B.Tech.

Fourth Semester Examination

Fluid Mechanics (ME-208-F)

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) What is meant by stability of a floating body ? Explain the stability of a floating body with reference to its metacentric height. Give neat sketches. 6

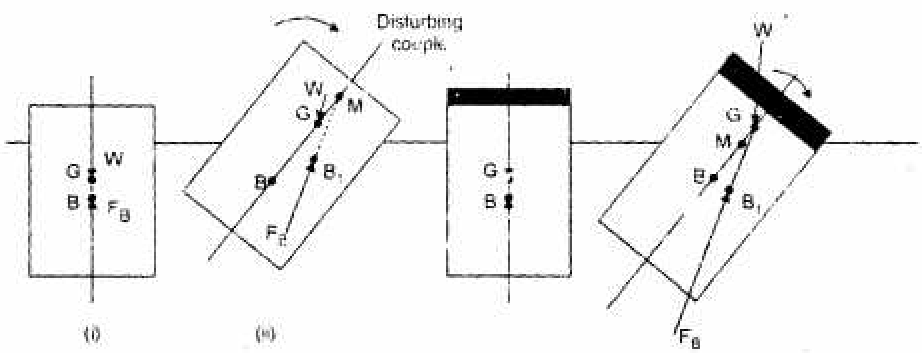
Ans. A submerged or a floating body is said to be stable if it comes back to its original position after a slight disturbance.

Stability of Floating Body : The stability of a floating body is determined from the position of meta centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(a) Stable Equilibrium : If the point M is above G, the floating body will be in stable equilibrium as shown in fig (a). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.

(b) Unstable Equilibrium : If the point M is below G, the floating body will be in unstable equilibrium as shown in fig. (b). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(c) Neutral Equilibrium : If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

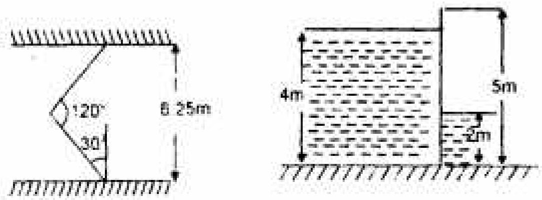


(a) Stable Equilibrium M is above G (b) Unstable Equilibrium M is below G

Fig. : Stability of Floating Bodies

Q. 1. (b) The end gates of a lock are 5m high and when closed include an angle of 120°. The width of the lock is 6.25 m. Each gate is carried on two hinged placed at the top and the bottom of the gate. If the water levels are 4m and 2m on the upstream and downstream sides, respectively. Determine the magnitudes of the forces on the hinges due to the water pressure. 14

Ans. Width at each lock, $l = \dfrac{6.25}{2 \times \cos 30} = 3.6 \text{ m}$



Water pressure on upstream side

$$F_1 = \rho g h_1 A_1$$

Where $A_1 = l \times H_1 = 3.6 \times 4 = 14.43\ m^2$

$$h_1 = \frac{H_1}{2} = 2m$$

$\therefore$ $$F_1 = 1000 \times 9.8 \times 2 \times 14.43 = 282901.6\ N$$

Water pressure on down stream side

$$F_2 = \rho g h_2 A_2$$

$$A_2 = l \times H_2 = 3.6 \times 2 = 7.2\ m^2\ h_2 = 1\ m$$

$\therefore$ $$F_2 = 1000 \times 9.8 \times 1 \times 7.2 = 70632\ N$$

Resultant water pressure $= F_1 - F_2$

$$= 282901.6 - 70632$$

$$= 212269\ N = 212.269\ KN$$

The reaction between gates is given by

$$P = \frac{F}{2 \sin \theta} = \frac{212.269}{2 \sin 30} = 212.269\ KN$$

If R_T and R_B are reactions at top and bottom hinges then,

$$R_T + R_B = R$$

But $$R = P = 212.269\ KN$$

The force F_1 is acting $\frac{4}{3} = 1.33$ m from bottom and F_2 at

$\frac{2}{3} = 0.67$ m from bottom, Resultant force at x distance given by

$$F \times x = F_1 \times 1.33 - F_2 \times 0.67$$

$\Rightarrow$ $$x = \frac{282.901 \times 1.33 - 70.632 \times 0.67}{212.269}$$

$$= 1.55\ m$$

Taking moments of R_T and R above bottom

$$R_T \times (5) = R \times (x)$$

$\Rightarrow$ $$R_T = \frac{212.269 \times 1.55}{5} = 65.79\ KN$$

$$R_B = 212.269 - 65.79 = 146.48\ KN$$

Q. 2. (a) Define and distinguish between stream line, path line and streak line. Also derive 2-D differential equation for stream line. **12**

