B.Tech.

Third Semester Examination

Computer Aided Design (ME-203-F)

Note : Attempt *all* questions.

Q. 1. Explain :

(i) **Homogeneous coordinate**

(ii) **Rational B-spline**

(iii) **Virtual Manufacturing cell**

(iv) **Numerical Control**

(v) **Programmable automation controller**

(vi) **Automation**

(vii) **G-code**

(viii) **Servomechanism**

(ix) **Positioning system**

(x) **Vertical spindle Milling machine.**

Ans. (i) Homogeneous Coordinate : Problem of homogeneity of transformation is involved. We must be able to represent the basic transformations as 3×3 homogeneous coordinate matrices do as to be compatible with the transformation matrix. This can be accomplished with the following procedure :

Representation of a point (x, y) in 2D Cartesian system is (x, y, 1) in homogeneous coordinate system. Further any point (x, y, w). When $w \neq 0$ is in homogeneous coordinate corresponding to the point (x/w, y/w) in 2D Cartesian system. Likewise there can be infinite number of revivalist homogeneous representations of any given point, as shown below :

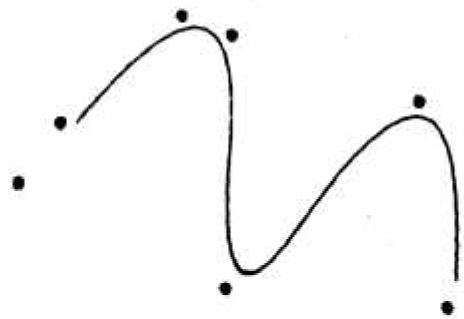
$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xw \\ yw \\ w \end{bmatrix} = w\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \text{ w is known as weight.}$$

If w = 1 then table (x, y, 1) represents homogeneous coordinate for representation of point positions (x, y) in xy-plane.

(ii) Rational B-spline : A B-spline curve is a rational curve if the control points are specified with homogeneous coordinates. Such curves are sometimes also known as non-uniform, rational B-spline (NURBS).

A rational B-spline has 4 tuples (x, y, z, w) as control points, curve q(u) is represented as weighted average of control points.

$$P(t) = \sum_{i=0}^{n} P_i B_{i,d}(t)$$



Rational B-Spline

(iii) Virtual Manufacturing Cell :

(i) In highly volatile manufacturing environments functional job shops and classical cellular manufacturing systems do not perform well.

(ii) In order to adapt cellular manufacturing systems to volatile manufacturing environments, the virtual cellular manufacturing concept was proposed in the 1980s by the National Bureau of Standards in USA.

(iii) Virtual cellular manufacturing systems are most suitable in production environments that experience frequent product mix changes.

(iv) This concept is similar to group technology where job families are processed in manufacturing cells.

(v) The virtual manufacturing cell concept allows the flexible reconfiguration of shop flows in response to changing requirements.

(iv) Numerical Control (NC) : Numerical control refers to the automation of machine tools that are operated by abstractly programmed commands encoded on a storage medium, as opposed to manually controlled via handwheels or levers, or mechanically automated via came alone. The first NC machines were built in the 1940s and 50's based on existing tools that were modified with motors that moved the controls to follow points fed into the system on punched tape. These early servomechanisms were rapidly argumented with analog and digital computers, creating the modern computer numerical controlled (CNC) machine tools that have revolutionized the manufacturing process.

(v) Programmable Automation Controller : A programmable automation controller (PAC) is a compact controller that combine the features and capabilities of a PC based control system with that of a typical programmable logic controller (PLC). PACs are most often used in industrial setting for process control, data acquisition, remote equipment monitoring, machine vision and motion control. Additionally, because they function and communicate over popular network interface protocol like TCP/IP, OLE for process control (OPC) and SMTP, PACs are able to transfer data from the machines they control to other machines and components in a networked control system or to application software and databases.

(vi) Automation : Automation is the use of control systems and information technologies to reduce the need for human work in the production of goods and services. In the scope of industrialization, automation is a step beyond mechanization. Whereas mechanization provided human operators with machinery to assist them with muscular requirements of work, automation greatly decreases the need for human sensory and mental requirements as well. Automation plays an increasingly important role in the world economy and in daily experience.

(vii) G-code : G-code is the common name for the most commonly used control (NC) programming language, which has many implementations. This general sense of the term, referring to the language overall is imprecise, because it comes metonymically from the literal sense of the term, referring to one letter address among many in the language (G address, for preparatory commands) and to the specific codes that can be formed with it.

Our standardized version of G-code, known as BCL, is used only on very few machines.

G-code began as a limited type of language that lacked constructs such as loops, conditional operators and programmer-declared variable with natural-word including names.

G-codes are also called preparatory codes, and are any word in a CNC program that begins with the letter "G". Generally it is a code telling the machine tool what type of action to perform, such as :

(i) Rapid move

(ii) Controlled feed move in a straight line or arc.

(iii) Set tool information such as offset.

There are other codes : the type codes can be thought of like registers in a computer.

(viii) Servomechanism : It will be helpful to understand the drive systems used on NC machinery. The drive motors on a particular machine will be one of four types : Stepper motors, dc servos, ac servos or hydraulic servos. Stepper motors move a set amount of rotation (a step) every time the motor receive an electrical pulses. DC and ac servos are widely used variable speed motors found on small and medium continuous-path machines. Unlike a stepper motor, a servo does not move a set distance. When current is applied, the motor starts to turn : When the current is removed, the motor stop turning. The ac servo is fairly recent development. Hydraulic servo, like ac or dc servos are variable speed motor. Because they are hydraulic motor, they can produce much more power than an electric motor. They are used on large NC machinery, usually with an electric or pneumatic control system.

(ix) Positioning System : There are two ways that machines position themselves with respect to their coordinate systems. These systems are called incremental positioning and absolute positioning. With incremental positioning each tool movement is made with reference to the prior or last tool position. Absolute positioning measures all tool movement from a fixed point, origin or zero point. Use absolute dimensioning where possible because a mistake on the dimensions at one point will not be carried over to the dimensions at other points. It is also easier to check for errors.

(x) Vertical Spindle Milling Machine : Many of the CNC vertical spindle milling machines being introduced to the Navy today. It is basically a vertical milling machine that has an onboard computer to control its motion. Most of these machines are manufactured with what is known as an R-8 spindle taper that employs a quick-change tool system.

The quick change tool system consists of a quick release check and a set of tool holders that hold the individual tool needed or a particular part program. The chuck is a separate tool holding mechanism that says in the spindle. During tool change, the tool holder is removed from the chuck and a tool holder containing the next required tool is installed in its place. Many varieties of quick-change tool systems are available on the marked.

Q. 2. Attempt any two :

(a) Define rotation and scaling in 2-D transformation. Prove that reflection along y = –x is equivalent to the reflection about y-axis followed by counter clockwise by 90°.

Ans. 2-D Rotation : A two-dimensional rotation is applied to an object by repositioning it along a circular path in the xy-plane. Points can be rotated through an angle θ about the origin. The sign of angle determine the direction of rotation. Positive values for the rotation angle defines counterclockwise rotations and negative values rotate objects in clockwise directions.

Suppose rotation by θ transforms the point P(x, y) into P' (x', y'). Because the rotation is about the origin the distances from the origin to P and to P' is r are equal.

$$x = r\cos\phi$$

$$y = r\sin\phi$$

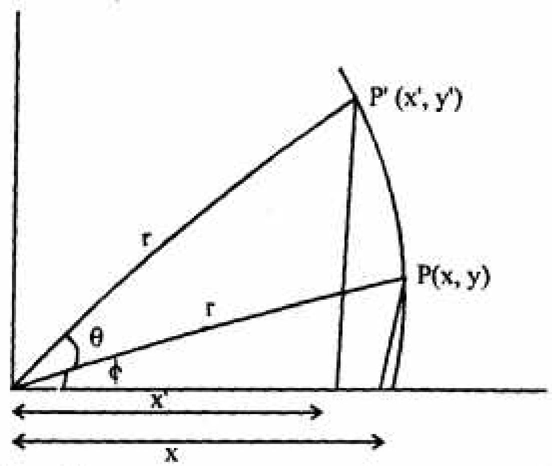
and
$$x' = r\cos(\theta+\phi) = r\cos\theta\cos\phi - r\sin\phi\sin\theta$$

$$y' = r\sin(\theta+\phi) = r\sin\theta\cos\phi + r\sin\phi\cos\theta$$

Put the values of x and y, we get

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$



In matrix form (row-major order)

$$[X'] = [X][T]$$

Or
$$[x'y'] = [x \;\; y][T]$$

Put the value of x' and y'. We get

$$[x\cos\theta - y\sin\theta \quad \sin\theta + y\cos\theta] = [x.y][T]$$

Hence,
$$[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$[T] = \cos^2\theta + \sin^2\theta = 1$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Suppose we want to rotate P' back to point P i.e., to perform the inverse transformation or rotation, the required angle $-\theta$. Then

$$[T]=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$[T^{-1}]=\begin{bmatrix}\cos(-\theta) & \sin(-\theta)\\ -\sin(-\theta) & \cos(-\theta)\end{bmatrix}$$

$$=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

Now,

$$[T][T^{-1}]=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos^2\theta+\sin^2\theta & -\cos\theta\sin\theta+\sin\theta\cos\theta\\ \cos\theta\sin\theta+\sin\theta\cos\theta & \sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=[I]$$

Thus, the transformation matrix for rotation in,

(i) Anticlockwise direction will be

$$\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

(ii) Clockwise direction will be

$$\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$$

2-D Scaling : Scaling is a transformation that changes the size or shape of an object. Scaling with respect to origin can be carried out by multiplying the co-ordinate value (x, y) of each vertex of a polygon, or each end point of a line by scaling factor S_x and S_y respectively to produce the coordinates (x', y').

