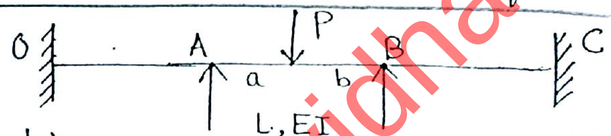
Slope Deflection Method

→ Given by G.A. Maney.
→ Based on stiffness concept
→ Suitable when $D_K < D_S$
→ Basic unknowns are taken joint displacements (θ &
→ To find joint displacements, joint equilibrium & shear equations are written.
→ To find moment in members - force displacement relations are written called slope deflection equations
→ Axial deformations are neglected but shear displacements are considered.

Derivation of slope-deflection equation-



Sign convention-

1. To compute end moments.

2. Slope

3. Deflection

Those displacement taken '+ve' which will produce clockwise rotation to the member and hence '+ve' displacement will produce '-ve' moments.

step 1→ Consider span AB → The effect of loading present

$$\overline{M}_{AB} = -\frac{Pab^2}{L^2}$$ (fixed end moment at A in span AB)

$$\overline{M}_{BA} = \frac{Pa^2b}{L^2}$$ (fixed end moment at B in span A

step 2→ If joint A rotates by angle θ_A, then F.E.M. developed at A in AB

$$= \frac{4EI}{L}\theta_A$$

and carry over moment at B due to θ_A

$$= \frac{2EI}{L} \cdot \theta_A$$

step 3 → Effect of θ_B

If joint B rotates by θ_B, then moment developed at B $= \frac{4EI}{L} \cdot \theta_B$

and carry over moment at A $= \frac{2EI}{L} \cdot \theta_B$

Step 4 → If support B settles down with respect to support A by Δ which causes rotation of AB in clockwise direction, then F.E.M. produce at A and B will be, $-\frac{6EI}{L^2}\Delta$ at A & $-\frac{6EI}{L^2}\Delta$ at B.

The final moment at end A and B due to above multiple effect will be,

$$M_{AB} = \bar{M}_{AB} + \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI}{L^2}\Delta$$

$$\boxed{M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)}$$

Slope deflection equation at A for AB.

OR

force displacement Relation

slope deflection equation at B,

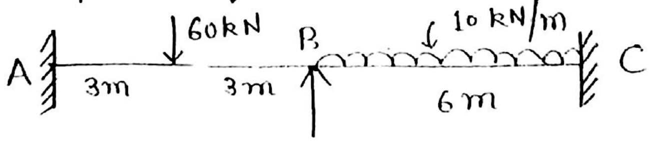
$$\boxed{M_{BA} = \bar{M}_{BA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)}$$

Step 5 → To find joint displacements (θ & Δ), moment equilibrium conditions at joints and shear equations are written.

No. of moment equi. conditions = No. of rotational dis^ment

No. of shear eqns = No. of linear disp. component.

Example :- Analyse the beam shown in figure using slope deflection method.



(EI = constt.)

$$\bar{M}_{AB} = -\frac{PL}{8} = -45$$

$$\bar{M}_{BA} = +45$$

$$\bar{M}_{BC} = -\frac{\omega l^2}{12} = -30$$

$$\bar{M}_{CB} = +30$$

$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\cancel{\theta_A}^{\,0} + \theta_B - \frac{3\cancel{\Delta}^{\,0}}{L}\right)$$

$$M_{AB} = -45 + \frac{EI}{3}\theta_B \quad \text{------ (1)}$$

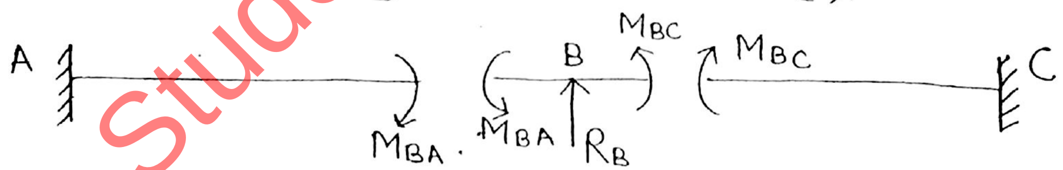
$$M_{BA} = +45 + \frac{2EI}{L}\left(2\theta_B + \cancel{\theta_A}^{\,0} - \frac{3\cancel{\Delta}^{\,0}}{L}\right)$$