Ans. Stream Line : A stream line is an instantaneous line drawn in a flow field in such a way that all the fluid particles falling on the line will have velocity vectors along the tangents drawn to the line at the particle locations at that instant.

Path Line : Path line is the line drawn through the path traced by a single fluid particle over a period of time.

Steak Line : The line, which at a given instant connects the temporary location of all the fluid particles that have traversed a fixed point in the flow field, is called a steak line.

Derivation : The equation of a stream line can be obtained by equating the slope of the stream line to the slope of the velocity vector.

Thus, at point P in I fig.

$$\frac{dy}{dx} = \frac{v}{u} \text{ or } \frac{dx}{u} = \frac{dy}{v}$$

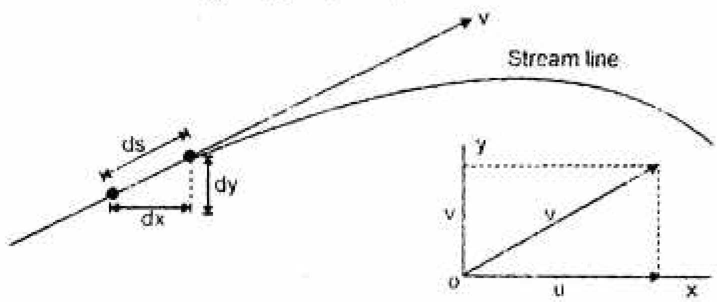


Fig. : Stream line

Q. 2. (b) Calculate unknown velocity component to satisfy the continuity equation. **8**

$$u = 2x^2 + 2xy,\ w = z^3 - 4xz - 2yz$$

Ans. The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(2x^2 + 2xy\right) = 4x + 2y$$

$$\frac{\partial w}{\partial z} = \frac{\partial}{\partial z}\left(z^3 - 4xz - 2yz\right) = 3z^2 - 4x - 2y$$

$\therefore$
$$4x + 2y + \frac{\partial v}{\partial y} + 3z^2 - 4x - 2y = 0$$

$\Rightarrow$
$$\partial v = (-3z^2)\partial y$$

Upon integrating, we get

$$v = -3z^2 y + c$$

Where c is constant at integration = F(x, z)

$\Rightarrow$
$$v = -3z^2 y + F(x, z)$$

Q. 3. (a) Start from the first principles obtain the following expression for discharge of a liquid through a orificemeter.

$$Q = Cd \cdot A \cdot \sqrt{h}$$

where Q is discharge in m^3/s, Cd-discharge coefficient and A-constant of orificemeter. **10**

Ans. Neglecting the small loss of head that occurs in the jet contraction and applying Bernoulli's equation between (1) and (c),

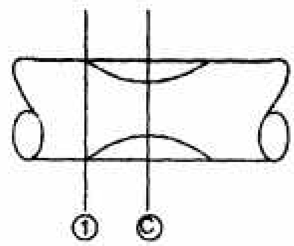
$$\frac{P_1}{W} + \frac{V_1^2}{2g} + z_1 = \frac{P_c}{W} + \frac{V_c^2}{2g} + Z_c$$

Where Z_1 and Z_c are the heights of centres of section (1) and (c) above an arbitrary datum. Denoting the difference between the piezometric head by h, i.e.,

$$h = \left(\frac{P_1}{W} + Z_1\right) - \left(\frac{P_c}{W} + Z_c\right)$$

So that,

$$h = \frac{V_c^2 - V_1^2}{2g}$$



Using the continuity equation :

$$V_1 A = V_c a_c = \frac{C_c a}{A} V_c$$

Where A = area of cross-section of pipe.