The mathematical expression for pure scaling is

$$x' = S_x * x$$

$$y' = S_y * y$$

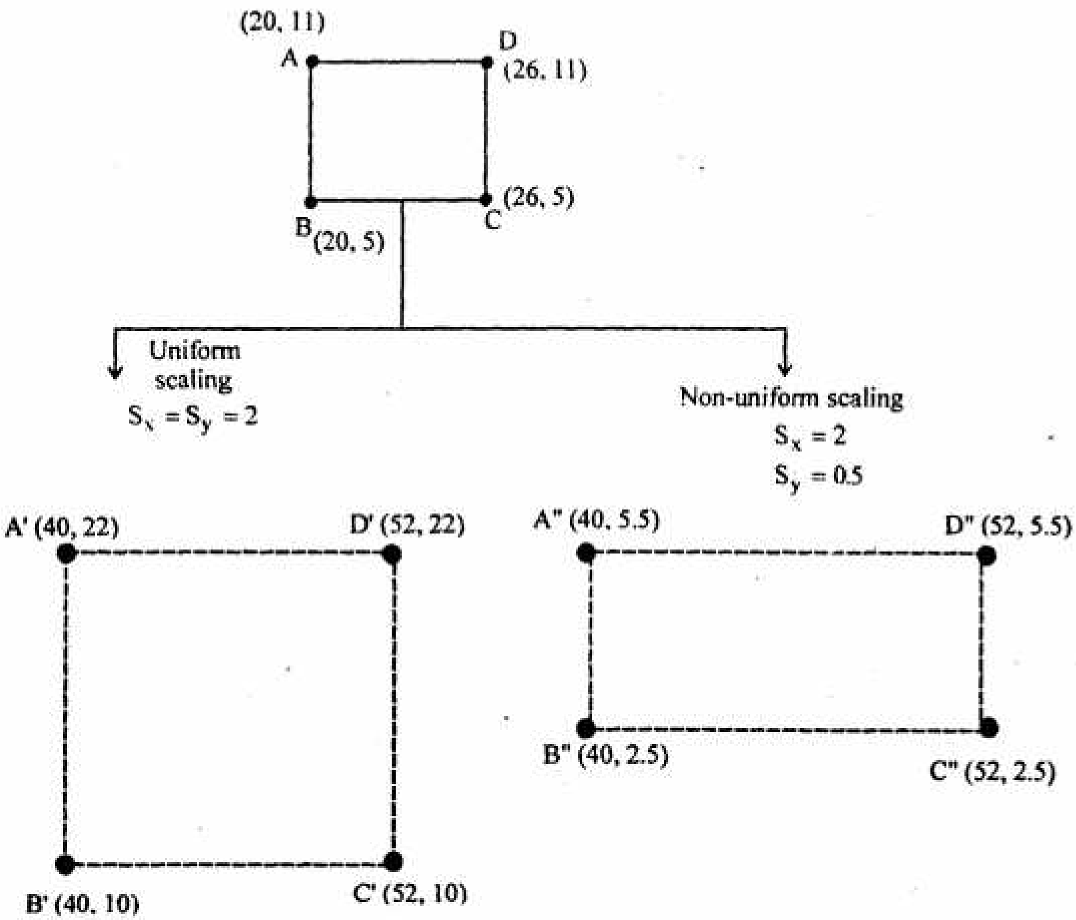
$$\begin{bmatrix}x'\\ y'\end{bmatrix}=\begin{bmatrix}S_x & 0\\ 0 & S_y\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$$

Or symbolically $\quad [X'] = [T][X]$.

After applying scaling factor or matrix an any object, coordinate is either increased or decreased depending on the value of scaling factors S_x and S_y. If it is greater than one, then enlargement of the object will occur if it is less than one compression will occur.

In general, for uniform scaling if $S_x = S_y > 1$, then a uniform compression occurs i.e., the object becomes larger. If $S_x = S_y < 1$, then a uniform compression occurs i.e., the object gets smaller. Non-uniform expansions or compressions occur, depending on whether S_x and S_y are individually > 1 or < 1 but unequal, such scaling is also known as differential scaling. While the basic object shape remains unaltered in uniform scaling, the shape and size both changes in differential scaling.

Thus, in pure uniform scaling with factors < 1 moves objects closer to the origin while factors > 1 moves object farther from origin, at the same time decreasing or increasing the object size.



Solution : Transformation matrix for reflection about line $y = -x$ is

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Transformation matrix for reflection relative to y axis is, $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and the transformation matrix for counterclockwise rotation is

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $\theta = 90°$

So, $$\begin{bmatrix} \cos 90° & \sin 90° & 0 \\ -\sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

When applied successively, we get

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Which is equal to the transformation matrix for reflection about line $y = -x$.

Q. 2. (b) Define CAD. Also give architecture and capabilities of CAD.

Ans. Computer Aided Design (CAD) is the use of a wide range of computer based tools that assist engineers, architects and other design professionals in their design activities. It is the main geometry authoring tool within the product lifecycle management process and involves both software and sometimes special-purpose hardware. Current packages range from 2D vector based drafting systems to 3D parametric surface and solid design modulers.

CAD is used to design and develop products, there can be good used by end consumers or intermediate goods used in other products. CAD is also extensively used in the design of tools and machinery used in the manufacturing of components. CAD is used throughout the engineering process from conceptual design and layout, through detailed engineering and analysis of components to definition of manufacturing process.

(i) Field of Use :

(i) Fashion design

(i) AEC (Architecture Engineering and Construction).

(i) MAC D Mechanical :

(i) Automotive

(ii) Aerospace

(iii) Consumer goods

(iv) Machinery

(v) Ship building

(i) ECAD electronic and electrical

(ii) Manufacturing process planning

Architecture : The software package may produce its result in several formats, but typically provides a graphically-based result which is then able to be used to create concept sketches for assessment and approval and eventually working drawings. An example would be a structural design package used to assess the integrity of a steel-framed building by performing all the calculations necessary to determine the size and strength of the components and the effect of such things as wind loading. The output commonly is a schedule of materials and some basic sketches which can be transferred to a computer aided drawing package for final production of construction working drawings.

Computer-aided drafting, however, commonly refers to the actual technical drawing component of the project, using a computer rather than a traditional drawing board. The input into this aspect of the design process may come from specialised calculation packages, from pre-existing component drawings, from graphical images such as maps, from photos and other media, or simply from hand drawn sketches done by the designer. The operator's task is to use the CAD software to meld all the relevant component together to produce drawings and specifications which can then be used to estimate quantities of materials, determine the cast of the project and ultimately provide the detailed drawing necessary to build it.

The spectrum of architectural and engineering projects commonly documented with computer-aided drafting is broad, and includes architectural, mechanical, electrical, structural, hydraulic, interior design, civil construction. CAD may also provide input to other forms of design communication such as 3D visualisations, model construction, animated fly through, to name a few.

Computer-aided drafting software is also a basic tool used in other disciplines related to architecture, for example Civil Engineering for site design, for instance roads, grading and drainage, in mapping and cartography, in the production plans and sketches for a variety of other purposes (such surveyor's plans and legal description of land), and as the input format to geographic and facilities information systems. Additionally, landscape architecture and interior design is often also commonly performed using CAD software.

Capabilities : The capabilities of modern CAD system include :

(i) Wireframe geometry creation.

(ii) 3D parametric feature based modelling, solid modelling.

(iii) Freedom surface modelling.

(iv) Automated design of assemblies, which are collections of parts and/or other assemblies.

(v) Create engineering drawings from the solid models.

(vi) Reuse of design components.

(vii) Ease of modification of designs of models and the production of multiple versions.

(viii) Automated generation of standard components of the design.

(ix) Validation/verification of designs against specification and design rules.

(x) Simulation of designs without building a physical prototype.

(xi) Output of engineering documentation, such as manufacturing drawings and Bills of Materials to reflect the BOM required to build the product.

(xii) Import/Export routines to exchange data with other software packages.

(xiii) Output of design data directly to manufacturing facilities.

(xiv) Output directly to a rapid prototyping or rapid manufacture machine for industrial prototypes.

(xv) Maintain libraries of parts and assemblies.

(xvi) Calculate mass properties of parts & assemblies.

(xvii) Aid visualization with shading, rotating, hidden line removal etc.

(xviii) Bi-directional parametric associatively.

(xix) Kinematics, interference and clearance checking of assemblies.

(xx) Sheet metal.

(xxi) Nose/cable routing.

(xxii) Electrical component packaging.

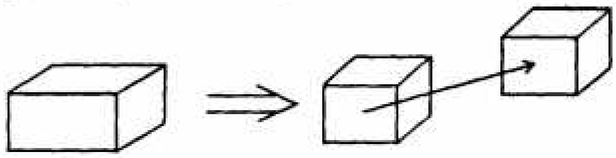
(xxiii) Inclusion of programming code in a model to control and relate desired attributes of the model.

(xxiv) Programmable design studies and optimization.

(xxv) Sophisticated visual analysis routine, for draft, armature curvature continuity.

Q. 2. (c) Derive the transformation matrices of translation, shearing & rotation in 3-D. Derive transformation matrix to scale a unit cube twice uniformly w.r.t. origin.

Ans. 3-D Translation : Translation is another word for moving an object to a new position. It is also sometimes called offsetting. Basically, it is the change of location of the object in space as shown in fig.



If a point P(x, y, z) in 3D is moved to $P'(x', y', z')$ by translated vector $= \langle t_x, t_y, t_z \rangle$. t_x, t_y, t_z are the displacements of P along three principle directions x, y, z respectively. Algebraically,

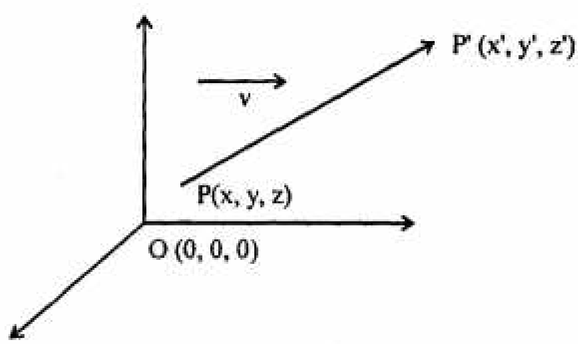
$$x' = x + t_x$$

$$y' = y + t_y$$

$$z' = z + t_z$$

Above equations can be expressed in column matrix formed as

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Translation of a Point

3-D Shearing : Twist in 3D graphics takes place along any one of the axis perpendicular to other two baxis.

(a) Along x-axis, perpendicular to y-axis and z-axis : $\left[Sh_{xy}, Sh_{xz}\right]$

$$x' = ySh_{xy} + zSh_{xz}$$
$$y' = y$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & Sh_{xy} & Sh_{xz} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(b) Along y-axis, perpendicular to the z-axis and x-axis : $\left(Sh_{yz}, Sh_{yx}\right)$

$$x' = x$$
$$y' = xSh_{yz} + ZSh_{yx}$$
$$z' = z$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ Sh_{yx} & 0 & Sh_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(c) Along z-axis, perpendicular to the x-axis and y-axis : $\left[Sh_{zx}, Sh_{zy}\right]$

$$x' = x$$
$$y' = y$$
$$z' = xSh_{zx} + ySh_{zy}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ Sh_{zx} & Sh_{zy} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3-D Rotation : A rotation specifies a pivoting or angular displacement about an axis. Objects are rotated by specifying an angle of rotation and a pivot point. Then trigonometric functions are used to determine the new position.

