Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
B. E. - SEMESTER – III • EXAMINATION – WINTER 2012

Subject code: 130001   **Date: 09-01-2013**
Subject Name: Mathematics - III
Time: 10.30 am – 01.30 pm   **Total Marks: 70**
Instructions:
   1. **Attempt all questions.**
   2. **Make suitable assumptions wherever necessary.**
   3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find a Fourier Series for f$(x) = x^2$, where $0 \le x \le 2\pi$ **05**

   (b) *If* $f(x) = \pi + x$, $-\pi < x < 0$ **05**

   $= \pi - x$, $0 < x < \pi$

   and $f(x) = f(x + 2\pi)$, *for all* x then expand f(x) in a Fourier Series.

   (c) Find a half range sine series for $f(x) = \pi - x, 0 < x < \pi$ **04**

OR

   (c) Find a Fourier Series for a periodic function f(x) with a period 2,where   **04**
   $f(x) = -1$, $-1 < x < 0$

   $= 1$, $0 < x < 1$

Q.2 **(a)** Find the inverse Laplace transforms of   **05**

   $(i) \dfrac{5s^2 + 3s - 16}{(s-1)(s+3)(s-2)}$ $(ii) \dfrac{s^3}{s^4 - 81}$

   (b) State the Convolution theorem on Laplace transforms and using it find   **05**
   $L^{-1}\left[\dfrac{1}{s(s^2 + 4)}\right]$

   (c) Find the Laplace transforms of (i) $\cos^2 2t$ and (ii) $t^3 \cosh 2t$ **04**

OR

   (c) Find the Laplace transforms of the half wave rectifier   **04**

   $f(t) = \sin \omega t$, $0 < t < \dfrac{\pi}{\omega}$ and $f(t) = f\left(t + \dfrac{2\pi}{\omega}\right)$

   $= 0$, $\dfrac{\pi}{\omega} < t < \dfrac{2\pi}{\omega}$

Q.3 **(a)** Define Beta function.   **05**

   Prove that (i) $\left|\dfrac{1}{2}\right. = \sqrt{\pi}$ (ii) B(m , n) = B(m , n + 1) + B(m + 1 , n)

   (b) State the Duplication formula.   **05**

   Show that $\displaystyle\int_0^\infty \sqrt{x}\, e^{-x^2}\, dx \times \int_0^\infty \dfrac{e^{-x^2}}{\sqrt{x}}\, dx = \dfrac{\pi}{2\sqrt{2}}$

   (c) Show that (i) $\displaystyle\int_{-a}^{a}(a+x)^{m-1}(a-x)^{n-1}\, dx = (2a)^{m+n-1}B(m,n)$ **04**

   (ii) $\displaystyle\int_0^1 \dfrac{x}{\sqrt{1-x^5}}\, dx = \dfrac{1}{5}B\left(\dfrac{2}{5}, \dfrac{1}{2}\right)$

Q.3 **(a)** State the necessary and sufficient conditions to be exact differential **05** equation. Using it, solve $x^2 y\, dx - (x^3 + y^3) dy = 0$

(b) Using the method of variation of parameters, solve $\dfrac{d^3 y}{dx^3} + \dfrac{dy}{dx} = \cos ec\ x$ **05**

(c) Solve (i) $\dfrac{dy}{dx} = \cos x \cos y - \sin x \sin y$ **04**

(ii) $\dfrac{dy}{dx} + y \tan x = \sin 2x$

Q.4 **(a)** Using the method of undetermined coefficients, solve **05**

$\dfrac{d^2 y}{dx^2} + \dfrac{dy}{dx} - 6y = 6x + 3x^2 - 6x^3$

(b) Solve $x^3 \dfrac{d^3 y}{dx^3} + 2x^2 \dfrac{d^2 y}{dx^2} + 2y = 10\left(x + \dfrac{1}{x} \right)$ **05**

(c) Solve $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + y = \dfrac{e^x}{x^2}$ **04**

OR

Q.4 **(a)** (i) If one of the solutions of $x^2 y'' - 4xy' + 6y = 0\ is\ y_1 = x^2, x > 0$ then **05** determine its second solution.

(ii) Solve : $y''' - y'' + 100y' - 100y = 0$

(b) Prove that, (i) $J_{\frac{1}{2}}(x) = \sqrt{\dfrac{2}{\pi\ x}} \sin x$ (ii) $J_{\frac{3}{2}}(x) = \sqrt{\dfrac{2}{\pi\ x}}\left(\dfrac{\sin x}{x} - \cos x \right)$ **05**

(c) Show that (i) $J_{n-1}(x) - J_n'(x) = \dfrac{n}{x} J_n(x)$ (ii) $J_0(0) = 1$ **04**

Q.5 **(a)** Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 3$ in terms of Legendre's polynomials. **05**

(b) Find a power series solution of $\dfrac{d^2 y}{dx^2} + y = 0$ **05**

(c) Classify the singularities for following differential equations **04**

(i) $2x^2 \dfrac{d^2 y}{dx^2} + 6x \dfrac{dy}{dx} + (x + 3) y = 0$

(ii) $x(x+1)^2 y'' + (2x - 1) y' + x^2 y = 0$

OR

Q.5 **(a)** Solve two dimensional Laplace's equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$, using the **05** method separation of variables.

(b) A rod of length L with insulated side is initially at uniform temperature **05** 100^0C. Its ends are suddenly cooled at 0^0C and kept at that temperature. Find the temperature $u(x, t)$.

(c) Find the Fourier transform of $f(x) = xe^{-x}, x > 0$ **04**
$= 0\quad, x < 0$

****