the function f from the relation

$$z = y^2 + 2\,f\left(\frac{1}{x} + \log\, y\right).$$

(b) Solve :

$$\frac{\partial^2 z}{\partial x^2} = a^2 z \text{ given that, when } x = 0,$$

$$\frac{\partial z}{\partial x} = a \sin y \text{ and } \frac{\partial z}{\partial y} = 0.$$

(c) Solve :

$$x\left(y^2 - z^2\right)p + y\left(z^2 - x^2\right)q = z\left(x^2 - y^2\right).$$

9. A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points a velocity $\lambda x\,(\ell - x)$, find the displacement of the string at any distance x from one end at any time t.

B. Tech 2nd Semester Examination,

May–2016

MATHEMATICS–II

Paper–Math–102 F

Common for all branches

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in total selecting at least one from each section.* ***Question No. 1 is compulsory.***

1. (a) Define Geometrical interpretation of gradient.

(b) State Green's theorem in the plane.

(c) Solve $y\,dx - x\,dy + 3\,x^2 y^2 e^{x^3}\,dx = 0$.

(d) Find the P. I. of $(D^2 - 4)\,y = e^{2x}$.

(e) Find the Laplace transform of cosh at sin at.

(f) Find the Laplace transform of

$$\frac{\sin^2 t}{t}.$$

(g) Solve $p^2 + q = q^2$.

(h) Solve :

$$\frac{\partial u}{\partial x} = 4\,\frac{\partial u}{\partial y}$$

by method of separation of variables.

Section–A

2. (a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point P (1, 2, 3) in the direction of the line PQ, where Q is the point (5, 0, 4).

(b) A vector field is given by

$$\vec{A} = (x^2 + xy^2)\,\hat{i} + (y^2 + x^2 y)\,\hat{j}.$$

Show that the field is irrotational and find the scalar potential.

3. Verify Green's theorem in the plane for

$$\oint_C (2xy - x^2)\,dx + (x^2 + y^2)\,dy,$$

where C is the boundary of the region enclosed by $y = x^2$ and $y^2 = x$.

Section–B

4. (a) State and prove the necessary and sufficient condition for the differential equation $M\,dx + N\,dy = 0$ to be exact.

(b) Find the orthogonal trajectory of the family of curves. $r = a(1 + \cos\theta)$.

5. (a) Solve :

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$$

(b) An uncharged condenser of capacity C is charged by applying an e.m.f. $E \sin \frac{t}{\sqrt{LC}}$, through leads of self-inductance L and negligible resistance. Find the charge on one of the plates at time t.

Section–C

6. (a) Find Laplace transform of the function f (t) defined as $f(t) = |t-1| + |t+1| + |t+2| + |t-2|,\ t \geq 0$.

(b) Find the inverse Laplace transform of

$$\log \frac{s^2+1}{(s-1)^2}.$$

(c) Find the inverse Laplace transform of

$$\frac{1}{(s^2+1)(s^2+9)}$$

by convolution theorem.

7. (a) Solve the integral equation

$$\int_0^t \frac{y(u)}{\sqrt{t-u}}\,du = 1 + t + t^2$$

by Laplace transform method.

(b) Find the Laplace transform of the triangular wave function of period 2 c given by

$$f(t) = t,\ 0 < t < c$$
$$= 2c - t,\ c < t < 2c.$$