$$\frac{V_c^2}{2g}\left[1 - \left(\frac{C_c a}{A}\right)^2\right] = h$$

Or $$V_c = \frac{\sqrt{2gh}}{\sqrt{1 - (C_c a/A)^2}} = \frac{\sqrt{2gh}}{\sqrt{1 - C_c^2 (d/D)^4}}$$

The actual velocity is given by

$$V_c = \frac{C_v \sqrt{2gh}}{\sqrt{1 - C_c^2 (d/D)^2}}$$

Where C_v is a coefficient of velocity that accounts for the small velocity reduction due to friction. The rate of flow is :

$$Q = a_c V_c = C_c a_c V_c \frac{C_c C_v \sqrt{2gh}\,(\pi d^2/4)}{\sqrt{1 - C_c^2 (d/D)^4}}$$

But $C_c C_v = C_d$

Taking $$A = \sqrt{\frac{2g\,(\pi d^2/4)}{1 - C_c^2 \left(\frac{d}{D}\right)^4}}$$ we get, $Q = C_d A\sqrt{h}$

Q. 3. (b) Water under a constant head of 3m discharges through an external cylindrical mouthpiece 50 mm diameter, for which q = 0.82 find :

(i) discharge

(ii) absolute pressure at vena contracta and

(iii) the maximum head for the mouthpiece to flow full. 10

Ans. Area of mouthpiece

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2$$

$$= 1.96 \times 10^{-3}\ m^2$$

Discharge $= c_d \times A \times \sqrt{2gh}$

$$= 0.82 \times 1.96 \times 10^{-3} \times \sqrt{2 \times 9.81 \times 3}$$

$$= 0.0123\ m^3/s \text{ Ans.}$$

Pressure lead at vena contracta

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + Z_A = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c$$

But $\frac{P_A}{\rho g} = H_a + H,\ V_A = 0,\ Z_A = Z_c$

$\therefore$ $H_a + H = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} = H_c + \frac{V_c^2}{2g}$

$\Rightarrow$ $H_c = H_a + H - \frac{V_c^2}{2g}$

Assuming $C_c = 0.62$

$$V_c = \frac{V_1}{0.62} \text{ and } H = 1.375 \frac{V_1^2}{2g} \Rightarrow \frac{V_1^2}{2g} = \frac{H}{1.375}$$

Taking $H_a = 10.3$ m

$\therefore$ $H_c = H_a + H - \frac{V_1^2}{2g} \times \left(\frac{1}{0.62}\right)^2$

$$= 10.3 + 3 - \frac{3}{1.375} \times \frac{1}{(0.62)^2}$$

$$= 7.62 \text{ m Ans.}$$

Max. lead to flow full

$$H_{max} = H_{atm} - H_c$$

$$= 10.3 - 7.62 = 2.67 \text{ m Ans.}$$

Q. 4. The nose of a solid strut 10 cm wide is to be placed in an infinite two-dimensional air stream of velocity 20m/s and density 1.22 kg/m^3 and is to be designed in the shape of a Rankine half body. Calculate :

(i) strength of the corresponding source

(ii) distance between the stagnation point and the source

(iii) equation of the surface in rectangular co-ordinates based on source located at the origin.

(iv) pressure difference between the stagnation point and a point on the strut where it is 5cm wide. **20**

Ans. For the flow past a Rankine half body.

Width of nose = 0.1 m

Maximum ordinate $$y_{max} = \frac{0.1}{2} = 0.05 \text{ m}$$

But $$y_{max} = \frac{m}{2U_0} \Rightarrow 0.05 = \frac{M}{2 \times 20}$$

$$\Rightarrow \quad m = 2 \text{ m}^2/\text{s Ans.}$$

$$r_s = \frac{m}{2\pi U_0} = \frac{2}{2\pi \times 20} = 1.59 \times 10^{-2} \text{m}$$

The contour of Rankine half body

$$\frac{m}{2} = U_0 y + \frac{m\theta}{2\pi}$$

$$\Rightarrow \quad \frac{m}{2} = U_0 y + \frac{m}{2\pi} \sin^{-1} \frac{y}{\sqrt{x^2 + y^2}}$$

$$\Rightarrow \quad \frac{2}{2} = 20y + \frac{2}{2\pi} \sin^{-1} \frac{y}{\sqrt{x^2 + y^2}} = 1$$

