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## GUJARAT TECHNOLOGICAL UNIVERSITY BE - SEMESTER-VII • EXAMINATION - WINTER • 2014

## Subject Code: 171901

Date: 25-11-2014
Subject Name: Operation Research
Time: 10:30 am - 01:00 pm
Total Marks: 70 Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q:-1 (a) Write the dual of the following linear programming problem.
Minimize, $Z=20 X_{1}+23 X_{2}$
Subjected to,

$$
\begin{aligned}
& -4 \mathrm{X}_{1}-\mathrm{X}_{2} \leq-8 \\
& 5 \mathrm{X}_{1}-3 \mathrm{X}_{2}=-4 \\
& \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0
\end{aligned}
$$

Solve the Dual problem using simplex method and predict the value of variables $\mathrm{X}_{1}$, $\mathrm{X}_{2}$ from the solution of dual linear programming problem.
(b) Optimum simplex table of the following linear programming problem has been given in the table:-1.

Maximize, $Z=60 \mathrm{X}_{1}+20 \mathrm{X}_{2}+80 \mathrm{X}_{3}$
Subjected to,

$$
\begin{aligned}
& 6 X_{1}+3 X_{2}+5 X_{3} \leq 750 \\
& 3 X_{1}+4 X_{2}+5 X_{3} \leq 600 \\
& X_{1}, X_{2}, X_{3} \geq 0
\end{aligned}
$$

|  | Basic <br> Varian |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 60 | $\mathrm{X}_{1}$ | $\mathrm{X}_{2}$ | $\mathrm{X}_{3}$ | $\mathrm{~W}_{1}$ | $\mathrm{~W}_{2}$ |  |
| 80 | 1 | $-1 / 3$ | 0 | $1 / 3$ | $-1 / 3$ | 50 |

(i) If the RHS of the constraints changes to [750, 900] ${ }^{\mathrm{T}}$, does it affect the optimum solution? If yes, then find the optimum solution using sensitivity analysis approach.
(ii) If coefficient of $X_{2}$ in the constraints change to [1, 1] ${ }^{\mathrm{T}}$, does it affect the optimum solution? If yes, obtain the optimum solution using sensitivity analysis approach.
(iii) If new constraint $X_{1}+X_{2}+X_{3} \leq 90$ is added to the LP Problem, does it affect the optimum solution? If yes, obtain the optimum solution using sensitivity analysis approach.

Q:-2 (a) A Manufacturer wants to ship 8 loads of his product as shown in following matrix.
The matrix gives the mileage from origins, O to the destinations, D . The shipping cost is Rs. 10 per load per mile. What will be the optimal schedule and optimal cost? Use Vogel's approximation method to find initial basis feasible solution and MODI method to obtain optimal solution.

|  | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | Supply |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{1}$ | 50 | 30 | 220 | 1 |
| $\mathrm{O}_{2}$ | 90 | 45 | 170 | 3 |
| $\mathrm{O}_{2}$ | 250 | 200 | 50 | 4 |
| Demand | 4 | 2 | 2 |  |

(b) (i) Using dynamic programming solve the following L.P.P.,

Maximize, $Z=X_{1}+9 X_{2}$
Subjected to,

$$
\begin{aligned}
& \mathrm{X}_{2} \leq 11 \\
& 2 \mathrm{X}_{1}+\mathrm{X}_{2} \leq 25 \\
& \mathrm{X}_{1}, \mathrm{X}_{2} \geq 0
\end{aligned}
$$

(ii) In brief, explain characteristics of operation research.

OR
(b) (i) A student of engineering wants to appear in the three competitive exam and he has three days left before examination. He wânts to revise the whole syllabus of the subjects before examination by devoting a single day, two days or not a single day to any subject based on given estimate of expected grade points as shown in matrix. How he should plan his study?

(ii) Write the definition of operation research given by author Churchman, Ackoff and

## Arnoff.

Q:-3 (a) The owner of a small machine shop has four machinists available. To assign jobs for the days. Five jobs are offered with the expected profit in rupees for each machinist on each job has been shown in matrix below. Find the assignment of machinists to jobs that will result in a maximum profit. Which job should be declined?

|  |  | Jobs |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C | D | E |
|  | 1 | 6.20 | 7.80 | 5.00 | 10.10 | 8.20 |
|  | 2 | 7.10 | 8.40 | 6.10 | 7.30 | 5.90 |
|  | 3 | 8.70 | 9.20 | 11.10 | 7.10 | 8.10 |
|  | 4 | 4.80 | 6.40 | 8.70 | 7.70 | 8.00 |

(b) Determine the approximate solution of following game problem (do minimum 10 iteration)

|  |  | B |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 1 | 2 | 3 | 4 |
| A | 1 | 3 | 2 | 4 | 0 |
|  | 2 | 3 | 4 | 2 | 4 |
|  | 3 | 4 | 2 | 4 | 0 |
|  | 4 | 0 | 4 | 0 | 8 |

OR
Q:-3 (a) A company is facing the problem of assigning six different machines to five different jobs. The estimated costs are given in matrix as below.

