B.Tech.

Third Semester Examination

Discrete Structure (CSE-203-F)

Note : Attempt any *FIVE* questions. All questions carry equal marks.

Q. 1. (a) Define the following terms and give example in each case :

Universal set, Multi set, Power set, Countable set.

Ans. Universal Set : In any application of the theory of sets, the members of all sets under investigation usually belong to some fixed-large set called the universal set. For example in plane geometry, the universal set consists of all the points in a plane. We will let the symbol.

U denote the universal set unless otherwise stated or implied.

For a given set U and a property P, there may not be any elements of U which have property P. For example the set

$$S = \left\{x : x \text{ is a positive integer } x^2 = 3\right\}$$

Has no elements since no positive integer has the required property.

Multiset :

Let $$E = \left\{x : x^2 - 3x + 2 = 0\right\},\ F = \{2, 1\} \text{ and } G = \left\{1, 2, 2, 1, \frac{6}{3}\right\}$$

Then $$E = F = G$$

Observe that a set does not depend on the way in which its elements are displayed. A set remains the same if its elements are repeated or rearranged. Hence G is a mulitset.

Power Set : For a given sets, we may speak of the class of all subsets of S. This class is called the power set of S and will be denoted by power(S). If S is finite then so power (S). In fact, the number of elements in Power (S) is 2 raised to the cardinality of S; that is

$$n(\text{Power}(S)) = 2^{n(s)}$$

Countable Sets : A set having finite no. of elements known as countable sets.

Q. 1. (b) Let A and B be any two sets, prove :

(i) A is disjoint union of (A–B) and $(A \cap B)$

(ii) $(A \cup B)$ is disjoint union of (A–B), $(A \cap B)$, (B – A).

Ans. x be any arbitrary element of A–B then

$$x \in (A - B) \Rightarrow x \in A\ \ x \notin B \Rightarrow x \notin A \text{ and } x \notin B',\ x \in A \cap B'$$

$$A - B \subseteq A \cap B'$$

Again y be an arbitrary element of $A \cap B'$. Then,

$$y \in A \cap B' \Rightarrow y \in A \text{ and } y \in B'$$

$$y \in A \text{ and } y \in B'$$

(v) Bijection Mapping : A mapping $f:A \to B$ is said to be bijective (or bijection) if f is both injective and surjective.

e.g., Let $f:z \to z$ be defined by $f(x) = x, z, x \in I \in z$. The mapping f is both injective and surjective. So, it is bijective.

(vi) Many One Mappings : A mapping $f : A \to B$ is said to be many one if two or more distinct elements in A have the same image i.e., $f(x) = f(x)' \Rightarrow x = x'$.

(vii) Identity Mapping : Let A be a set, let the functions $f:A \to A$ be defined by the formula f(x). Then f is called the identity function or the identity mapping or the identity transformation on A.

(viii) Inverse Mapping : Let $f:A \to B$ be a one-one onto mapping. Then the mapping $f^{-1}:B \to A$ which associates to each element $b \in B$ the element $a \in A$ such that $f(a) = b$ is called the inverse mapping of the mapping $f:A \to B$.

Hence, if $f:A \to B$ be one-one onto mapping then $f^{-1}:B \to A$. The mapping f^{-1} is called the inverse mapping of the mapping f.

Q. 2. (b) If R is a relation on the set of integers such that $(a, b) \in R$ if and only if '$3a + 4b = 7n$' or some integer n. Prove that R is an equivalence relation.

Ans. Consider a non-empty set S. A relation R on S is an equivalence relation if R is reflexive, symmetric and transitive. That is R is an equivalence relation on S if it has the following three properties :

(i) For every $a \in S$, aRa.

(ii) If aRb then bRa.

(iii) If aRb and bRc then aRc.

R is a relation on the set of integers such that $(a, b) \in R$ if and only if $3a + 4a = 7n$ for some integer n.

Reflexive : Let R is a relation in a set A (set of integer) $(a, a) \in R$ for all $a \in A$ so R is reflextive.

