UNIT - 3

COLUMNS

1. Column :-
A vertical member of a structure, which is subjected to axial compressive load and is fixed at both of its ends, is known as column.

2. Strut :-
It is member of a structure which is not vertical or whose one or both of its ends are hinged or pin jointed.

3. Types of stresses causes for failure in column.
* Direct compressive stress
* Buckling stress
* Combined Direct compressive and Buckling stress.

4. Slenderness ratio :-
The ratio of unsupported length (effective length) of the column to its least radius of gyration is known as slenderness ratio.

5. Factors which affect the strength of the column.
* Slenderness ratio
* End conditions.

6. Buckling / crippling.
A long column when subjected to direct load effects in lateral dimensions is known as buckling.

7. Buckling Load / critical load / crippling Load : (P)
The load at which the column just buckles is known as buckling load or critical load or crippling load.

8. Assumptions in Euler's Theory (Long Column).

* The material of the column is homogeneous.
* The section of the column is uniform throughout.
* The column initially straight and loaded axially.
* Effect of direct axial stress is neglected.
* The column fails by buckling only.

9. End conditions in Euler's Theory.

* Both ends hinged. $P = \frac{\pi^2 EI}{l^2}$ $[l_e = l]$
* One end fixed and other end free. $P = \frac{\pi^2 EI}{4l^2}$ $[l_e = 2l]$
* Both ends fixed $P = \frac{4\pi^2 EI}{l^2}$ $[l_e = l/2]$
* One end fixed and other end hinged. $P = \frac{2\pi^2 EI}{l^2}$ $[l_e = l/\sqrt{2}]$

10) Limitations in Euler's Theory :-

* It is not valid for steel column having slenderness ratio < 80
* It does not take the direct stress. But in case of load it can withstand compression only.

11). Crippling load by Euler's formula in terms of Radius gyration.

$$P_{Euler} = \frac{\pi^2 EA}{\left(\frac{l_e}{K}\right)^2} \Rightarrow \frac{\pi^2 EA}{\left(\frac{l_e}{K}\right)^2} \Rightarrow \frac{\pi^2 EI}{l_e^2}$$

$\left[\because K = \sqrt{\frac{I}{}}\right]$

Crippling load by Rankine's formula.

$$P = \frac{f_c \times A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

13) Beam column:

Column having transverse load in addition to the axial compressive load is termed as beam column.

14. Thick cylinders:

Thick cylinders are vessels, containing fluid ~~vessels~~ under pressure and whose wall thickness is not small

$(t \geq d/20)$

15). Assumptions in Lame's equation:

* The material of the column is homogeneous
* Plane section is perpendicular to the longitudinal axis of the cylinder remain plane after the application of internal pressure.
* The material is stressed within the limit.
* All the fibers of the material are free to expand or contract independent without being constrained by the adjacent fibres.

16) Lames equation.

Radial pressure $\sigma_r = \frac{b}{x^2} - a$

Hoop stress. $\sigma_c = \frac{b}{x^2} + a$

a, b constants - obtained from boundary conditions.

* The hoop's stress is maximum at the inner circumference and minimum at the outer circumference of thick cylinder.

18. Reduction of hoop's stress in a thick cylinder ?

It can be reduced by shrinking one cylinder over another cylinder.

19). Thin cylinders :-

Thin cylinders are vessels, containing fluid under pressure and whose wall thickness is less than 1/20 of its internal diameter.

20) Buckling Factor:

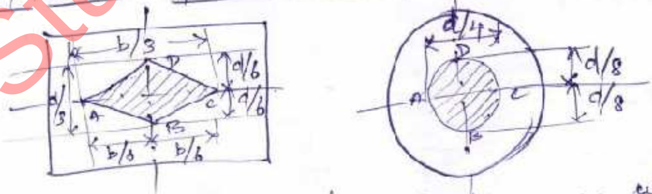
It is the relation between equivalent length and minimum radius of gyration.

21. Safe load :-

It is the load ~~at the~~ to which a column is actually subjected to and is well below the buckling load.

$$\text{Safe Load} = \frac{\text{Buckling load}}{\text{Factor of safety}}$$

22. Core / Kernal of the section :-



kernel of the section is the area within which the line of action of the eccentric load P must cut the c/s if the stress not to become tensile.

Problems:

1. A solid round bar 3m long and 5 cm in diameter is used as a strut with following end conditions. Determine crippling load. Take $E = 2\times10^5\ N/mm^2$.

(i) Both ends hinged.

(ii) one end fixed other end free.

(iii) Both ends fixed.

(iv) one end fixed other end hinged.

Solution:

Given: $l = 3m = 3000mm$

$d = 5cm = 50mm.$

$E = 2\times10^5\ N/mm^2.$

Formula:

$$P_{euler} = \frac{\pi^2 EI}{l_e^2}$$

$$I = \frac{\pi}{64}(d^4)$$

$$= \pi/64\,(50^4) \Rightarrow 30.68\times10^4\ mm^4$$

(i) Both ends Hinged.

$$P_{euler} = \frac{\pi^2 EI}{l_e^2} \quad [l_e = l]$$

$$= \frac{\pi^2 EI}{l^2}.$$

$$P = \frac{\pi^2\times2\times10^5\times30.68\times10^4}{3000^2}.$$

(ii) One end fixed other end free

$$P_{euler} = \frac{\pi^2 EI}{l_e^2} \quad [l_e = 2l].$$

$$P = \frac{\pi^2 EI}{4l^2}.$$

$$= \frac{\pi^2\times2\times10^5\times30.68\times10^4}{4\times3000^2}$$

$$= 16822\ N.$$

(iii) Both end fixed.