|  |  | Jobs |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 |
|  | 1 | 2.5 | 5 | 1 | 6 | 1 |
|  | 2 | 2 | 5 | 1.5 | 7 | 3 |
|  | 3 | 3 | 6.5 | 2 | 8 | 3 |
|  | 4 | 3.5 | 7 | 2 | 9 | 4.5 |
|  | 5 | 4 | 7 | 3 | 9 | - 6 |
|  | 6 | 6 | 9 | 5 | 10 | 6 |

Solve the problem assuming that the objective function is to minimize total cost. Is there any alternate optimal solution exist? If yes, find the possible alternate solution.
(b) For a game shown below, if $\mathrm{X}_{1}: \mathrm{X}_{2}=(1 / 2):(2 / 3)$ and $\mathrm{Y}_{1}: \mathrm{Y}_{2}=(3 / 4):(1 / 4)$.

Find expected payof Are these strategies optimal for player I and II? Why?

|  | II |  |
| :---: | :---: | :---: |
| I | 1 | 4 |
|  | 3 | 2 |

Q:-4 (a) The activities A to H of a new project having relationships and timings shown in table 07 below.

|  | Duration (in days) |  |  | Relationship between |
| :---: | :---: | :---: | :---: | :---: |
| Activity | $\mathrm{t}_{0}$ | $\mathrm{t}_{\mathrm{m}}$ | $\mathrm{t}_{\mathrm{p}}$ |  |
| A | 2 | 2 | 8 | $\mathrm{C}<\mathrm{F}$ |
| B | 2 | 5 | 8 | D < F |
| C | 3 | 6 | 15 | E, $\mathrm{F}<\mathrm{H}$ |
| D | 2 | 5 | 14 |  |
| E | 1 | 1 | 7 |  |
| F | 2 | 2 | 8 |  |
| G | 2 | 2 | 8 |  |
| H | 2 | 5 | 14 |  |

(1) Draw the network.

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(2) Find the critical path and expected time of completion of the project.
(3) What will be the standard deviation of the project completion duration?
(4) What will be the probability of completing the project in expected time of completion?
(b) (i) Explain in brief Monte carlo simulation.
(ii) Automobile arrives at a petrol pump having one petrol unit in poisson fashion with an average of 10 units per hour. The service time is distributed exponentially with a mean of 3 min . Find following:-
a. Average number of unit in system
b. Average waiting time for customer in queue.
c. Probability that number of units in system is 2 .
d. Probability that waiting time exceeds 30 min .

OR
Q:-4 (a) Following table shows jobs, normal and crash time, normal and crash cost of a
project. Indirect cost for the project is 300 Rs ./day.

| Jobs <br> $\mathrm{i}-\mathrm{j}$ | Normal <br> Time <br> (Days) | Normal <br> Cost <br> (Rs.) | Crash <br> Time <br> (Days) | Crash <br> Cost <br> (Rs.) |
| :---: | :---: | :---: | :---: | :---: |
| $1-2$ | 6 | 1400 | 4 | 1900 |
| $1-3$ | 8 | 2000 | 5 | 2800 |
| $2-3$ | 4 | 1100 | 2 | 1500 |
| $2-4$ | 3 | 800 | 2 | 1400 |
| $3-4$ | --- | --- | -- | --- |
| $2-5$ | 6 | 900 | 3 | 1600 |
| $4-6$ | 10 | 2500 | 6 | 3500 |
| $5-6$ | 3 | 500 | 2 | 800 |

Draw the network and find the critical path.
(ii) What is normal duration and cost of project.
(iii) Find optimal cost and duration.
(b)(i) Explain Kendall's notation for representing Queuing model.
(ii) Generate random numbers using (1) Mixed congruence method and (2) Additive 04 congruence method for the data: $\mathrm{r}_{0}=2, \mathrm{a}=14, \mathrm{~b}=12$ and $\mathrm{m}=32$.

## Q:-5 (a)

(i) Explain importance of replacement in brief.
(ii) The value of the money is $10 \%$ per year. Machine-1 is to be replaced every 3 years and Machine -2 is to be replaced for every 6 years with yearly expenditure as given below. Which machine costs less?

|  | Expenditure (in rupees) |  |
| :---: | :---: | :---: |
| Year | Machine:-1 | Machine:-2 |
| 1 | 2000 | 3400 |
| 2 | 400 | 200 |
| 3 | 800 | 400 |
| 4 | --- | 600 |
| 5 | --- | 800 |
| 6 | --- | 1000 |

(b) Derive the expression of optimal production lot size and optimum level of shortage for the inventory model with gradual supply and shortage is allowed.

OR
Q:-5 (a) (i) Explain in brief the reason for replacement.
(ii) As new automobile vehicle costs of Rs. 10000 and it can be sold at the end of any year with the selling price as shown below. The operating and maintenance cost are given year wise in following table. Find when the automobile vehicle needs to be replaced because of wear and tear.

|  | Expenditure (in rupees) |  |
| :---: | :---: | :---: |
| Year | Selling <br> Price (Rs.) | Operating and <br> maintenance <br> cost (Rs.) |
|  | 7000 | 1000 |
|  | 2 | 5000 |
|  | 3 | 3000 |

(b) Explain ABC analysis.