$$M_{BA} = +45 + \frac{EI}{3}(2\theta_B) \quad \text{------ (2)}$$

$$M_{BC} = -30 + \frac{2EI}{6}\left(2\theta_B + \cancel{\theta_C}^{\,0} - \frac{3\cancel{\Delta}^{\,0}}{L}\right)$$

$$M_{BC} = -30 + \frac{EI}{3}(2\theta_B) \quad \text{------ (3)}$$

$$M_{CB} = +30 + \frac{EI}{3}\theta_B \quad \text{------ (4)}$$



Joint equilibrium equation at B,

$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$45 + \frac{EI}{3}(2\theta_B) + (-30) + \frac{EI}{3}(2\theta_B) = 0$$

$$\frac{EI}{3} \times 4\theta_B = -15$$

$$\boxed{\theta_B = -\frac{11.25}{EI}}$$

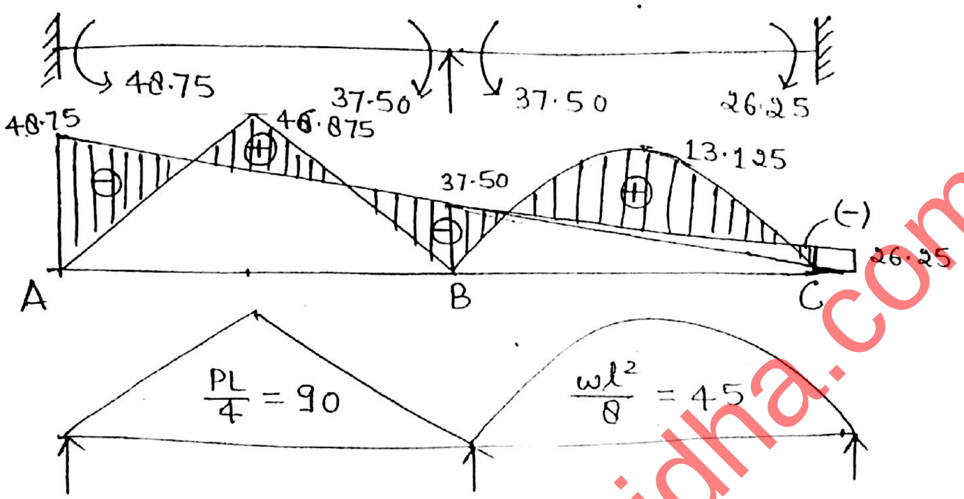
From equation (1), (2), (3) and (4).

$$M_{AB} = -45 - \frac{11.25}{3} = -48.75$$

$$M_{BA} = 45 - \frac{2}{3} \times 11.25 = +37.50$$

$$M_{BC} = -37.50$$

$$M_{CB} = 30 - \frac{11.25}{3} = +26.25$$



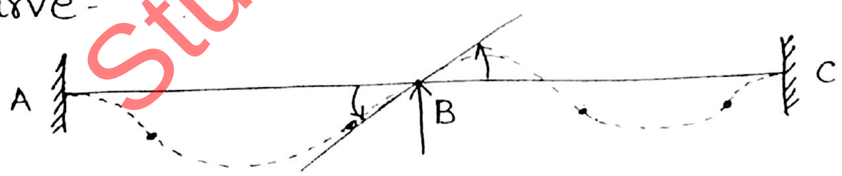
Net BM at centre of AB $= 90 - \left(\frac{48.75 + 37.50}{2}\right)$

$= \underline{\underline{46.875}}$ kN-m

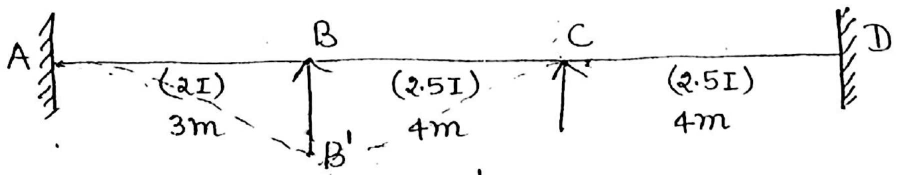
Net BM at centre of BC $= 45 - \left(\frac{37.50 + 26.25}{2}\right)$