With source at origin, the coordinates of point on sturt where it is 5 cm wide i.e. $y = 0.025$ m can be estimated as

$$\frac{m}{2} = U_0 y + \frac{m\theta}{2\pi}$$

$$\Rightarrow \quad \frac{2}{2} = 20 \times 0.025 + \frac{2\theta}{2\pi}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} = 90°$$

$$r = \frac{y}{\sin\theta} = \frac{0.025}{\sin 90} = 0.025 \text{ m}$$

$$\psi = U_0 r \sin\theta + \frac{m\nu}{2\pi}$$

The velocity components

$$V_r = \frac{1}{r}\left(\frac{\partial\psi}{\partial\theta}\right) = U_0 \cos\theta + \frac{m}{2\pi r}$$

and $$V_\theta = -\frac{\partial\psi}{\partial r} = -U_0 \sin\theta$$

At point $(0.025, \pi/2)$

$$V_r = U_0 \cos\frac{\pi}{2} + \frac{2}{2\pi \times 0.025} = 12.73 \text{ m/s}$$

$$V_\theta = -20 \sin\frac{\pi}{2} = -20 \text{ m/s}$$

Resultant velocity $\quad V = \sqrt{(12.73)^2 + (20)^2} = 23.71\ m/s$

Applying Bernolli's equation between stagnation point $\left(0.025, \frac{\pi}{2}\right)$ or strut,

$$P_0 + \frac{1}{2}\rho U_0^2 = P + \frac{1}{2}\rho V^2$$

$$\Rightarrow \quad P_0 + 0 = P + \frac{1}{2} \times 1.22 \times (23.71)^2$$

$$\Rightarrow \quad P_0 - P = \frac{1}{2} \times 1.22 \times (23.71)^2 = 342.9\ N/m^2$$

Q. 5(a) Two fixed parallel plates kept 8 cm apart have laminar flow of oil between them with a maximum velocity 1.5 m/s. Taking dynamic viscosity of oil to be 2 NS/m² find

(i) discharge per m width

(ii) shear stress at the plates

(iii) pressure difference between two points 25 m apart

(iv) velocity at 2 cm from the plate

(v) the velocity gradient at the plates end. **14**

Ans. Discharge per m width Q = Mean velocity × Area

Where, mean velocity $= \frac{2}{3} U_{max}$

$$U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x} t^2$$

$$1.5 = -\frac{1}{8 \times 2} \times \frac{\partial p}{\partial x} \times (0.08)^2$$

$$\Rightarrow \quad \frac{\partial p}{\partial x} = -3750\ N/m^2 \text{ per m}$$

$$Q = \frac{2}{3} \times 1.5 \times (t \times 1) = \frac{2}{3} \times 1.5 \times (0.08 \times 1)$$

$$= 0.08\ m^3/s \quad \textbf{Ans. (i)}$$

Shear stress

$$\tau_0 = -\frac{1}{2}\frac{\partial p}{\partial x} \times t$$

$$= -\frac{1}{2} \times -(3750) \times 0.08$$

$$= 150\ N/m^2 \quad \textbf{Ans. (ii)}$$

$$P_1 - P_2 = \frac{12\mu \bar{u} L}{t^2} = \frac{12 \times 2 \times \left(\frac{2}{3} \times 1.5\right) \times 25}{(0.08)^2}$$

$$= 93750\ N/m^2 \quad \textbf{Ans (iii)}$$

Velocity distribution is given by

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left[ty - y^2\right]$$

$$= -\frac{1}{2\times 2}\times(-3750)\left[(0.08)\times(0.02)-(0.02)^2\right]$$

$$= 1.125 \text{ m/s} \quad \textbf{Ans. (iv)}$$

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x}y + \left(-\frac{1}{2\mu}\frac{\partial p}{\partial x}t\right)$$