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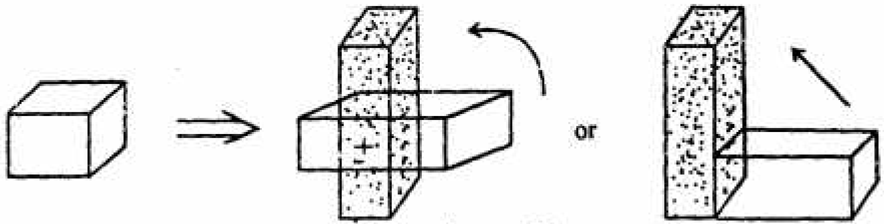
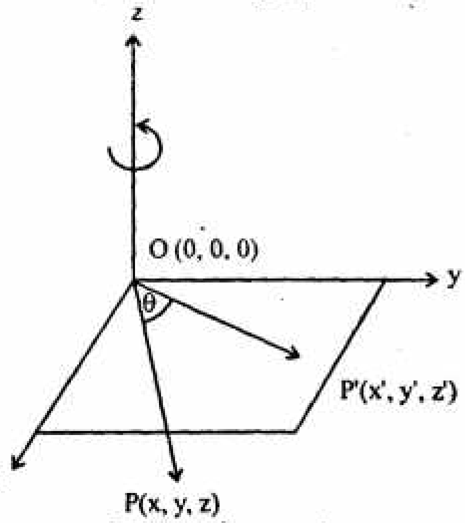


90° around centroid

(a) Rotation About z-axis : If the rotation is carried out about the z-axis, the z-coordinates remain unchanged (because rotation occurs in planes perpendicular to the z-axis) while x and y coordinates behave exactly the same way as in two dimensions, as shown in fig. (2).



Rotation about z-axis.

Rotation of any point P(x, y, z) about z-axis by an amount θ is represented by

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$$z' = z$$

The corresponding transformation matrix in 4×4 form is,

$$R_{\theta,\hat{K}} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_{\theta,\hat{K}}$ represents rotational transformation matrix about z-axis. $\hat{K}$ is the unit vector along z-axis.

$R_{-\theta,\hat{K}}$ represents rotational transformation matrix about z-axis. When rotation is taken in clockwise, $\hat{K}$ is the unit vector along z-axis.

(b) Rotation About x-axis : Here rotation takes place in the planes perpendicular to x-axis, hence x-coordinate doesn't change after rotation while y and z coordinates are transformed. The expression can be derived similarly as before if we replace the z-axis as in fig. (2) with x-axis and the other two axis accordingly, maintaining right handed coordinate system.

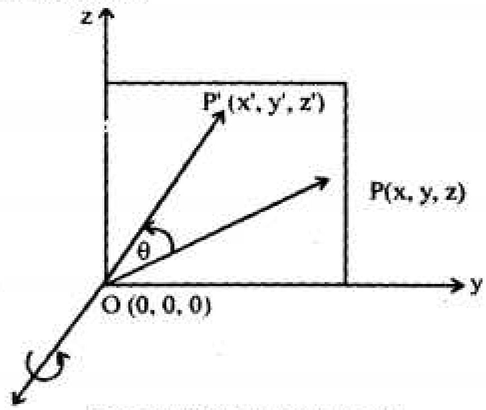


Fig. 2. Rotation about x-axis.

Rotation of any point P(x, y, z) about x-axis by an amount θ is represented by

$$\left.\begin{aligned} y' &= y\cos\theta - z\sin\theta \\ z' &= y\sin\theta + 2\cos\theta \\ x' &= x \end{aligned}\right\}$$

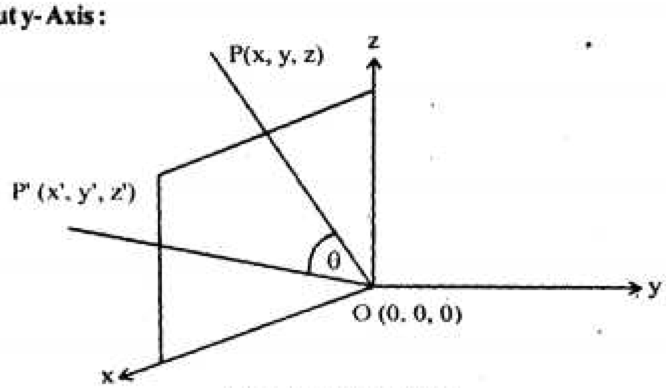
The corresponding transformation matrix is in 4×4 form is,

$$R_{\theta\hat{i}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_{\theta\hat{i}}$ represents rotational transformation matrix about x-axis, $\hat{i}$ is the unit vector along x-axis.

$R_{-\theta\hat{i}}$ represents rotational transformation matrix about x-axis when rotation is taken in clockwise, $\hat{i}$ is the unit vector along x-axis.

(c) Rotation About y- Axis :



Rotation about y-axis

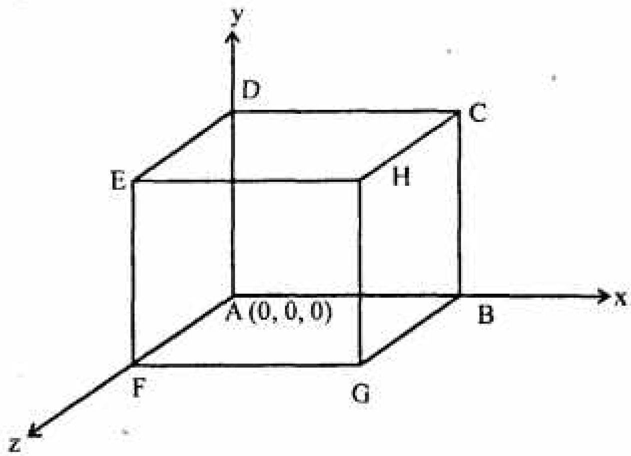
$R_{\theta \hat{j}}$ represent rotational transformation matrix about y-axis, $\hat{j}$ is the unit vector along y-axis.

$R_{-\theta, \hat{j}}$ represents rotational transformation matrix about y-axis when rotation is taken in clockwise $\hat{j}$ is the unit vector along y-axis.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & S_z \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Co-ordinates of all vertices of the cube are,

A (0, 0, 0) B (1, 0, 0)
C (1, 1, 0) D (0, 1, 0)
E (0, 1, 1) F (0, 0, 1)
G (1, 0, 1) H (1, 1, 1)



$$A'B'C'D'E'F'G'H' = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Q. 3. Attempt any two :

(a) Define Bezier curve. Also give their properties. Find equation of Bezier curve which passes through point (0, 0) and (–2, 1) and is controlled through points (7, 5) and (2, 0).

Ans. Bezier Curve : A Bezier curve can be fitted to any number of control points. Without necessitating tangent vector specification at any of the control points, a set of characteristics polynomial approximately functions, called Bezier blending functions, blend the control points to produce a Bezier curve segment. However, the degree of a Bezier curve segment is determined by the number of control points to be fitted with that curve segment.

Given a set of n+1 control points P_0, P_1, ..., P_n, a parametric Bezier curve segment that will fit to those points is mathematically defined by,

$$P(u) = \sum_{i=0}^{n} P_i B_{i,n}(u) \text{ where } 0 \le u \le 1$$

Where $B_{i,n}(u)$ are the Bezier blending functions, also known as Bernstein Basis functions defined as

$$B_{i,n}(u) = C(n,i)u^i(1-u)^{n-i}$$

and $C(n,i)$ is the binomial coefficient.

$$C(n,i) = \frac{n!}{(n-i)!i!}$$

Expanding the equation, we get

$$P(u) = P_0B_{0,n}(u) + P_1B_{1,n}(u) + \ldots\ldots P_{n-1}B_{n-1}(u) + P_nB_{n,n}(u)$$

$$= P_0\left\{C(n,0)u^0(1-u)^n\right\} + P_1\left\{C(n,1)u^1(1-u)^{n-1}\right\} + \ldots\ldots\ldots\ldots$$

$$+P_{n-1}\left\{C(n,n-1)u^{n-1}(1-u)\right\} + P_n\left\{C(n,n)u^n(1-u)^0\right\}$$

From this equation we discover two very important facts about Bezier curves, if we minutely observe the blending functions in curly brackets.

(i) n, the degree of polynomial functions and thus of the Bezier curve segment is one less than the number of control points used.

(ii) The Bezier curve in general passes through only the first control point P_0 and the last control point P_n as because $P(u=0) = P_0$ and $P(u=1) = P_1$.

From Bezier curve of degree $z(n=3)$, we find that four $(n+1)$ control points are required to specify a cubic Bezier curve segment.

Thus, for a parametric cubic Bezier curve,

$$P(u) = \sum_{i=0}^{3} P_iB_{i,3}(u)$$

$$0 \le u \le 1$$

$\Rightarrow$

$$P(u) = P_0B_{0,3}(u) + P_1B_{1,3}(u) + P_2B_{2,3}(u) + P_3B_{3,3}(u)$$

Where,

$$B_{0,3}(u) = \left[\frac{3!}{0!3!}\right]u^0(1-u)^3 = (1-u)^3$$

$$B_{1,3}(u) = \left\{\frac{3}{1!2!}\right\}u^1(1-u)^2 = 3u(1-u)^2$$

$$B_{2,3}(u) = \left\{\frac{3!}{2!1!}\right\}u^2(1-u)$$

$$= 3u^2(1-u)$$

$$B_{3,3}(u) = \left\{\frac{3!}{3!0!}\right\} u^3 (1-u)^0 = u^3$$

$$\Rightarrow \quad P(u) = (1-u)^3 P_0 + 3u(1-u)^2 P_1 + 3u^2(1-u)P_2 + u^3 P_3$$

$$\Rightarrow \quad P(u) = \left(-u^3 + 3u^2 - 3u + 1\right)P_0 + \left(3u^3 - 6u^2 + 3u\right)P_1 + \left(-3u^3 + 3u^2\right)P_2 + u^3 P_3$$

This expression can be translated into the matrix form i.e.,

$$P(u) = [F][B]$$

Or

$$P(u) = [T][M][B]$$

Here the basis function matrix $\quad [F] = \left[B_{0,3}(u) B_{1,3}(u) B_{2,3}(u) B_{3,3}(u)\right]$

The Bezier geometry matrix,

$$[B] = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$[T] = \begin{bmatrix} u^3 & u^2 & u \end{bmatrix}$$

& the Bezier basis matrix

$$[M] = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Such that

$$[F] = [T][M]$$

Therefore, a cubic Bezier curve controlled by the points P_0, P_1, P_2, P_3 is

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Properties of Bezier Curve :

(i) The basis functions are real.