$= \underline{\underline{13.125}}$ kN-m

Elastic Curve -



Question - Analyse the continuous beam shown in figure by slope deflection method if support B settles down by 5 mm. Given that $E = 2 \times 10^5$ N/mm² and $I = 36 \times 10^6$ mm⁴



$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 0 + \frac{2E(2I)}{L}\left(0 + \theta_B - 3\times\frac{5\times10^{-3}}{3}\right)$$

$$M_{AB} = \frac{4EI}{3}\left(\theta_B - 5\times10^{-3}\right) \quad \text{—— (1)}$$

$$M_{BA} = \frac{4EI}{3}\left(2\theta_B - 5\times10^{-3}\right) \quad \text{—— (2)}$$

$$M_{BC} = 0 + \frac{2E}{4}(2.5I)\left(2\theta_B + \theta_C + \frac{3\times5\times10^{-3}}{4}\right)$$

$$M_{BC} = \frac{5}{4}EI\left(2\theta_B + \theta_C + \frac{15}{4}\times10^{-3}\right) \quad \text{—— (3)}$$

$$M_{CB} = \frac{5}{4}EI\left(2\theta_C + \theta_B + \frac{15}{4}\times10^{-3}\right) \quad \text{—— (4)}$$

$$M_{CD} = \frac{2E(2.5I)}{4}\left(2\theta_C + \cancel{\theta_D}^{0} - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$M_{CD} = \frac{5}{4}EI\left(2\theta_C\right) \quad \text{—— (5)}$$

$$M_{DC} = \frac{5}{4}EI\left(\theta_C\right) \quad \text{—— (6)}$$

Joint equations,

$\Sigma M_B = 0,$

$$M_{BA} + M_{BC} = 0$$

$$\frac{4EI}{3}\left(2\theta_B - 5\times10^{-3}\right) + \frac{5}{4}\cancel{EI}\left(2\theta_B + \theta_C + \frac{15}{4}\times10^{-3}\right) = 0$$

$$16\left(2\theta_B - 5\times10^{-3}\right) + 15\left(2\theta_B + \theta_C + \frac{15}{4}\times10^{-3}\right) = 0$$

$$62\theta_B + 15\theta_C - 23.75\times10^{-3} = 0 \quad \text{—— (A)}$$

$\Sigma M_C = 0,$

$$M_{CB} + M_{CD} = 0$$

$$\cancel{\frac{5}{4}EI}\left(2\theta_C + \theta_B + \frac{15}{4}\times10^{-3}\right) + \cancel{\frac{5}{4}EI}\left(2\theta_C\right) = 0$$