At $y = t$

$$\frac{\partial u}{\partial y} = \frac{1}{2\mu}\frac{\partial p}{\partial x}t$$

$$= \frac{1}{2\times 2}\times(-3750)\times(0.08)$$

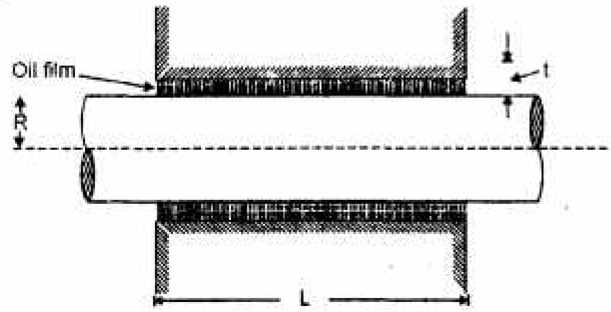
$$= -75 \text{ m/s per m} \quad \textbf{Ans. (v)}$$

Q. 5. (b) Work out a relation prescribing the power absorbed due to viscous resistance in a bush bearing. **6**

Ans. Let N = speed of shaft in r.p.m.

t = thickness of oil film

L = length of oil film



Journal bearing

$\therefore$ Angular speed of the shaft, $\omega = \frac{2\pi N}{60}$

$\therefore$ Tangential speed of the shaft $= \omega \times R$

Or $$V = \frac{2\pi N}{60}\times\frac{D}{2} = \frac{\pi DN}{60}$$

The shear stress in the oil is given by,

$$\tau = \mu\frac{du}{dy}$$

$$\frac{du}{dy} = \frac{V-0}{t} = \frac{V}{t} = \frac{\pi DN}{60\times t}$$

$\therefore$ $$\tau = \mu\frac{\pi DN}{60\times t}$$

$\therefore$ Shear force or viscous resistance $= \tau \times$ Area of surface of shaft

$$= \frac{\mu \pi DN}{60\,t} \times \pi DL$$

$$= \frac{\mu \pi^2 D^2 NL}{60\,t}$$

$\therefore$ Torque required to overcome the viscous resistance,

$$T = \text{Viscous Resistance} \times \frac{D}{2}$$

$$= \frac{\mu \pi^2 D^2 NL}{60\,t} \times \frac{D}{2} = \frac{\mu \pi^2 D^3 NL}{120\,t}$$

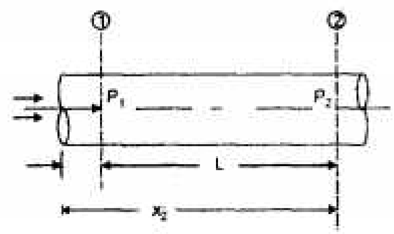
$\therefore$ Power absorbed in overcoming the viscous resistance

$$P = \frac{2\pi NT}{60}$$

$$= \frac{2\pi N}{60} \times \frac{\mu \pi^2 D^3 NL}{120\,t}$$

$$= \frac{\mu \pi^3 D^3 N^2 L}{60 \times 60 \times t} \text{ watts}$$

Q. 6. (a) Start from the first principles, derive Hagen-Poiseuille equation for fully developed flow through pipe. 12



Ans.

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\delta P}{\delta x}\right) R^2$$

or

$$= \left(\frac{-\delta P}{\delta x}\right) = \frac{8\mu \bar{u}}{R^2}$$

integrating the above equation w.r.t.x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu \bar{u}}{R^2}\,dx$$

$\therefore$

$$-[P_1 - P_2] = \frac{8\mu \bar{u}}{R^2}[x_1 - x_2]$$

or

$$(P_1 - P_2) = \frac{8\mu \bar{u}}{R^2}[x_1 - x_2]$$

or

$$= \frac{8\mu \bar{u}}{R^2} L \quad \{\therefore x_2 - x_1 = L \text{ From fig.}\}$$

$$= \frac{8\mu \bar{u} L}{(D/2)^2} \qquad \{\because R = \frac{D}{2}\}$$

Or $$(P_1 - P_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $P_1 - P_2$ is the drop of pressure.