(ii) Bezier curve always passes through the first and last control points i.e., curve has same end points as the guiding polygon.

(iii) The degree of the polynomial defining the curve segment is one less than the number of defining polygon point. Therefore, for 4 control points, the degree of the polynomial is three i.e., cubic polynomial.

(iv) The curve generally follows the shape of the defining polygon.

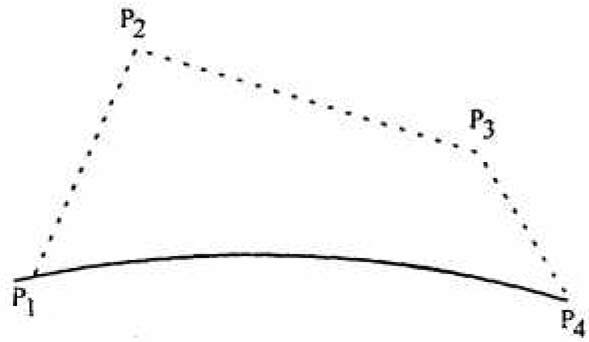
(v) The direction of the tangent vector at the end points is the same as that of the vector determined by first and last segments.

(vi) The curve lies entirely within the convex hull formed by four control points.

(vii) The convex hull property for a Bezier curve ensures that the polynomial smoothly follows the control points.

(viii) The curve exhibits the variation diminishing property. This means that the curve does not oscillates about any straight line more often than the defining polygon.

(ix) The curve is invariant under an off-line transformation.



This means that if we want to connect two Bezier curves, we have to made the first control point of the second Bezier curve match the last control point of the first curve. We can also observe that at the start of the curve, the curve is tangent to the line connecting first and second control points. Similarly at the end of curve, the curve is tangent to the line connecting the third and fourth control point. This means that, to join two Bezier curves smoothly we have to place the third and the fourth control point of the first curve on the same line specified by the first and the second control points of the second curve.

Solution : Using four given points, as control points a cubic Bezeir curve can be defined. But as the cubic Bezier passes through (0, 0) and (–2, 1) there two points should be considered as the end control points while the other two points (7, 5) and (2, 0) should be considered as intermediate control points. But depending on the sequence in which we consider the successive control points, we obtain two different equations of cubic Bezier curve.

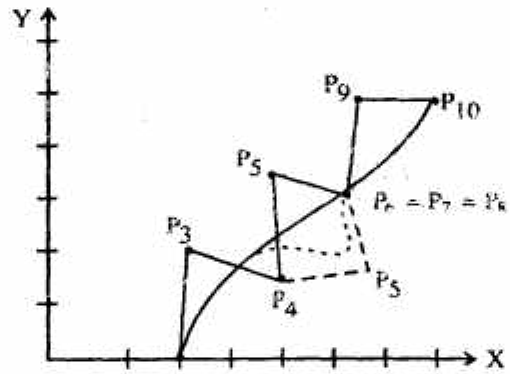
(i) Let
$$P_0=(0,0);\ P_1=(7,5)$$
$$P_2=(2,0);\ P_3=(-2,1)$$

The corresponding cubic Bezier curve is given by

$$P(u)=\left[u^3 u^2 u^1\right]=\begin{bmatrix}-1 & 3 & -3 & 1\\ 3 & -6 & 3 & 0\\ -3 & 3 & 0 & 0\\ 1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0\\ 7 & 5\\ 2 & 0\\ -2 & 1\end{bmatrix} \qquad (0\le u\le 1)$$

$$=\left[u^3 \quad u^2 \quad u \quad 1\right]\begin{bmatrix}13 & 16\\ -36 & -30\\ 21 & 15\\ 0 & 0\end{bmatrix}$$

$$=\left(13u^3-36u^2+21u\right)\left(16u^3-30u^2+15u\right)$$



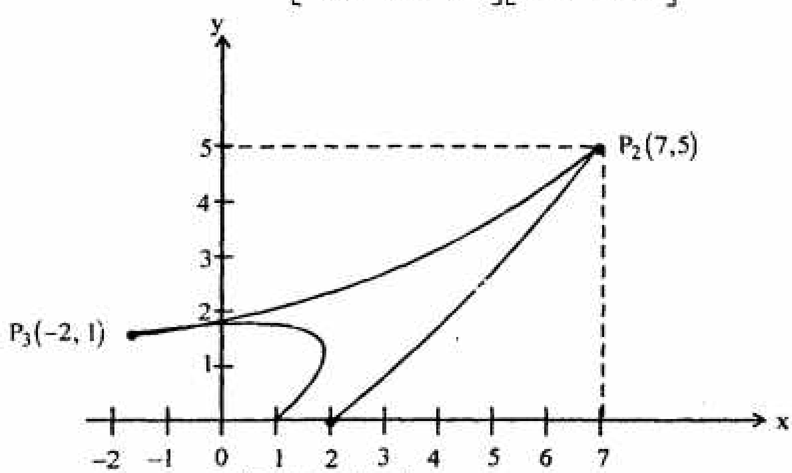
(ii) If
$$P_0 = (0,0);\ P_1 = (2,0);$$
$$P_2 = (7,5);\ \&\ P_3 = (-2,1)$$

Than the resulting cubic Bezier curve is given by

$$P(u) = \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \\ 7 & 5 \\ -2 & 1 \end{bmatrix} \quad (0 \le u \le 1)$$

$$= \begin{bmatrix} u^3 & u^2 & u & 1 \end{bmatrix} \begin{bmatrix} -17 & -14 \\ 9 & 15 \\ 6 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \left[-17u^3 + 9u^2 + 6u\right]\left[-14u^3 + 15u^2\right]$$



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Q. 3. (b) What is blending function? Explain in detail. Also give their properties.

Ans. Blending Function : In numerical analysis and approximation theory, basis functions are also called blending functions, because of their use in interpolation : In this application, a mixture of basis function provides an interpolating function (with the "blend" depending on the evaluation of the basis functions at the data points).

In mathematics, a basis function is an element of a particular basis for a function space. Every function in the function space can be represented as a linear combination of basis functions, just as every vector in a vector space can be represented as a linear combination of basis vectors.

The K^{th} order blending function is defined by convolving the k-Ist order blending function with the first order blending function. This convolution can be seen to be the integral.

$$N_k(t) = (N_{k-1} * N_i)(t)$$

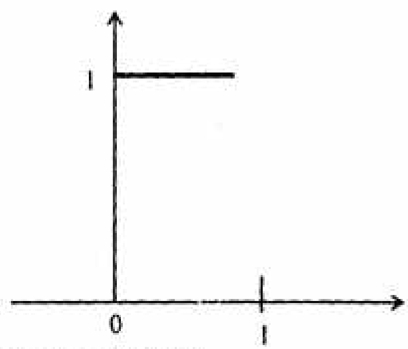
$$= \int_{-\infty}^{\infty} N_{k-1}(x) N_1(t-x) dx$$

$$= \int_{t-1}^{t} N_{k-1}(x)\, dx$$

(i) The First Order Blending Function : The first order blending function & just the floor scaling function.

$$N_1(t) = \begin{cases} 1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

and is shown by the graph



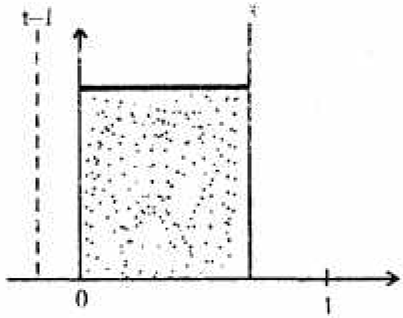
The support of this function is the interval [0, 1].

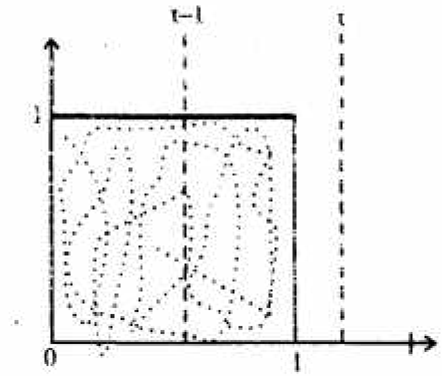
(ii) The Second Order Blending Function : To calculate the second order blending function we must calculate

$$N_2(t) = \int_{t-1}^{t} N_1(x) dx$$

The function $N_1(x)$ is non-zero only when $0 \le x \le 1$. Thus, we can get non-zero values in the integrand $N_1(x)$ for any t where $0 < t < 2$. The integral splits naturally into the two cases shown below for $0 \le t \le 1$

and $1 \le t \le 2$.