Symmetric Relation : R be a relation on a set A

$$a, b, \in A$$

$$(a, b) \in R \Rightarrow (b, a) \in R$$

So, R is symmetric.

Transitive Relation : R is a transitive relation because for a, b, $c \in A$,

$$(a, b) \in R, \ (b, c) \in R$$

$$(a, c) \in R \text{ also holds}$$

Hence R is transitive relation.

R is symmetric, reflexive, transitive relation hence R is equivalence relation.

Q. 3. (a) Show that the number of n-bit strings having exactly KO's, with no. two O's consecutive, is $C(n - k + 1, k)$.

Ans. String is of n bit length having exactly K O's with exactly KO's with no two O's consecutive is Consider K' O's corresponding to K positions. In each position takes an expression given by,

$$x^0 + x^1 + x^2 \ldots\ldots x^n$$

Here the various powers of x viz; 0, 1, 2, ... n correspond to the number of items each position can have in distributing. Since the total number of alphabet is n. So, the required number of ways is the coefficients of x^n in the product.

$$\left(x^0 + x^1 + x^2 \ldots\ldots x^n\right)\left(x^0 + x^1 + x^2 \ldots\ldots x^n\right)$$

$$\ldots\ldots \left(x^0 + x^1 + x^2 \ldots\ldots x^n\right) \text{ (K-bracket)}$$

$$= \text{coefficient of } x^n\left(x^0 + x^1 + x^2 + \ldots\ldots x^n\right)^r$$

$$= \text{coefficient of } x^n \text{ in } \left(\frac{1-x^{n+1}}{1-x}\right)^r$$

$$= \text{coefficient of } x^n \text{ in } \left(1-x^{n+1}\right)^r (1-x)^{-K}$$

$$= \text{coefficient of } x^n \text{ in } (1-x)^{-K}$$

$$= \frac{(K+1)(K+2)(K+3)\ldots\ldots(K+n-1)}{1.2.3\ldots.K}$$

$$= {}^{n-K+1}C_k = C(n-K+1, K) \qquad \textbf{Hence Proved}$$

Q. 3. (b) Seven car accidents occur in a week. What is the probability that they all occurred on the same day.

Ans. Seven car accidents occur in a week. Probability that they all occurred on the same day.

By product rule, there are seven ways of distributing seven accidents in span of seven days.

Among those are seven when all accidents occurred on the same day.

$\therefore$ P (seven accidents in week, all on same day)

$$= \frac{7}{7^7} = \frac{1}{7^6} \qquad \textbf{Ans.}$$

Q. 3. (c) How many permutation of 10 digits from {0, 1, 2, ..., 9} are there in which first digit is greater than 1 and last digit is less than 8.

Ans. Permutation of 10 digits from (0, 1, 2, ... 9) first digit is greater than 1 and last digit is less than 8.

First digit can be filled in 8 ways end last digit can be filled in 7 ways.

So, number of permutations are :

$$= 8 \times 19 \times 7$$

Q. 4. (a) Define field, integral domain and Ideal. Find ideal for $(z_6, \oplus, \otimes)$.

Ans. Integral Domains and Fields : R is called a commutative ring if ab = ba for every $a, b \in R$.

R is called a ring with an identity element 1 if the element 1 has property that $a.1 = 1.a = a$ for every element $a \in R$. In such cases an element $a \in R$ is called a unit if a has multiplicative inverse that is an element a^{-1} in R such that $aa^{-1} = a^{-1}a = 1$.

"A commutative ring R is an integral domain if R has no zero divisors if $ab = 0$ implies $a = 0$ or $b = 0$.

"A commutative ring R with an identity element 1 (not equal to 0) is a field if every non-zero $a \in R$ is a unit i.e. has a multiplicative inverse.

Ideal : A subset J of a ring R is called an Ideal in R if the following properties hold :

(i) $0 \in J$

(ii) For any a, $b \in J$ we have $(a - b) \in J$.