$\therefore$ Loss of pressure head $= \dfrac{P_1 - P_2}{\rho g}$

$$\frac{P_1 - P_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

This equation is called **Hagen Poiseuille Formula.**

Q. 6. (b) At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow. **8**

Ans. Hydraulic gradient $= 0.01$ m

$$= \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1 - V_2)^2}{2g}$$

But $$V_1 = V_2 \frac{A_2}{A_1} = V_2 \left(\frac{d_2}{d_1}\right)^2$$

$$= V_2 \left(\frac{0.48}{0.24}\right)^2 = 4\,V_2$$

$\Rightarrow$ $$0.01 = \frac{4\,V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{(4\,V_2 - V_2)^2}{2g}$$

$\Rightarrow$ $$0.01 = \frac{6V_2^2}{2g}$$

$\Rightarrow$ $$V_2 = 0.1808 \text{ m/s}$$

& discharge $$Q = A_2 V_2$$

$$= \frac{\pi}{4}(0.48)^2 \times 0.1808$$

$\Rightarrow$ $$Q = 0.0327 \text{ m}^3/\text{s}$$ **Ans.**

Q. 7. (a) Find the displacement thickness, the momentum thickness and energy thickness for the velocity distribution in the boundary layer given by

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

Where u is the velocity at a distance y from the plate and δ is boundary layer thickness. **10**

Ans. Velocity distribution $$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$$

(i) Displacement thickness δ^* is given by equation

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U}\right) dy$$

Substituting the value of $\dfrac{u}{U} = 2\left(\dfrac{y}{\delta}\right) - \left(\dfrac{y}{\delta}\right)^2$ we have

$$\delta^* = \int_0^\delta \left\{1 - \left[2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2\right]\right\} dy$$

$$= \int_0^\delta \left\{1 - 2\left(\frac{y}{\delta}\right) + \left(\frac{y}{\delta}\right)^2\right\} dy = \left[y - \frac{2y}{2\delta} + \frac{y^3}{3\delta^2}\right]_0^\delta$$

$$= \delta - \frac{\delta^2}{\delta} + \frac{\delta^3}{3\delta^2} = \delta - \delta + \frac{\delta}{3} = \frac{\delta}{3} \text{ Ans.}$$

(ii) Momentum thickness θ, is

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left[1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right]\left[1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{4y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^2}{\delta^2} + \frac{2y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy$$

$$= \int_0^\delta \left[\frac{2y}{\delta} - \frac{5y^2}{\delta^2} + \frac{4y^3}{\delta^3} - \frac{y^4}{\delta^4}\right] dy = \left[\frac{2y^2}{2\delta} - \frac{5y^3}{3\delta^2} + \frac{4y^4}{4\delta^3} - \frac{y^5}{5\delta^4}\right]_0^\delta$$

$$= \left[\frac{\delta^2}{\delta} - \frac{5\delta^3}{3\delta^2} + \frac{\delta^4}{\delta^3} - \frac{y^5}{5\delta^4}\right] = \delta - \frac{5\delta}{3} + \delta - \frac{\delta}{5}$$

$$= \frac{15\delta - 25\delta + 15\delta - 3\delta}{15} = \frac{30\delta - 28\delta}{15} = \frac{2\delta}{15} \quad \textbf{Ans.}$$

(iii) Energy thickness δ^{**}

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left[1 - \frac{u^2}{U^2}\right] dy = \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{2y}{\delta} - \frac{y^2}{\delta^3}\right]\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \left[\frac{4y^2}{\delta^2} + \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right]\right) dy$$

$$= \int_0^\delta \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{4y^2}{\delta^2} - \frac{y^4}{\delta^4} - \frac{4y^3}{\delta^3}\right) dy$$

$$= \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{8y^2}{\delta^3} - \frac{2y^5}{\delta^5} + \frac{8y^4}{\delta^4} - \frac{y^2}{\delta^2} + \frac{4y^4}{\delta^4} + \frac{y^6}{\delta^6} - \frac{4y^5}{\delta^5} \right) dy$$

$$= \int_0^{\delta} \left[\frac{2y}{\delta} - \frac{y^2}{\delta^2} - \frac{8y^3}{\delta^3} + \frac{12y^4}{\delta^4} - \frac{6y^5}{\delta^5} + \frac{y^6}{\delta^6} \right] dy$$

$$= \left[\frac{2y^2}{2\delta} - \frac{y^3}{3\delta^2} - \frac{8y^4}{4\delta^3} + \frac{12y^5}{5\delta^4} - \frac{6y^6}{6\delta^5} + \frac{y^7}{7\delta^6} \right]_0^{\delta}$$

$$= \frac{\delta^2}{\delta} - \frac{\delta^3}{3\delta^2} - \frac{2\delta^4}{\delta^3} + \frac{12\delta^5}{5\delta^4} - \frac{\delta^6}{\delta^5} + \frac{\delta^7}{7\delta^6}$$