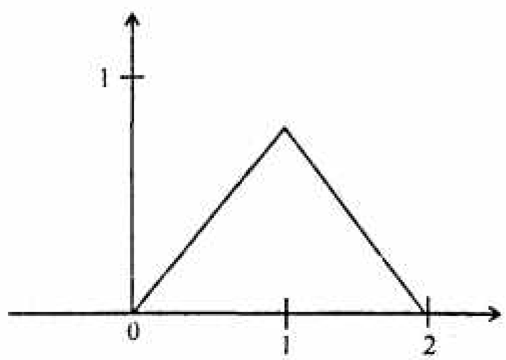
Where in each case we have shaded the areas between the limits of integration 0 and 1.
So, we have that,

$$N_2(t) = \int_{t-1}^{t} N_1(x)\,dx$$

$$= \begin{cases} \int_0^t dx & \text{if } 0 \le t \le 1 \\ \int_{t-1}^{1} dx & \text{if } 1 \le t \le 2 \end{cases}$$

$$= \begin{cases} t & \text{if } 0 \le t \le 1 \\ 2-t & \text{if } 1 \le t \le 2 \end{cases}$$

Which is illustrated by



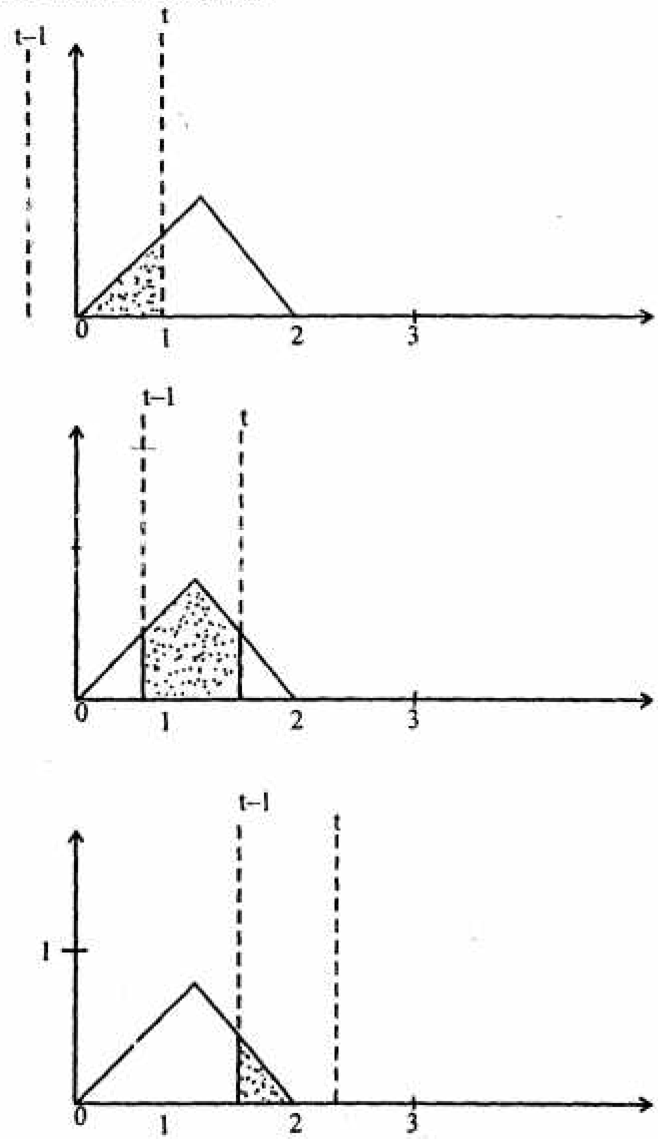
It is clear that the support of $N_2(t)$ is the interval [0, 2].

(iii) The Third Order Blending Function : To calculate the third order blending function, we must calculate

$$N_3(t) = \int_{t-1}^{t} N_2(x)\,dx$$

The function $N_2(x)$ is non-zero only when $0 \le x \le 2$. So, we get non-zero values in the integrated for any t where, $0 < t < 3$. This is straight forward to calculate once the reader sees that there are three cases, each depending on t.

These three cases are illustrated below as,



In each case the section of the curve $N_2(x)$ that lies between the integration bounds of 0 and 1 has been shaded.

So, now we can calculate the integral by

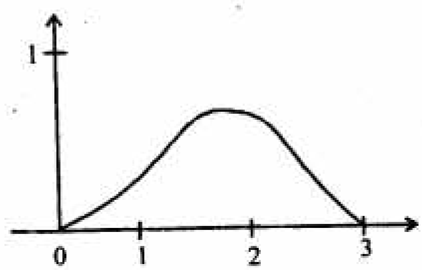
$$N_3(t) = \int_{t-0}^{t} N_2(x)dx$$

$$= \begin{cases} \int_0^t N_2(x)dx & \text{if} \quad 0 \le t \le 1 \\ \int_{t-1}^{1} N_2(x)dx + \int_1^t N_2(x)dx & \text{if} \quad 1 \le t \le 2 \\ \int_{t-2}^{1} N_2(x)dx & \text{if} \quad 2 \le t \le 3 \end{cases}$$

$$= \begin{cases} \int_0^t xdx & \text{if} \quad 0 \le t \le 1 \\ \int_{t-1}^{1} 2x\,dx + \int_1^t x\,dx & \text{if} \quad 1 \le t \le 2 \\ \int_{t-2}^{1} 2 - xdx & \text{if} \quad 2 \le t \le 3 \end{cases}$$

$$= \begin{cases} \frac{1}{2}t^2 & \text{if} \quad 0 \le t \le 1 \\ \left(\frac{1}{2}\left(-2t^2 + 6t - 3\right)\right) & \text{if} \quad 1 \le t \le 2 \\ \left(\frac{1}{2}\left(t^2 - 6t + 9\right)\right) & \text{if} \quad 2 \le t \le 3 \end{cases}$$

This curve is a piecewise quadratic –i.e., it has quadratic pieces that are smoothly joined together. The curve is drawn as,



It is clear that the support of $N_3(t)$ is the interval [0, 3]

Properties of Blending Function : We have noted that a degree n Bezier curve always begins at P_0 and ends at P_n. Also, the curve is always tangent to the control polygon at P_0 and P_n.

Other popular blending functions exist for defining curves. In fact, you can easily make up your own set of blending functions. And by following a few simple rules, you can actually create a new type of free form curve which has desirable properties.

Consider a set of control points P_i, i = 0, ..., n and blending functions $f_i(t)$ which define the curve,

$$P(t) = \sum_{i=0}^{n} f_i(t) P_i$$

We can select our blending functions such that the curve has any or all of the following properties :

(i) Coordinate System Independence : This means that the curve will not change if the coordinate system is changed. In other words, imagine that the control points are drawn on a piece of paper and we move that the piece of paper around so that the (x, y) coordinates of the control points change. It would be nice if the curve did not change relative to the control points. Actually, if we were to pick an arbitrary set of blending functions, the curve would change. In order to provide co-ordinate system independence, the blending functions must identically sum to one,

$$\sum_{i=0}^{n} f_i(t) \equiv 1$$

(ii) Convex Hull Property : The convex hull property exists in curves which are coordinate system independent and for which the blending functions are all non-negative.

$$\sum_{i=0}^{n} f_i(t) \equiv 1;$$

$$f_i(t) \geq 0,\ 0 \leq t \leq 1,\ i = 0, \ldots\ldots, n$$

(iii) Symmetry : Curves which are symmetric do not change if the control points are ordered in reverse sequence :

$$\sum_{i=0}^{n} f_i(t) P_i \equiv \sum_{i=0}^{n} f_i(1-t) P_{n-i}$$

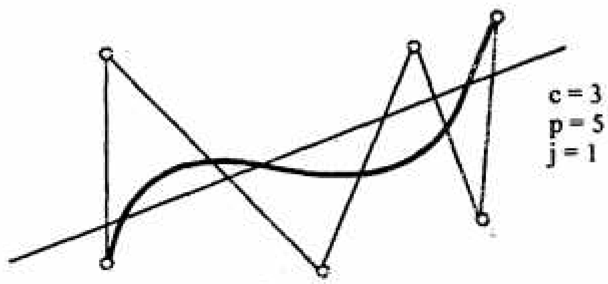
This holds if

$$f_i(t) = f_{n-i}(1-t)$$

(iv) Variation Diminishing Property : This is a property which is obeyed by Bezier curves and B-spline curves. It states that if a given straight line intersects the curve in C number of points and the control polygon in p number of points, then it will always hold that

$$C = p - 2j$$

Where j is zero or a positive integer. This has the practical interpretation that a curve which obeys the variation diminishing property will "wiggle" no more than the control polygon.



Variation diminishing property

The conditions under which a curve will obey the variation diminishing property are rather complicated. Suffice it to say that Bezier curves and B-spline curves obey this property, and most other curves do not.

5. Linear Independence : It is very desirable that the blending functions are linearly independent. If they are not linearly independent, then it is possible to express one blending function in terms of the other ones. This has the practical disadvantage that for certain control point arrangements, the curve collapses to a single point.

(vi) Endpoint Interpolation : If a curve is to pass through the first and last control points, as in the case of Bezier curves, the following conditions must be meet :

$$f_0(0) = 1;\ f_i(0) = 0,\ i = 1, \ldots\ldots\ldots, n$$

$$f_n(1) = 1,\ f_i(1) = 0,\ i = 0, \ldots\ldots\ n-1$$

The Bezier and B-spline curves are currently the most popular curve forms. Historically, other curve forms evolved independently at several different industrial sites, each faced with the common problem of making free-from curve accessible to designers without a mathematical background :

Q. 3. (c) Define Bezier surface and B-spline surface-differentiate between Bezier spline curve and B-spline curve. Find the cubic B-spline curve with the following control points.

A (0, 40), B (40, 40), C(60, 20), D (60, –10)?

Ans. Bezier Surface : The Bezier surface is formed as the Cartesian product of the blending functions of two orthogonal Bezier curves. The simplest way to construct a Bezier surface is as the tensor product of Bezier curves. A tensor product Bezier surface of order n+1 is defined by $(n+1)^2$ control points. It is called a Bezier patch.

$$B(u, v) = \sum_{i=0}^{N_i} \sum_{j=0}^{N_j} P_{i,j} \frac{N_i!}{i!(N_i - i)!} u^i (1-u)^{N_i - i} \frac{N_j!}{j!(N_j - j)} v^j (1-v)^{N_j - j} \qquad 0 \le u \le 1$$

$$0 \le v \le 1$$

Where, $P_{i,j}$ is the i, jth control points. There are N_{i+1} and N_{j+1} control points in the i and j directions, respectively.

You can think about this as moving the control points of one Bezier curve along a set of Bezier curves to sweep out a surface. Continuity across a boundary between two Bezier patches is only guaranteed if each of the Bezier curves across the join obeys the curve continuity conditions.

The corresponding properties of the Bezier curve apply to the Bezier surface.