(iii) For any $r \in R$ and $a \in J$ we have $ra \to J$.

Note first that J is subring of R also J is a subgroup (necessarily) if the adaptive group of R. Thus, we can form the collection of cosets

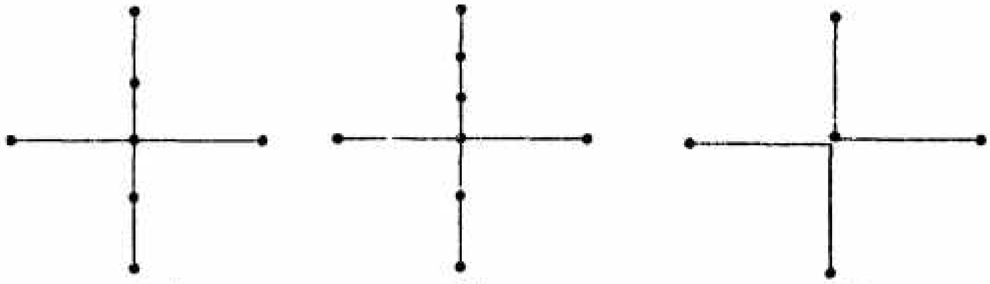
$$\{a + J : a \in R\}$$

Which forms a partition of R

Q. 4. (b) Explain Isomorphism, homomorphism and automorphism with the help of suitable example.

Ans. Homomorphism : Given any graph G, we can obtain a new graph by dividing an edge G with additional T vertices. Two graphs are said to be homomorphic if they can be obtained from the same graph or isomorphic graphs by this method.

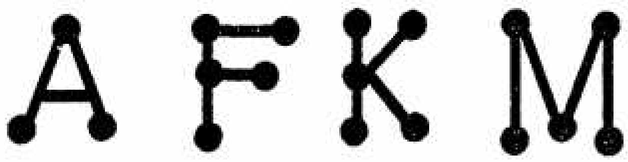
The graphs (a) and (b) are not isomorphic but they are homomorphic since they can be obtained from the graph (c) by adding appropriate vertices.

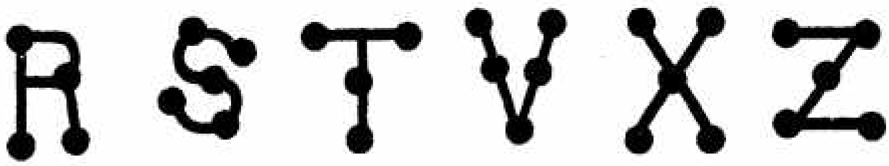


(a) (b) (c)

Isomorphism : Graphs G(V, E) and $G^*(V^*, E^*)$ are said to be isomorphic if there exist a one to one correspondence $f : V \to V^*$ such that $\{u, v\}$ is an edge of G if and only if $[f(u), f(v)]$ is an edge of G^*.

Normally we do not distinguish between isomorphic graphs.





A, R are isomorphic

F, T, K, X are isomorphic

M, S, V, Z are isomorphic

Q. 5. (a) Check the validity of the following argument :

$(p \vee q), (p \rightarrow r), (q \rightarrow r) \vdash r$.

Ans. $p \rightarrow q, \neg p \vdash \neg q$

Construct the truth table for $[p \rightarrow q \wedge \neg(p) \rightarrow \neg q] \rightarrow \neg q$ since the proposition $(p \rightarrow q) \wedge \neg p \rightarrow \neg q$ is not a a tautology the argument is fallacy. Equivalently the argument is fallacy since in third line of the truth table $p \rightarrow q$ and $\neg p$ are true but $\neg q$ is false.