$$4\theta_C + \theta_B + 3.75\times10^{-3} = 0 \quad \text{—— (B)}$$

eqn (A) × 4 − eqn (B) × 15,

$$(62\times4 - 15)\theta_B - (23.75\times4 + 3.75\times15)\times10^{-3} = 0$$

$$\boxed{\theta_B = 6.49\times10^{-4}}$$

from eqns (B),

$$4\theta_C = -4.399\times10^{-3}$$

$$\boxed{\theta_C = -1.10\times10^{-3}}$$

Final moments,

$M_{AB} = -41.77$

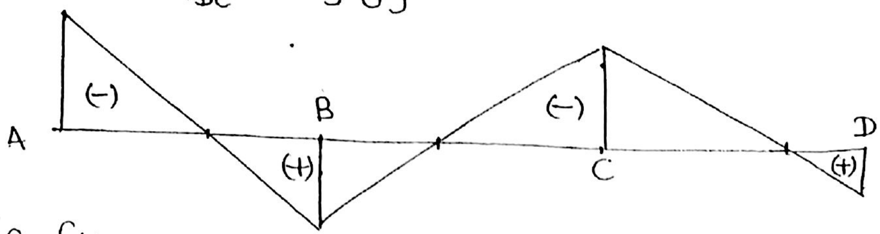
$M_{BA} = -35.53$

$M_{BC} = +35.53$

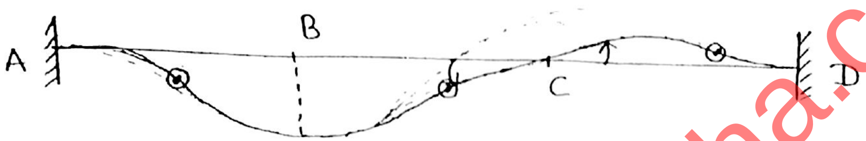
$M_{CD} = -19.78$

$M_{DC} = -9.89$

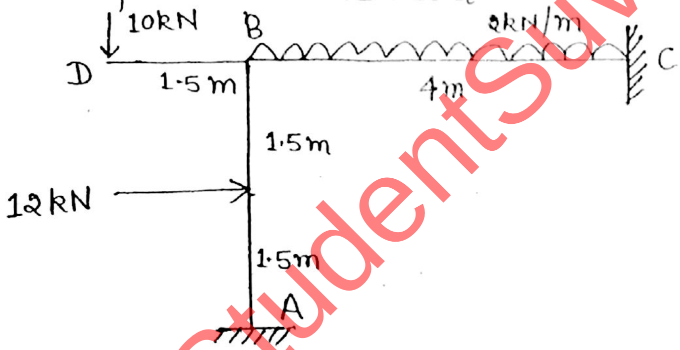
BMD



Elastic Curve,



Question- Analyse the frame shown in figure by slope deflection method.



EI = constt.

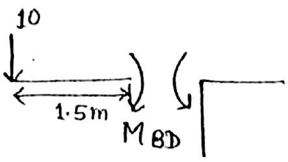
Fixed end Moments,

$\bar{M}_{AB} = -\dfrac{12 \times 3}{8} = -4.5$

$\bar{M}_{BA} = +4.5$

$\bar{M}_{BC} = -32$

$\bar{M}_{CB} = +32$



$\bar{M}_{BD} = \underline{15 \text{ kN-m}}$

$\therefore M_{AB} = -4.5 + \dfrac{2EI}{3}\left(2\cancel{\theta_A} + \theta_B - \dfrac{3\cancel{\Delta}}{\cancel{L}}\right)$

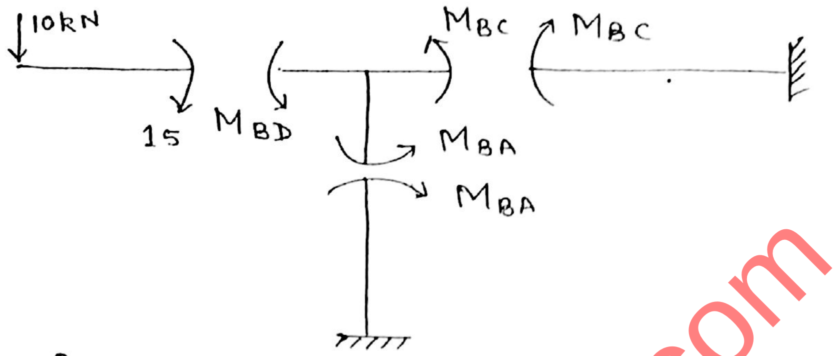
$M_{AB} = -4.5 + \dfrac{2EI}{3} \cdot \theta_B$ ——— (1)

$$M_{BA} = +4.5 + \frac{2EI}{3}(2\theta_B) \quad \text{——— (2)}$$

$$M_{BC} = -32 + \frac{2EI}{4}\left(2\theta_B + \cancel{\theta_C} - \cancel{\frac{3\Delta}{L}}\right)$$