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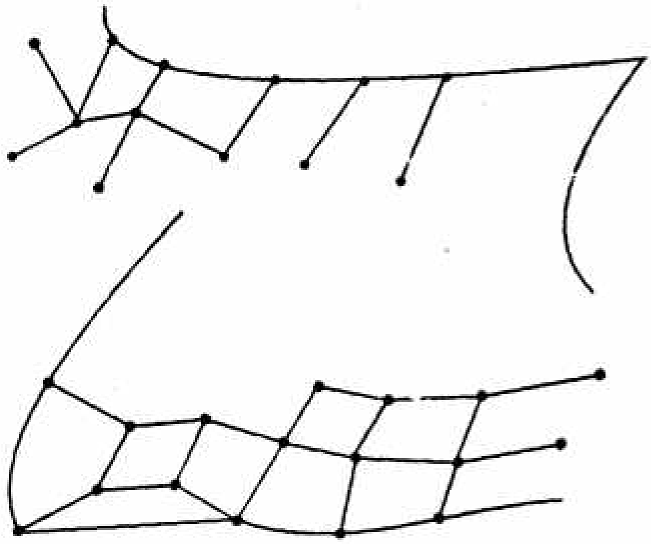
(i) The surface does not, in general, pass through the control points except for the corners of the control point grid.

(ii) The surface is contained within the convex hull of the control points.

Along the edges of the grid patch the Bezier surface matches that of a Bezier curve through the control points along that edge.



Two Bezier patches joined along the edges P_{14}, P_{24}, P_{3_4} and P_{44}.

Closed surfaces can be formed by setting the last control point equal to the first. If the tangents also match between the first two and last two control points, then the closed surface will have first order continuity while a cylinder/cone can be formed from a Bezier surface, it is not possible to form a sphere.

B-Spline Surface : We can create a B-spline surface using a similar method to the Bezier surface. For B-spline curves, we used two phantom knots to clamp the ends of the curve. For a surface, we will have phantom knots all around the real knots as shown below for an M+1 by N+1 knot surface.

$P_{-1,-1}$	$P_{0,-1}$	$P_{1,-1}$	$\cdots$	$P_{M-1,-1}$	P_{M-1}	$P_{M+1,-1}$
$P_{-1,0}$	$P_{0,0}$	$P_{1,0}$	$\cdots$	$P_{M-1,0}$	$P_{M,0}$	$P_{M+1,0}$
$P_{-1,1}$	$P_{0,1}$	$P_{1,1}$	$\cdots$	$P_{M-1,1}$	$P_{M,1}$	$P_{M+1,1}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$	$\vdots$
$P_{-1,N-1}$	$P_{0,N-1}$	$P_{1,N-1}$	$\cdots$	$P_{M-1,N-1}$	$P_{M,N-1}$	$P_{M+1,N-1}$
$P_{-1,N}$	$P_{0,N}$	$P_{1,N}$	$\cdots$	$P_{M-1,N}$	$P_{M,N}$	$P_{M+1,N}$
$P_{-1,N+1}$	$P_{0,N+1}$	$P_{1,N+1}$	$\cdots$	$P_{M-1,N+1}$	$P_{0,N+1}$	$P_{M+1,N+1}$

There are two extra rows and two extra columns of knot in parametric space surrounding the real knots. Where we place these knots determines the shape of the surface at the edges. The method described here gives similar results to the method used for Bezier surface; that is, the edges of the surface form a B-spline curve of the edge knots. This means some of the boundary conditions are

$$P_{m,-1} = 2P_{m,0} - P_{m,1}$$

$$P_{m,N+1} = 2P_{m,N} - P_{m,N-1}$$

$$P_{-1,n} = 2P_{0,n} - P_{1,n}$$

$$P_{m+1,n} = 2P_{m,n} - P_{m-1,n}$$

For $0 <= m <= M$ and $0 <= n <= N$. These conditions are essentially the same as the two-dimensional case. They mean the weighting of a sample taken at the boundary $m = 0$ is dependent only on knots along the $m = 0$ boundary (the phantom knots at $m = -1$ balance out the real knots at $m = 1$). The remaining boundary conditions make the surface corners and the corners knots coincide. The coordinate of the corner as set by $P_{0,0}$ (and hence the parametric knot at $\{-1, -1\}$) is

$$P_{0,0} = \frac{4}{9}P_{0,0} + \frac{1}{9}\left(P_{-1,0} + P_{0,-1} + P_{1,0} + P_{0,1}\right) + \frac{1}{36}\left(P_{-1,-1} + P_{1,-1} + P_{-1,1} + P_{1,1}\right)$$

$$P_{-1,-1} = 4P_{0,0} - 2P_{1,0} - 2P_{0,1} + P_{1,1}$$

This gives us a surface that interpolates the corners knots and forms B-spline curves down each side. To explore the B-spline surface further, see the surface credited from a 4×4 or a 5×5 mesh.

Bezier Spline Curve vs B-Spline Curve :

(i) Bezier curves are special cases of B-spline curves.

(ii) In Bezier curve change in a portion of a curve causing changes to whole curve. In contrast B-spline has local control over curve. This property allows a designer or artist to edit one portion of a curve without causing changes to the remaining portion of the curve.

(iii) Unlike Bezier curve, B-spline curves do not necessarily interpolate their first and last control points.

(iv) B-spline curves are more flexible and intuitive than Bezier curves with large number of control points.

(v) Curve designer has much flexibility in adjusting the curvature of B-spline curve than Bezier curves.

(vi) B-splines can be designed with sharp bend and even "corners" than Bezier curves.

Solution : Equation of cubic B-spline curve passes through A, B, C, D is given by

$$B(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 40 \\ 40 & 40 \\ 60 & 20 \\ 60 & -10 \end{bmatrix}$$

Where, $(0 \le t \le 1)$

$$= \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \frac{1}{6} \begin{bmatrix} 0 & 10 \\ -60 & -60 \\ 180 & -60 \\ 220 & 220 \end{bmatrix}$$

$$B(t) = \left[\frac{1}{6}\left(-60t^2 + 180t + 220\right) \frac{1}{6}\left(10t^3 - 60t^2 - 60t + 220\right)\right]$$

Q. 4. Attempt any two :

(a) Define flexible automation. Also give comparison between PAC & PLC.

Ans. Flexible Automation : Flexible automation is an operational response to this need. "Manufacturing is shifting from attempting to exploit economies of scale to exploiting economics of scope, "Raid Michael Higgins, marketing director of ABB Flexible Automation In c in Auburn Hills, Mich. "As a result, the ability to adapt to changing market requirements, product designs, and technological development is becoming a key factor in the competitiveness of an organization. Viewed from the shop floor, this translates into dealing efficiently with frequent changeovers of parts and small production batches."

The ideal system uses reprogrammable processing, material handling machinery, and computer coordination of cycles to enables the simultaneous production of different part types with zero-on-line setup time and costs. No system has thus for lived upto this ideal completely, still, new developments in hardware and software are bringing state-of-the art systems closer all the time.

For years now, industry has successfully implemented computer-driven, flexibly automated manufacturing systems. These systems tend to be small in both size and scope. Large scale flexible manufacturing systems have inherently different problems. Given the high off-line setup costs, large scales of operation cannot be economically mapped to a set of independently functioning small systems. To support manufacturing operations of large scale and scope, flexible automation systems must be able to scale up. Large systems will also require these devices to be organized into subsystems. The subsystems will have to be dynamically configured, and their operations must be properly coordinated. The make system-level flexibility the central problem, rather than device flexibility.

The interaction between automation systems and computer-aided design systems is also increasing. Given a set of instructions in the traditional way, a computer has no real knowledge of the size or shape of the part it is working on; it just knows that it needs to move the tool along a prescribed path. Interfacing with a CAD system is another shortcut that spares programmers from having to write code. By importing the object geometry from a CAD system, software can automatically write positioning code and process code to achieve the result specified. Systems can handle either two or three dimensional drawings.

The area in which flexible automation is most pervasive is the automotive industry. The industry not only has a great need for automation systems but also has the budget to afford such expensive and elaborate systems.

PAC-PLC Comparison : Generally, PACs and PLCs serve the same purpose. Both are primarily used to perform automation, process control, and data acquisition functions such as digital and analog control, serial string handling, PID, motion control, and machine vision. The parameters within which PACs operate to achieve this, however, sometimes run counter to how a PLC functions.

Unlike PLCs, PACs offer open, modular architecture, the rational being that because most industrial applications are customized, the control hardware used for them needs to allow engineers to pick and choose the other components in the control systems architecture without having to worry whether or not they will be compatible with the controller.

PACs and PLCs are also programmed differently. PLCs are often programmed in ladder logic, a graphical programming language resembling the rails and rings of ladder that is designed to emulate old electrical relay wiring diagrams. PAC control programs are usually developed with more generic software tools that permit the designed program to be shared across several different machines, processors, HMI terminals or other components in the control system architecture.

PAC processing and input/output scanning is also very different. Unlike PLCs, which constantly scan all the input/output, inputs in the control system at very high rates of speed.

PACs utilize a single tagnams database and a logical address system to identify and map input/output points as needed.

Q. 4. (b) Explain Group technology. Give the type of layout explain GT application in manufacturing leads to focused factories and cellular manufacturing.

Ans. Group Technology : Group technology or GT is a manufacturing philosophy in which the parts having similarities (Geometry, manufacturing process and/or function) are grouped together to achieve higher level of integration between the design and manufacturing functions of a firm. The aim is to reduce work-in-progress and improve delivery performance by reducing lead times. GT is based on a general principle that many problems are similar and by grouping similar problems, a single solution can be found to a set of problems, thus saving time and effort. The group of similar parts is known as part family and the group of machineries used to process an individual part family is known as machine cell. It is not necessary for each part of a part family to be processed by every machine of corresponding machine cell. This type of manufacturing in which a part family is produced by a machine cell is known as cellular manufacturing. The manufacturing efficiencies are generally increased by employing GT because the required operations may be confined to only a small cell and thus avoiding the need for transportation of in-process parts.

General Features of Group Technology :

(i) Group Technology (GT) is based on the principle that similar things should be done similarly.

(ii) GT is a philosophy with central objective of increasing production efficiency by grouping various parts are and products with similar and/or production processes together.

(iii) GT has application in many areas related with manufacturing such as product design, process planning, fabrication, assembly and production control.

(iv) GT does not seek to reduce variety in the kinds of products offered to customers.

(v) GT seeks to reduce variety in the kinds of products produced by the manufacturer.

Types of Group Layout :

(i) GT Flow Line : All parts assigned to group follow the same machine sequence and requires relatively proportional time requirement on each machine.

(ii) GT Cell : Allows parts to move from any machine to any other machine.

(iii) GT Centre : It is a logical arrangement. Machine may be located as in a process layout by using functional departments, but each machine is dedicated to producing only certain part families–virtual manufacturing cell.