(p)	(q)	$(p \rightarrow q)$	$\neg p$	$(p \rightarrow q \wedge \neg p)$	$(\neg q)$	$[p \rightarrow q \wedge p] \rightarrow \neg q)$
T	T	T	F	F	F	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	T	T	T	T

Q. 5. (b) Find the discrete numeric function corresponding to generating function

$$G(x) = \frac{x}{1 - 4x + x^2}$$

Ans. Discrete Functions : Functions are mappings from one Manifold to another. Discrete Functions are functions which can be represented using a finite number of values. Given the finite extent of computer memory, algorithms which compute a function that satisfies some special properties are computing a discrete function which approximates a continuous function. Computing the function involves writing a set of equations that may be solved for the values representing the function.

May be the simplest Discrete Functions are those whose domain is a Discrete Manifold, and which store a value from the range for each vertex in the domain. These we call Table Functions.

The more general discrete function may be written :

f(x) = sum v_i b_i (x)

For Table Functions, the b_i(x) are simply delta functions with origins at the ith vertex of the domain, and the values v _i are the value of the function when evaluated at the vertex. Discrete Functions provide all of the member functions that the Function provides, e.g. evaluate, evaluateDerivative, etc. In addition, the following functions are supported :

int getDimension( ) const;

double getValue(const int) const;

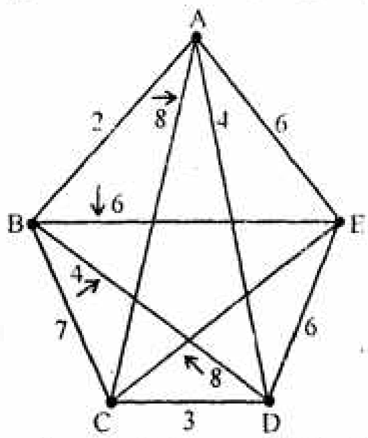
void setValue(const int, const double) const;

The dimension of a Discrete Function is the number of values used to represent it. These values can be retrieved and set with the getValue and setValue member functions. The significance of the values is left to the specific classes, although we have plans for adding the ability to provide a basis (the b_i(x)) to the FunctionSpace.

Some examples of discrete functions are :

* Table function, v _i=f(x-i)
* Fourier sine series, f(x)=sum v _i sin(i pi x).
* Finite element series, f(x)=sum v_i phi_i (x)
* Piecewise linear function.
* Splines.

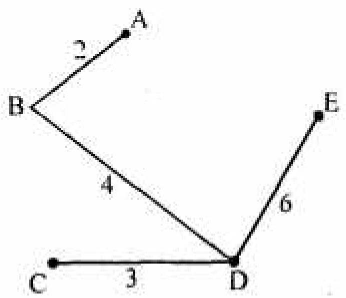
Q. 6. (a) Find minimum spanning tree for the following weighted graph using Kruskal's Algorithm :



Ans. Using Kruskal's Algorithm, edges of minimum weight that do not form a circuit are chosen a and added in the tree we arrange all the weights of edges in ascending order. Starting from edge with minimum weight that is first in the list.

An edge is added if it does not form a circuit.

Edge Selected	Decision	Weight
A B	Add	2
C D	Add	3
A D	Not added	4
B D	Add	4
E D	Add	6
B E	Not Added	6
A E	Not Added	6
B C	Not Added	7
C E	Not Added	8
A C	Not Added	8



Weigh of spanning tree = 15

Q. 6. (b) Describe Travelling salesperson problem and give nearest neighbour method of solving it.

Ans. The travelling salesman problem is a well-known problem in the area of network and combinatorial optimization. This problem is easy to state : "Starting from his home base, node I, a salesman wishes to visit each of several cities represented by nodes 2,...,n, exactly once and return home, doing so at the lowest travel cost". The simplicity of this problem and its complexity to solve have attracted the attention of many researchers over a long period of time. The first mathematical model related to the travelling salesman problem was studied in the 1800s. Researchers have paid attention to this problem because it is a generic core model that captures the combinatorial essence of most routing problems.

The purpose of this project is to create a decision support system that enables the user to solve the travelling salesman problem. We give a mixed-integer programming formulation of this problem and describe two heuristics that can be used to find feasible solutions. To learn more about the travelling salesman problem, we refer the students to Ahuja et al. (1993) and Winston (1994).