$$M_{BC} = -32 + \frac{2EI}{4}(2\theta_B) \quad \text{——— (3)}$$

$$M_{CB} = +32 + \frac{EI}{2}\theta_B \quad \text{——— (4)}$$



$\Sigma M_B = 0,$

$$-15 - M_{BA} - M_{BC} = 0$$

$$+15 + 4.5 + \frac{2EI}{3} \times 2\theta_B + (-32) + EI\theta_B = 0$$

$$\left(\frac{4}{3}\theta_B + \theta_B\right)EI = 12.5$$

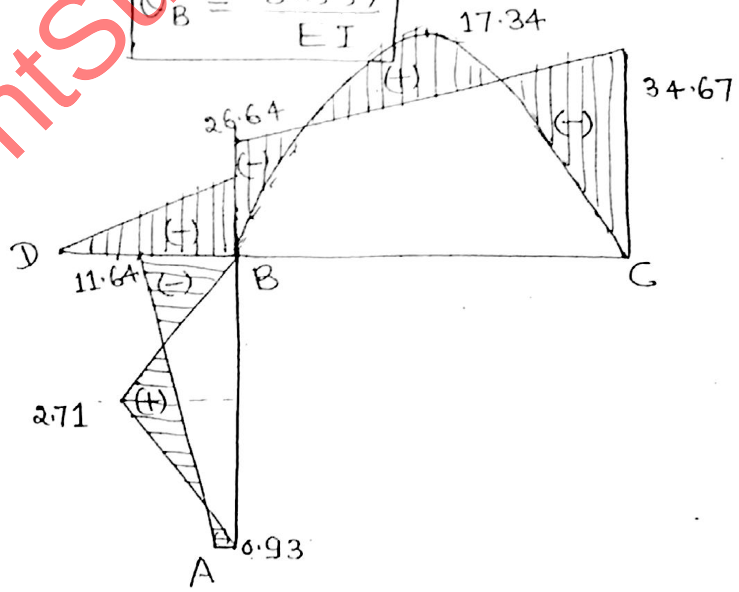
$$\boxed{\theta_B = \frac{5.357}{EI}}$$

$M_{AB} = -0.93$

$M_{BA} = +11.64$

$M_{BC} = -26.64$

$M_{CB} = +34.67$

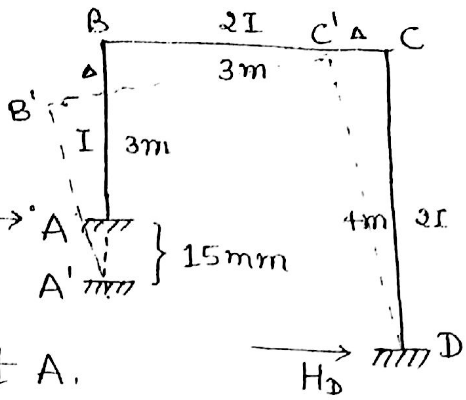


Question For the frame shown in figure, support A settles by 15 mm vertically down. Analyse the frame by slope deflection method and draw BMD. $EI = 48000$ kN-m². Draw deflected shape of the frame.

$\Delta = 15$ mm $\downarrow$

Horizontal component of displacement at B and C are equal $= \Delta$

vertical component of disp. at B = Vertical displacement at A.



$$M_{AB} = \bar{M}_{AB} + \frac{2EI}{3}\left(2\theta_A + \theta_B + \frac{3\Delta}{3}\right)$$

$$M_{AB} = \frac{2EI}{3}(\theta_B + \Delta) \quad \text{——— (1)}$$

$$M_{BA} = \frac{2EI}{3}(2\theta_B + \Delta) \quad \text{——— (2)}$$

$$M_{BC} = 0 + \frac{2E}{3}(2I)\left(2\theta_B + \theta_C + \frac{3 \times 15 \times 10^{-3}}{3}\right)$$

$$M_{BC} = \frac{4EI}{3}(2\theta_B + \theta_C + 15 \times 10^{-3}) \quad \text{——— (3)}$$

$$M_{CB} = \frac{4EI}{3}(2\theta_C + \theta_B + 15 \times 10^{-3}) \quad \text{——— (4)}$$

$$M_{CD} = 0 + \frac{2E}{4}(2I)\left(2\theta_C + \cancel{\theta_D}^{0} + \frac{3}{4}\Delta\right)$$