(iv) GT Application in Manufacturing Leads to Focused Factories and Cellular Manufacturing :

(i) Focused factory strives for a narrow range of products, customers and processes. The result is a factory that is smaller, simpler and totally focused on one or two key manufacturing tasks.

(ii) Focused factory is also called a plant within a plant.

(iii) Each focused factory is a portion of a plant devoted to making a group of several or numerous some what similar products.

(iv) The key manufacturing task provides the focus in focused factories.

(v) Cellular manufacturing system focuses on a group of part (part) family.

(vi) Cellular manufacturing relates to the organization of the manufacturing facility on the basis of dedicated cells of dissimilar machines, which process similar parts called part families.

(vii) This kind of facility layout is called GT layout or cellular layout or group layout.

(viii) This configuration is most appropriate for medium variety, medium volume environment.

(ix) With GT, each part type flows only through its specific group area.

A study shows that by applying GT, 150 similar parts were placed into a group of 8 dedicated machines. Previously the same parts had been made on 51 different machines with 87 routing.

Q. 4. (c) Explain in brief numerical control system. Give the type of numerical control system with figure.

Ans. Numerical Control System : Numerical Control (NC) refers to the automation of machine tools that are operated by abstractly programmed commands encoded on a storage medium, as opposed to manually controlled via handwheels or levers or mechanically automated via came alone. The first NC machines were in the 1940s and 50s, based on existing tools that were modified with motors that moved the controls to follow points fed into the system on punched tape. These early servomechanisms were rapidly augmented with analog & digital computers, creating the modern computer numerical controlled (CNC) machine tools that have revolutionized the manufacturing process.

In modern CNC systems, end-to-end component design is highly automated using CAD/CAM programs. The programs produce a computer file that is interpreted to extract the commands needed to operate a particular machine via a post processor, & then loaded into the CNC machines for production. Since any particular component might require the use of a number of different tools-drills, saves etc. modern machines often combine multiple tools into a single 'cell'. In other cases, a number of different machines are used with an external contollers & human or robotic operator that move the component from machine to machine. In either case, the complex series of steps needed to produce any part is highly automated & produces a part that closely matches the original CAD design.

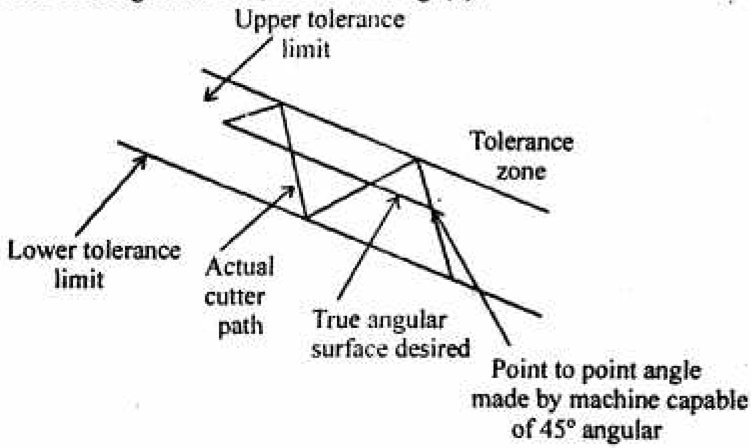
A CNC machine consists of two major components—the machine tool and the controller, which is an on board computer. These components may or may not be manufactured by the same machine. Each controller is manufactured with a standard set of built-in codes-other codes are added by the machine tool builders.

Every CNC machine, regardless of manufacturer is a collection of systems coordinated by the controller.

Types of Control System : There are two types of control system used on NC machines :

Point-to-point systems and continuous path systems :

Point-to-point machines move only in straight lines. They are limited in a practical sense to hole operations (drilling, reaming, boring and so on) and straight milling cuts parallel to a machine axis. When making an axis move; all affected drive motors run at the same speed. When one axis motor has moved the instructed amount, it stops while the other motor continues until its axis has reached its programmed location. This makes the cutting of 45-degree angles possible, but not arcs or angles other than 45 degrees. Arc and angles must be programmed as a series of straight line cuts, as shown in fig. (1).



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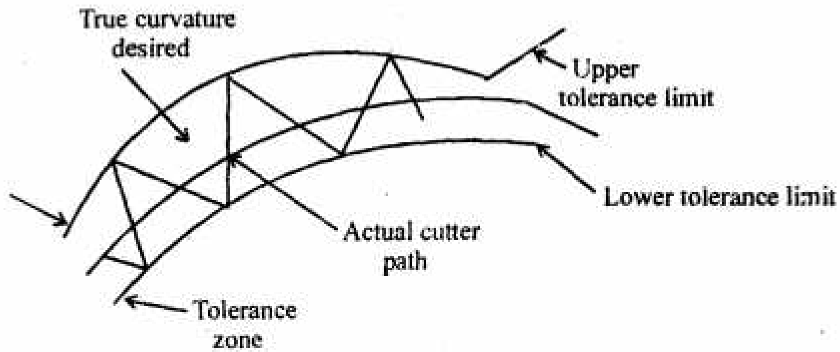

Fig. 1 : Point-to-point Arc as made by machine capable of only straight line cuts.

A continuous-path machine can move its drive motors at varying rates of speed while positioning the machine. Therefore, it can more easily cut arcs and angles as shown in fig. (2). Most newer NC machines are the continuous-path type.

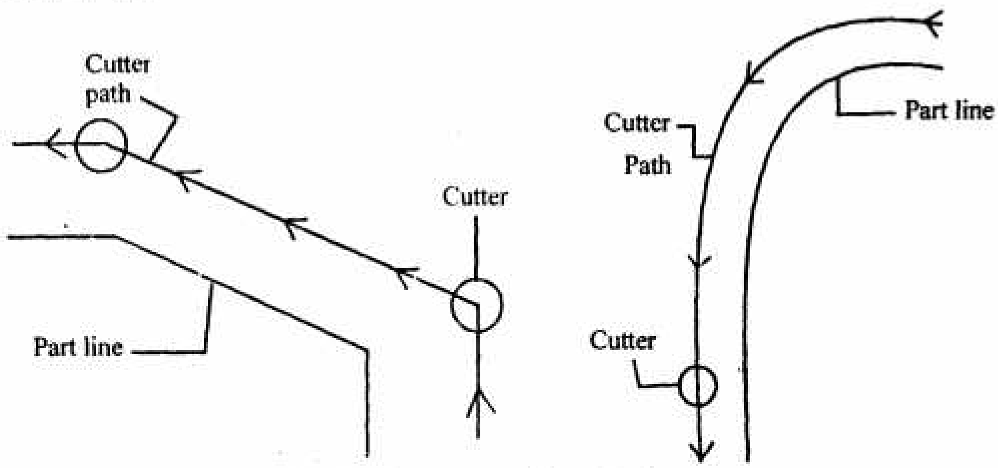

Fig. 2 : Continuous path angles & arcs.

Q. 5. Attempt any two :

(a) Define flexible manufacturing system. Also give objectives and limitation of FMS.

Ans. Flexible Manufacturing Systems [FMS] : A flexible manufacturing system [FMS] is a group of numerically controlled machine tools, interconnected by a central control system. A flexible manufacturing system is a manufacturing system in which there is some amount of flexibility that allows the system to react in the case of changes, whether predicted or unpredicted. This flexibility is generally considered to fall into two categories, which both contain numerous subcategories :

(i) The first category, machine flexibility, covers the system's ability to be changed to produce new product types and ability the change to order of operations executed on a part.

(ii) The second category is called routing flexibility, which consists of the ability to use multiple machines to perform the same operation on a part, as well as the system's ability to absorb large scale changes, such as in volume, capacity or capability.

The various machining cells are interconnected, via loading and unloading stations, by an automated transport system. Operation flexibility is enhanced by the ability to execute all manufacturing tasks on numerous product design in small quantities and with factor delivery.

It has been described as an automated job shop and as a miniature automated factory. Simply stated, it is an automated production system that produce one or more families of parts in a flexible manner. Today, this prospect of automation and flexibility presents the possibility of producing non-standard parts to create a competitive advantages.

The concept of flexible manufacturing systems evolved during 1960s. When robots, programmable controllers and computerized numerical controls brought a controlled environment to the factory floor in the form of numerically-controlled and direct-numerically-controlled machines.

For most part. FMS is limited to firms involved in batch production or job ship environments. Normally, batch producers have two kinds of equipment from which to choose : dedicated machinery or unautomated, general-purpose tools. Dedicated machinery results in cost savings but lack flexibility. General purpose machines such as lathes, milling machines, or drill presses are all costly and may not teach full capacity. Flexible manufacturing system provide the batch manufacturer with another option–one that can make batch manufacturing just as efficient and productive as mass production.

Flexible manufacturing systems are flexible in the sense that their device controllers and central control computer can be reprogrammed to make new parts or old parts in new ways. They can also often make a number of different types of parts at the same time. However, this flexibility is limited to a certain family of parts, for example, axles.

Objectives of FMS : Stated formally, the general objectives of an FMS are to approach the efficiencies and economis of scale normally associated with mass production, and to maintain the flexibility required for small and medium-lot-size production of a variety of parts. Two kinds of manufacturing systems fall within the FMS spectrum. There are assembly systems, which assemble components into final products and forming systems, which actually form components or final products.

A generic FMS is said to consist of the following components :

(i) A set of work stations containing machine tools that do not require significant set-up timer or change over between successive jobs. Typically, these machines perform milling, boring, drilling, tapping, reaming, turning and grooving operations.

(ii) A material-handling system that is automated and flexible in that it permits job to move between any pair of machine so that any job routing can be followed.

(iii) A network of supervisory computers and microprocessors that perform some or all of the following tasks : (a) directs the routing of jobs through the system : (b) tracks the status of all jobs in progress so it is known where each job is to go next. (c) passes the instructions for the processing of each operation to each station and ensures that the right tools are available for the job; and (d) provides essential monitoring of the correct performance of operations and signals problems requiring attention.

(iv) Storage, locally at the network stations, and/or centrally at the system level.

(v) The jobs to be processed by the system. In operating an FMS, the worker enter the job to be run at the supervisory computer, which then downloads the part programs to the cell control or NC controller.

Benefits of FMS : The potential benefits from the implementation and utilization of a flexible manufacturing system have been detailed by numerous researchers on the subject. A review of the literature reveals many tangible and intangible benefits that FMS were extol. These benefits include :

(i) Less waste

(ii) Fewer workstations
(iii) Quicker changes of tools, dies and stamping machinery
(iv) Reduced downtime
(v) Better control over quality
(vi) Reduced labour
(vii) More efficient use of machinery
(viii) Work in process inventory reduced
(ix) Increased capacity
(x) Increased production flexibility.