The following notation is used :

A	the set of arcs of the network
n	the total number of nodes.
N	the node-arc incidence matrix.
V	the set of nodes of the network
c_{ij}	the cost of using arc (i, j) $((i, j) \in A)$
b	the demand vector

The decision variables are :

x_{ij}	the flow on arc (i, j)
Y_{ij}	takes the value 1 if the salesman travels from city i to city j.

The TSP has several applications even in its purest formulation, such as planning, logistics and the manufacture of microchips. Slightly modified, it appears as a subproblem in many areas, such as genome sequencing. In these applications, the concept city represents, for example, customers, soldering points, or DNA fragments, and the concept distance represents travelling times or cost, or a similarity measure between DNA fragments. In many applications, additional constraints such as limited resources or time windows make the problem considerably harder.

In the theory of computational complexity, the decision version of TSP belongs to the class of NP-complete problems. Thus, it is assumed that there is no efficient algorithm for solving TSP problems. In other words. it is likely that the worst case running time for any algorithm for TSP increases exponentially with the number of cities, so even some instances with only hundreds of cities will take many CPU years to solve exactly.

Q. 6. (c) Show that number of odd degree vertices in an undirected graph is even.

Ans. Suppose G is undirected graph. For any vertex v of G the trail enters and leaves V the same number of times without repeating any edge. Hence, v has even degree.

Suppose conversely that each vertex of G has even degree. We construct as trail. We begin a trail T_1 at any edge e. We extend T_1 by adding one edge after the other. If T_1 is not closed at any step. Say T_1 begins of u but ends at $v \neq u$ then only an odd number of the edges incident on v appear in T_1 hence we can extend T_1 by another edge incident of v. Thus, we can continue to extend T_1 until T_1 returns to its initial vertex u. i.e., until T_1 is closed if T_1 includes all the edges of G T_1 is own trail.

Hence, the number of odd degree vertices in an undirected graph is even.

Q. 7. Prove Demorgan's laws of Boolean Algebra.

Ans. Prove Demorgan's Law of Boolean Algebra : Express $E(x, y, z) = x(y'z)'$ in the complex sum of products form,

$$E = x(y'z)' = x(y + z') = xy + xz'$$

$$E = xy(z + z') + xz'(y + y')$$

$$= xyz + xyz' + xyz' + xy'z'$$

$$= xyz + xyz' + xy'z'$$

Suppose Q is the consensus of P_1 and P_2.

Then $$P_1 + P_2 + Q = P_1 + P_2$$

Consensus Q of P_1 and P_2 where,

$$P_1 = xyz's \ \& \ P_2 = xy't$$

Delete y and y' and then multiply the literals of P_1 and P_2 to obtain $Q = xz's.t$

$$P_1 = xy' \qquad P_2 = y$$

Delete y and y' yields Q = x

$$P_1 = x'yz \qquad P_2 = x'yt$$

No variable appears uncomplemented in one of the products and complemented in the other. Hence P_1 and P_2 have no consensus.

$$P_1 = x'yz \qquad P_2 = xy'$$

Each of x and z appear complemented in one of the products and uncomplemented in the other. Hence P_1 and P_2 have no consensus.

Vertex file

		1	2	3	4	5	6	7	8
START [4]	Vertex	B		F	D	A		C	E
	Next V	3		5	1	8		0	7
	PTR	9		4	7	6		5	12

Edge File

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Adj	4	4	1	8	8	1	5	3	5	8	4	7		
Next	8	0	10	0	0	2	2	0	11	0	0	1		

Since start = 4 the list begins with the vertex D. The next V tells us to go to 1(B) then 3(F) then 5(A) then 8(E) and then 7 (C) that,