$$= \delta - \frac{\delta}{3} - 2\delta + \frac{12}{5}\delta - \delta + \frac{\delta}{7}$$

$$= -2\delta - \frac{\delta}{3} + \frac{12}{5}\delta + \frac{\delta}{7}$$

$$= \frac{-210\delta - 35\delta + 252\delta + 15\delta}{105}$$

$$= \frac{-245\delta + 267\delta}{105}$$

$$= \frac{22\delta}{105} \textbf{ Ans.}$$

Q. 7. (b) For the velocity profile given in Q.7. (a), find the thickness of boundary layer at the end of the plate and the drag force on one side of a plate 1 m along and 0.8 m wide when placed in water flowing with a velocity of 150 mm/s. Calculate the value of coefficient of drag. Take dynamic viscosity of water 0.01 poise. **10**

Ans. Lenght of plate, L = 1m

Width of Plate, b = 0.8 m

Velocity of fluid (water), U = 150 mm/s = 0.15 m/s

μ for water $= 0.01 \text{ poise} = \frac{0.01}{10} \frac{N-S}{m^2} = 0.001 \frac{N-S}{m^2}$

Reynold number at the end of the plate i.e., at a distance of 1 m from leading edge is given by

$$R_{e1.} = \frac{PUL}{\mu} = 1000 \times \frac{0.15 \times 1.0}{0.001} \quad (\because P = 1000)$$

$$= \frac{1000 \times .15 \times 1.0}{.001} = 150000$$

As laminar boundary layer exists upto Reynold number $= 2 \times 10^5$. Hence this is the case of laminar boundary layer. Thickness of boundary layer at x = 1.0 m is given by

$$\delta = 5.48 \frac{x}{\sqrt{Re_x}} = \frac{5.48 \times 1.0}{\sqrt{150000}} = 0.01415 \text{ m} = 14.15 \text{ mm} \textbf{ Ans.}$$

(ii) Drag force on one side of the plate is given by

$$F_D = 0.73\, b\mu U \sqrt{\frac{PUL}{\mu}}$$

$$= 0.73 \times 0.8 \times 0.001 \times 0.15 \times \sqrt{150000} \qquad \left\{\because \frac{PUL}{\mu} = R_{el.}\right\}$$

$$= 0.0338 \text{ N} \quad \textbf{Ans.}$$

(iii) Co-efficient of drag, C_D is given by

$$C_D = \frac{1.46}{\sqrt{R_{eL}}} = \frac{1.46}{\sqrt{150000}}$$

$$= .00376 \quad \textbf{Ans.}$$

Q. 8. A smooth pipe line of 100 mm diameter carries 2.27 m^3/min of water at 20° C with kinematic viscosity of 0.0098 stokes. Calculate the friction factor, maximum velocity as well as shear stress at the boundary. **20**

Ans. Mean velocity $V = \frac{\varphi}{A} \qquad = \frac{2.27}{60 \times \frac{\pi}{4} \times (0.1)^2}$

$$= 4.817 \text{ m/s}$$

$$R_e = \frac{Vd}{v} = \frac{4.817 \times 0.1}{0.0098 \times 10^{-4}} = 491539$$

Hence flow is turbulant

$$\therefore \qquad 4F = \frac{0.3164}{(491539)^{0.25}} = 0.0119$$

Friction factor $\qquad F = 2.98 \times 10^{-3}$ **Ans.**

Shear velocity $\qquad u^* = V\sqrt{\frac{4F}{8}} = 0.186 \text{ m/s}$

$$\frac{4}{u*} = 5.5 + 5.75 \log_{10} \frac{u \times y}{v}$$

$$t\ y = R = 0.1 \text{ m}$$

$$u = u^*\left[5.5 + 5.75 \log_{10}\left(\frac{0.186 \times 0.1}{0.0098 \times 10^{-4}}\right)\right]$$

$$= 5.6 \text{ m/s} \quad \textbf{Ans.}$$

The wall shearing stress is given by

$$T_0 = u^{*2} \times \rho = (0.186)^2 \times 1000$$

$$= 34.596 \text{ N/m}^2 \quad \textbf{Ans.}$$