Limitations of FMS : Despite these benefits, FMS does have certain limitations. In particular, this type of system can only handle a relative-narrow range of part varieties, so it must be used for similar parts (family of parts) that require similar processing. Due to increased complexity and cost, an FMS also require a large planning and development period than traditional manufacturing equipment.

Equipment utilization for the FMS sometimes is not as high as one would expect.

Other problem can result from a lack of technical literacy, management incompetence, and poor implementation of the FMS process. If the firm misidentifies its objectives and manufacturing missions and does not maintain a manufacturing strategy that is consistent with the firm's overall strategy, problems are inevitable. It is crucial that a firm's technology acquisition decisions be consistent with its manufacturing strategy.

However, an FMS may not be appropriate for some firms. Since new technology is costly and requires several years to install and become productive, it requires a supportive infrastructure and the allocation of scarce resources for implementation. Therefore, it is difficult to give an accurate indication of whether flexible manufacturing is justified.

For other forms, their product may not require processes at the technological level of an FMS. Potential FMS users should also consider that some of the costs traditionally incurred in manufacturing may actually be higher in a flexible automated system than in conventional manufacturing. Although the system is continually self-monitoring, maintenance costs are expected to be higher. Energy costs are likely to be higher despite more efficient use of energy. Increased machine utilization can result in faster deterioration of equipment, providing a shorter than average economic life. Finally, personnel training costs may prove to be relatively high.

Q. 5. (b) What is finite element analysis? Explain how does it works.

Ans. Finite Element Analysis : Finite element analysis consists of a computer model of a material or design that is stressed and analyzed for specific results. It is used in new product design and existing product refinement. A company is able to verify a proposed design will be able to perform to the client's specifications prior to manufacturing or construction. Modifying an existing product or structure is utilized to quality the product or structure for a new service condition. In case of structural failure, FEA may be used to help determine the design modifications to meet the new conditions.

There are generally two types of analysis that are used in industry : 2-D modeling, and 3-D modeling. While 2-D modeling conserves simplicity and allows the analysis to be run on a relatively normal computer, it tends to yield less accurate results. 3-D modeling, however, produces more accurate results while sacrificing the ability to run all but the fastest computers effectively. Within each of there modeling schemes, the programmer can insert numerous algorithms (functions) which may make the system behave linearly or non-linearly. Linear systems are far less complex and generally do not take into account plastic deformation. Non-linear systems do account for plastic deformation and many also are capable of testing a material all the way to fracture.

How does Finite Element Analysis Work?

Finite element analysis uses a complex system of points called nodes which make a grid called a mesh. This mesh is programmed to contain the material and structure properties which define how the structure will react to certain loading conditions. Nodes are assigned at a certain density throughtout the material depending on the anticipated stress levels of a particular area. Regions which will receive large amounts of stress usually have a higher node density than those which experience little or no stress. Points of interest may consist of : fracture point of previously tested material, fillets, corners, complex detail and high stress areas. The mesh acts like a spider web in that from each node, there extends a mesh element to each of the adjacent nodes. This web of vectors is what carries the material properties to the object, creating many elements.

A wide range of objective functions (variable within the system) are available for minimization or maximization :

(i) Mass, volume, temperature

(ii) Strain energy, stress strain

(iii) Force, displacement, velocity, acceleration

(iv) Synthetic (user defined).

There are multiple loading conditions which may be applied to a system :

(i) Thermal loads from solution of heat transfer analysis

(ii) Enforced displacements

(iii) Heat flux and convection

(iv) Point pressure and gravity dynamic loads

Each FEA program may come with an element library or one is constructed over time. Some sample elements are :

(i) Rod elements

(ii) Beam elements

(iii) Plate/shell/composite elements

(iv) Shear panel

(v) Solid elements

(vi) Spring elements

(vii) Mass elements

(viii) Rigid elements

(ix) Viscous damping elements.

Many FEA programs also are equipped with the capability to use multiple materials within the structure such as :

(i) Isotropic, identical throughout.

(ii) Orthotropic, identical at 90 degrees.

(iii) General anistropic, different throughout.

Result of Finite Element Analysis : FEA has become a solution to the task of predicting failure due to unknown stresses by showing problem areas in a material and allowing designers to see all of the theoretical stresses within. This method of product design and testing is far superior to the manufacturing costs which would accrue if each sample was actually built and tested.

Q. 5. (c) How will we illustrate the FEM? Also give general form of FEM.

Ans. We will illustrate the finite element method using two sample problems from which the general method can be extrapolated. It is assumed that the reader is familiar with calculus and linear algebra.

P_1 is a one-dimensional problem

$$P_1 : \begin{cases} u''(x) = f(x) \text{ in } = (0,1) \\ u(0) = u(1) = 0 \end{cases}$$

Where f is given, u is an unknown function of x and u'' is the second derivative of u with respect to x.

The two-dimensional sample problem is the Dirichlet problem

$$P_2 : \begin{cases} u_{xx}(x,y) + u_{yy}(x,y) = f(x,y) \text{ in } \Omega \\ u = 0 \qquad\qquad \text{on } \partial\Omega \end{cases}$$

Where Ω is a connected open region in the (x, y) plane whose boundary $\partial\Omega$ is 'nice' (e.g., a smooth manifold or a polygon), and u_{xx} and u_{yy} denote the second derivatives with respect to x and y, respectively.

The problem P_1 can be solved "directly" by computing antiderivatives.

However, this method of solving the boundary values problem works only when there is only one spatial dimension and does not generalize to higher-dimensional problems or to problems like u + u" = f. For this reason, we will develop finite element method for P_1 and outline its generalization to P_2.

Our explanation will proceed in two steps, which mirror two essential steps one most take to solve a boundary value problem (BVP) using the FEM [Finite Element Method].

(i) In the first step, one rephrases the original BVP in its weak, or variational form. Little to no computation is usually required for this step. The transformation is done by hand on paper.

(ii) The second step is the discretization, where the weak form is discretized in a finite dimensional space.

Variational Formulation : The first step is to convert P_1 and P_2 into their variational equivalents, or weak formulation. If u solves P_1, then for any smooth function v that satisfies the displacement boundary conditions. i.e.,

v = 0 at x = 0 and x = 1, we have

$$\int_0^1 f(x)v(x)dx = \int_0^1 u''(x)v(x)dx$$

Conversely, if u with $u(0) = u(1) = 0$ satisfies (1) for every smooth function v(x) then one may show that this is will solve P_1. The proof is easier for twice continuously differentiable u (mean value theorem), but may be proved in a distributional sense as well.

By using integration by parts on the right-hand-side of (1), we obtain

$$\int_0^1 f(x)v(x)dx = \int_0^1 u''(x)v(x)dx$$

$$= u(x)v(x)|_0^1 - \int_0^1 u'(x)v'(x)dx$$

$$= -\int_0^1 u'(x)v'(x)dx = -\phi(u,v)$$

Where we have used the assumption that $v(0) = v(1) = 0$.

Discretization : A function in H_0^1, with zero values at the end points (blue), and a piecewise linear approximation (red). The basic idea is to replace the infinite dimensional linear problem :

Find $u \in H_0^1$ such that

$$\forall v \in H_0^1, -\phi(u,v) = \int fv$$

With a finite dimensional version :

Find $u \in v$ such that

$$\forall v \in V, -\phi(u,v) = \int fv$$

Where V is a finite dimensional subspace of H_0^1. There are many possible choices for V (one possibility leads to the spectral method). However, for the finite element method we take V to be a space of piecewise polynomial functions.

For problem P_1, we take the interval (0, 1), choose n values of x with $0 = x_0 < x_1 < ...... < x_n < x_{n+1} = 1$ and we define V by

$$V = v:[0,1] \to R : vE \text{ is continuous, } v|_{[x_k, x_{k+1}]} \text{ is linear for } k = 0, ...n \text{ and } v(0) = v1 = 0\}$$

Where we define $x_0 = 0$ and $x_{n+1} = 1$. Observe that functions in V or not differentiable according to the elementary definition of calculus. Indeed, if $v \in V$ then the derivative is typically not defined at any $x = x_k$, $k = 1$ ......, n. However, the derivatives exists at every other value of x and one can use this derivative for the purpose of integration by parts.

A piecewise linear function in two dimensions.

For problem P_2, we need V to be a set of functions of Ω. One often reads V_h instead of V in the literature. The reason is that one hopes that as the underlying triangular grid become finer and finer, the solution of the discrete problem (3) will in some sense converge to the solution of the original boundary value problem P_2. The triangulation is then indexed by a real valued parameter h > 0 which one takes to be very small.

General Form of the Finite Element Method : In general, the finite element method is characterized by the following process :

(i) One choose a grid for Ω. In the preceding treatment, the grid consisted of triangles, but one can also use squares or curvilinear polygons.

(ii) Then, one chooses basis functions.

A separate consideration is the smoothness of the basis functions. For second order elliptic boundary value problems, piecewise polynomial basis function that are merely continuous suffice (i.e., the derivatives are discontinuous). For higher order partial differential equations, one must use smoother basis functions, for instance, for a fourth order problem such as $u_{xxxx} + y_{yyyy} = f$, one may use piecewise quadratic basis functions that are C'. Another consideration is the relation of the finite dimensional space V to its infinite dimensional counterpart. A conforming element method is one in which the space V is a subspace of the element space for the continuous problem. If this condition is not satisfied, we obtain a non-conforming element method, an example of which is the space of piecewise linear functions over the mesh which are continuous at each edge midpoint. Since these functions are in general discontinuous along the edges, this finite dimensional space is not a subspace of the original $H_0^{'}$.

Typically, one has an algorithm for taking a given mesh and subdividing it. If the main method for increasing precision is to subdivide the mesh, one has an h-method (h is customarily the diameter of the largest element in the mesh). In this manner, if one shows that the error with a grid h is bounded above by Ch^p, for some $C < \infty$ and $p > 0$, then one has an order p method.

If instead of making h smaller, one increases the degree of the polynomials used in the basis function, one has a p-method. If one combines these two refinement types, one obtains an hp-method. In the hp-FEM, the polynomial degrees can vary from element to element. High order methods with large uniform p are called spectral finite element method (SFEM).

For vector partial differential equations, the basis functions may tape values in R".