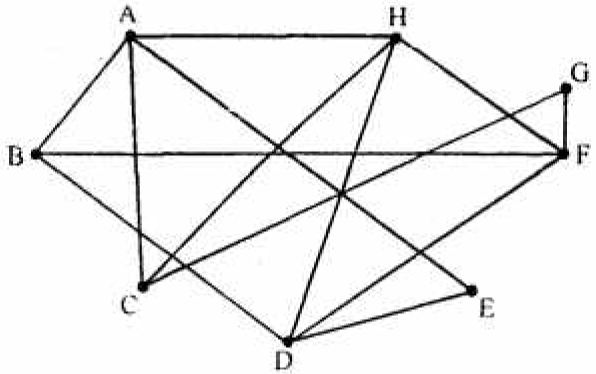
D B F A E C

Here, $\text{Adj}(D) = [5(A), 1(B), 8(E)]$

Specifically PTR 4[D] = 7 and $\text{Adj}[7] = 5[A]$

$$G = 2\{A:B:D;\ B:A,D;\ \ C;\ E,\ D;\ A,\ B,\ E;\ \ \ E:C,\ D,\ F;\ F:E\}$$

Q. 8. (a) What is Welch Powel's algorithm, explain. Use this algorithm to determine chromatic number of following graph :



Ans. Welch Powel's Algorithm : Consider a graph G. A vertex colouring or simply a colouring of G is an assignment of colours to the vertices of G such that adjacent vertices have different colours. We say that G is n colorable if there exist a colouring of G which uses n colours.

We give an algorithm by which Welch and Powell for a colouring of a Graph G. We emphasize that this algorithm does not always yield a minimal coloring of G.

Algorithm : The input is the graph G.

Step 1 : Order the vertices of G according to decreasing degrees.

Step 2 : Assign the first coulor G to the first vertex and then, in sequential order, assign C_1 to each vertex which is not adjacent to a previous vertex which was assigned C_1.

Step 3 : Repeat step 2 with a second colour C_2 and subsequences of non-coloured vertices.

Step 4 : Repeat step 3 with a third colour C_3, then a fourth colour C_4 and so on until all vertices are coloured. Degree of vertices in given graph :

A	4
B	3
C	3
D	4
E	2
F	4
G	2
H	4

Decreasing order (in terms of degree)

A	C_1
D	C_1
F	C_2
H	C_2
B	C_2
C	C_3
E	C_4
G	C_1

Four colours are required for the colouring of this graph so chromatic number is 4.

Q. 8. (b) If f, g, h : R → R are defined by

$$f(x) = x + 2g(x) = \frac{1}{x^2+1} \text{ and } h(x) = 3, \text{ find :}$$

(i) $g.h.f(x)$ **(ii)** $f^{-1}g.f(x)$

(iii) $f^{-1}g\,f(x)$ **(iv)** $g.f^{-1}f(x)$

(v) $f.g.h(x)$.

Ans. $f(x) = x+2$ $g(x) = \frac{1}{x^2+1}$

& $h(x) = 3$

(i) $g.h.f(x) = ?$

$f(x) = y$

/

$$x = f^{-1}(y)$$

$$y = x + 2$$

$$x = y - 2$$

$$f^{-1}(y) = (y - 2)$$

$$f^{-1}(x) = (x - 2)$$

$$g.h.f(x) = \frac{3}{x^2+1}(x-2)$$ **Ans.**

(ii) $$f^{-1}g\,f(x) = ?$$

$$g\,f(x)$$

$\Rightarrow$ $$gf(x+2) = \frac{1}{(x+2)^2+1}$$

$$f^{-1}gf(x)$$

$\Rightarrow$ $$f^{-1}\left(\frac{1}{(x+2)^2+1}\right)$$

$$= \frac{1}{(x+2)^2+1} - 2$$ **Ans.**

(iv) $$g.f^{-1}f(x) = ?$$

$$f^{-1}(x+2)$$

$$= x + x - 2$$

$$g.f^{-1}f(x)$$

$\Rightarrow$ $$g(x)$$

$\Rightarrow$ $$\frac{1}{x^2+2}$$ **Ans.**

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