$$M_{CD} = EI\left(2\theta_C + \frac{3}{4}\Delta\right) \quad \text{——— (5)}$$

$$M_{DC} = EI\left(\theta_C + \frac{3}{4}\Delta\right) \quad \text{——— (6)}$$

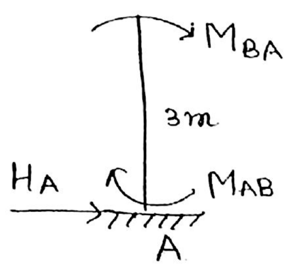
$\Sigma M_B = 0$,

$$M_{BA} + M_{BC} = 0 \quad \text{——— (A)}$$

$\Sigma M_C = 0$,

$$M_{CB} + M_{CD} = 0 \quad \text{——— (B)}$$

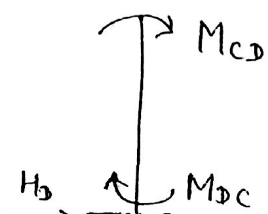
Shear equation,



$\Sigma F_x = 0$, $\quad H_A + H_D = 0$

$$-H_A \times 3 + M_{AB} + M_{BA} = 0$$

$$H_A = \frac{M_{AB} + M_{BA}}{3} \quad \text{——— (C)}$$



$$-H_D \times 4 + M_{CD} + M_{DC} = 0$$

$$H_D = \left(\frac{M_{DC} + M_{CD}}{4}\right) \quad \text{——— (D)}$$

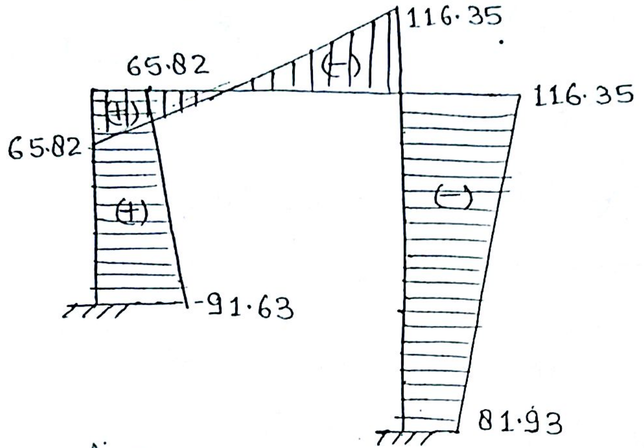
$$\therefore \left(\frac{M_{AB} + M_{BA}}{3}\right) + \left(\frac{M_{DC} + M_{CD}}{4}\right) = 0 \quad \text{——— (E)}$$

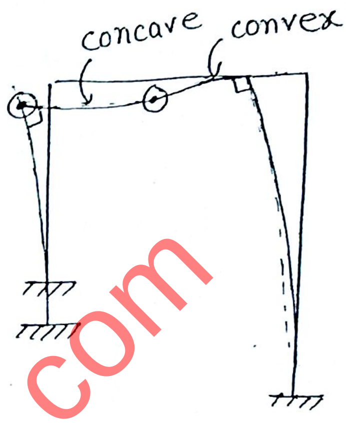
From joint equations and shear equations θ_B, θ_C, Δ are obtained and final values of moment are found as,

$M_{AB} = +91.63$ $\qquad$ $M_{CB} = +116.35$

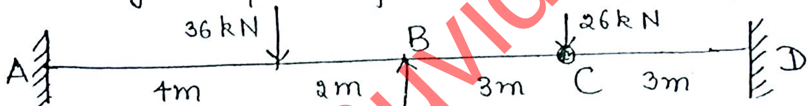
$M_{BA} = -65.82$ $\qquad$ $M_{CD} = -116.35$

$M_{BC} = +65.82$ $\qquad$ $M_{DC} = +81.93$





Question For the Beam shown in figure, Draw BMD using slope deflection method.



θ_B, θ_{C_1} (left of C), θ_{C_2} (right of C) (Just to right of C),

Δ_C = unknown.

$D_K = 3j - r_e + r_r$ - Axial deformation

$D_K = 3 \times 4 - 7 + 1 - 2$ (at B, at C)

$D_K = 4$

$\bar{M}_{AB} = -16$, $\bar{M}_{BA} = 32$

$\bar{M}_{BC} = \bar{M}_{CB} = 0$

$\bar{M}_{CD} = \bar{M}_{DC} = 0$

$$M_{AB} = -16 + \frac{2EI}{6}\left(2\cancel{\theta_A}^{\,0} + \theta_B - \frac{3\cancel{\Delta}^{\,0}}{6}\right)$$

$$M_{AB} = -16 + \frac{EI}{3}\theta_B \quad \text{———— (1)}$$

$$M_{BA} = +32 + \frac{EI}{3}(2\theta_B) \quad \text{———— (2)}$$

$$M_{BC} = 0 + \frac{2EI}{3}\left(2\theta_B + \theta_{C_1} - \frac{3\Delta_c}{3}\right)$$

$$M_{BC} = \frac{2EI}{3}\left(2\theta_B + \theta_{C_1} - \Delta_c\right) \quad \text{——— (3)}$$

$$M_{CB} = \frac{2EI}{3}\left(\theta_B + 2\theta_{C_1} - \Delta_c\right) \quad \text{——— (4)}$$

$$M_{CD} = 0 + \frac{2EI}{3}\left(2\theta_{C_2} + \cancel{\theta_D}^{\,0} + \frac{3\Delta_c}{3}\right) \quad \text{——— (5)}$$

$$M_{DC} = \frac{2EI}{3}\left(\theta_{C_2} + \Delta_c\right) \quad \text{——— (6)}$$

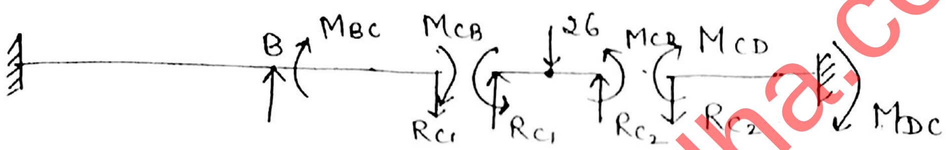
Joint conditions,

$\Sigma M_B = 0$,

$$M_{BA} + M_{BC} = 0 \quad \text{——— (A)}$$

$$M_{CB} = 0 \quad \text{——— (B)}$$

$$M_{CD} = 0 \quad \text{——— (C)}$$



Shear equation at C,

$\Sigma F_y = 0$,

$$R_{C_1} + R_{C_2} - 26 = 0$$

To find R_{C_1} pick moment just to right of B.

$$R_{C_1} \times 3 + M_{BC} + \cancel{M_{CB}}^{\,0} = 0$$

$$R_{C_1} = -\frac{M_{BC}}{3}$$

To find R_{C_2} take moment about D,

$\cancel{M_{CD}}^{\,0} + M_{DC} - R_{C_2} \times 3 = 0$

$$M_{DC} - R_{C_2} \times 3 = 0$$

$$R_{C_2} = \frac{M_{DC}}{3}$$

Shear equation,

$$-\frac{M_{BC}}{3} + \frac{M_{DC}}{3} = 26$$

$$M_{DC} - M_{BC} = 78 \quad \text{——— (D)}$$

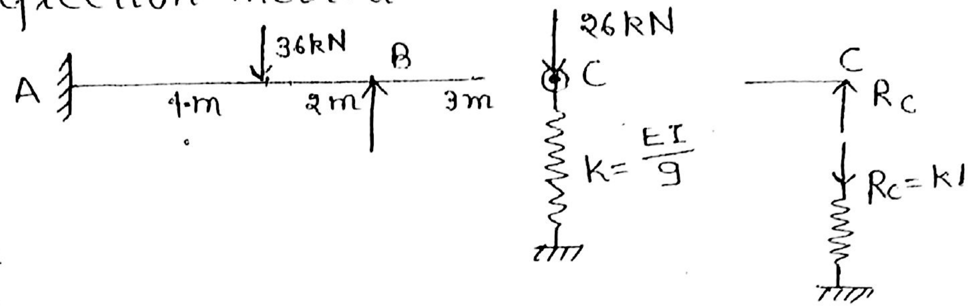
solving eqns (A), (B), (C) and (D), final moments;

$M_{AB} = -14$ kN-m $\quad M_{CB} = 0$

$M_{BA} = +36$ kN-m $\quad M_{CD} = 0$

$M_{BC} = -36$ kN-m $\quad M_{DC} = +42$ kN-m

Question - Analyse the beam section shown in figure by slope deflection method.



$\overline{M}_{AB} = -16$

$\overline{M}_{BA} = +32$

$\overline{M}_{BC} = \overline{M}_{CB} = 0$

$$M_{AB} = -16 + \frac{2EI}{6}\left(2\cancel{\theta_A}^{\,0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{\,0}\right)$$

$$M_{AB} = -16 + \frac{EI}{3}\theta_B \quad \text{———— (1)}$$

$$M_{BA} = +32 + \frac{EI}{3}\cdot 2\theta_B \quad \text{———— (2)}$$

$$M_{BC} = 0 + \frac{2EI}{3}\left[2\theta_B + \theta_C - \frac{3\Delta_C}{3}\right]$$

$$M_{BC} = \frac{2EI}{3}(2\theta_B + \theta_C - \Delta_C) \quad \text{———— (3)}$$

$$M_{CB} = \frac{2EI}{3}(2\theta_C + \theta_B - \Delta_C) \quad \text{———— (4)}$$

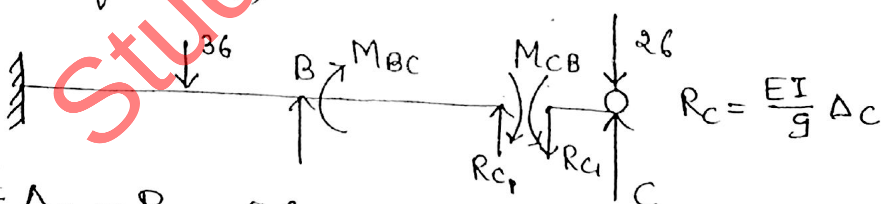
Joint equations at B,

$$M_{BA} + M_{BC} = 0 \quad \text{———— (A)}$$

Joint equations at C,

$$M_{CB} = 0 \quad \text{———— (B)}$$

Shear equation,



$$\frac{EI}{9}\Delta_C - R_{C_1} - 26 = 0$$

R_{C_1} can be found by taking moment just to the right of B,

$$-R_{C_1}\times 3 + M_{BC} = 0 \qquad (\because M_{CB} = 0)$$

Hence, shear equation is,

$$\frac{EI}{9}\Delta_C - \frac{M_{BC}}{3} - 26 = 0 \quad \text{———— (C)}$$

From equations (A), (B) and (C),
θ_A, θ_B and Δ_C are known and final moments are

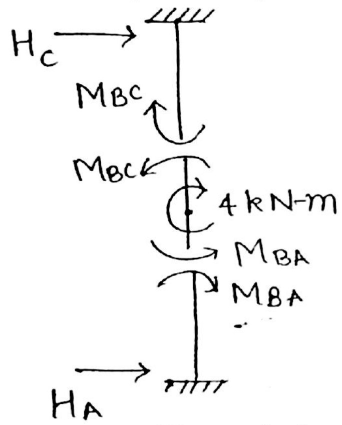
$M_{AB} = -14$ kN-m

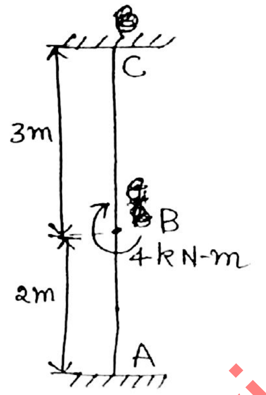
$M_{BA} = +36$ kN-m

$M_{BC} = -36$ kN-m

$M_{CB} = 0$

Question Analyse the frame shown in figure using slope deflection method.





For joint B,

$$\Sigma M = 0$$

$$-M_{BA} - M_{BC} + 4 = 0$$

$$M_{BA} + M_{BC} = 4 \quad \text{——— (1)}$$

Shear equation,

$$H_A + H_C = 0$$