$$P_{euler} = \frac{\pi^2 EI}{l_e^2} \quad [l_e = l/2)$$

$$= \frac{4\pi^2 EI}{l^2} = \frac{4\times\pi^2\times2\times10^5\times30.68\times10^4}{3000^2}$$

$$= 269152\ N.$$

(iv) one end fixed, other end hinged.

$$P_{euler} = \frac{\pi^2 EI}{l_e^2} \quad [l_e = l/\sqrt{2})$$

$$= \frac{2\pi^2 EI}{l^2} = \frac{2\times\pi^2\times2\times10^5\times30.68\times10^4}{3000^2}$$

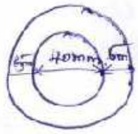
2. A hollow mild steel tube 6 m long 4 cm internal dia and 5mm thick is used as strut with both ends hinged. Find the crippling load and safe load, Taking Factor of Safety as 3. Take $E = 2\times10^5$ N/mm².

Given :-

$l = 6m = 6000 m.$

$d = 4 cm = 40mm$ $= d + 2t$

thick $= 5$ mm $\therefore D = 50mm.$



$FOS = 3$

$E = 2\times10^5$ N/mm².

End condition : Both ends hinged.

$P = ?$

Safe Load $= ?$

Solution :-

(i) $P_{euler} = \frac{\pi^2 EI}{l_e^2}$ $[l_e = l]$ (ii) Safe load.

$= \frac{\pi^2 EI}{l^2}$ $= \frac{\text{Buckling load}}{FOS}$.

$I = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(50^4 - 40^4)$

$= 18.11\times10^4 \text{ mm}^4.$

(i) $\therefore P = \frac{\pi^2 \times 2\times10^5 \times 18.11\times10^4}{6000^2}$.

$P = 9929.9 N \simeq 9930 N$ //

(ii) Safe load.

$= \frac{9930}{3} = 3310 N.$ //

A simply supported beam of length 4 m is subjected to a uniformly distributed load of 30 kN/m over the whole span and deflects 15 mm at the centre. Determine the crippling loads when this beam is used as a column with following conditions:

(i) One end fixed and other end hinged.
(ii) both the ends pin jointed.

Given :-

$l = 4\,m. = 4000\,mm.$
$w = 30\,kN/m. = 30\,N/mm.$
$\delta = 15\,mm.$
$P = ?$

End condition (i) One end fixed & other end hinged.
(ii) both ends pin jointed.

solution

(i) $P_{euler} = \frac{\pi^2 EI}{l_e^2}$ $[l_e = l/\sqrt{2}]$ (ii) $P_{euler} = \frac{\pi^2 EI}{l_e^2}$ $[l_e = l]$

$= \frac{2\pi^2 EI}{l^2}$ $= \frac{\pi^2 EI}{l^2}$

$EI = ?$

$$\delta = \frac{5}{384}\frac{wl^4}{EI}.$$

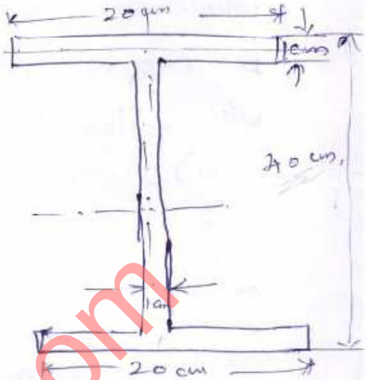
$$EI = \frac{5wl^4}{384\,\delta} = \frac{5 \times 30 \times (4000)^4}{384 \times 15}$$

$$= 2/3 \times 10^3\ N\,mm^2.$$

(i) $P = \frac{2\pi^2 EI}{l^2} = \frac{2 \times \pi^2 \times 2/3 \times 10^3}{4000^2} = 8224.5\ kN.$

(ii) $P = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 2/3 \times 10^3}{4000^2} = 4112.25\ kN.$

(4). Determine Euler's crippling load for an I-section joist 40 × 20 × 1 cm and 5 m long which is used as a strut with both ends fixed. Take $E = 2.1 \times 10^5\ N/mm^2$.



Given :-
40 × 20 × 1 cm.

$l = 5\ m = 5000\ mm.$

$E = 2.1 \times 10^5\ N/mm^2.$

$\cancel{I_{xx} = \frac{1}{12}(20 \times 1^3) + \frac{1}{12} \times 1 \times 38^3}$

$\cancel{2 \times 1.67 + 4572}$

$I_{xx} = \begin{Bmatrix} \text{MoI of Rectangle of dimension} \\ 20 \times 40\ cm. \end{Bmatrix} - \begin{Bmatrix} \text{MoI of Rectangle of} \\ \text{dimension } [20-1,\ 40-1-1] \end{Bmatrix}$

$= \frac{1}{12} bd^3 - \frac{1}{12} b_1 d_1^3$ $\quad \begin{bmatrix} b = 20 & b_1 = 19 \\ d = 40 & d_1 = 38 \end{bmatrix}$

$= \frac{1}{12}(20 \times 40^3) - \frac{1}{12}(19 \times 38^3)$

$= 19786\ cm^4$

$I_{YY} = \begin{Bmatrix} \text{MoI of rectangle of dimension} \\ (38 \times 1) \end{Bmatrix} + \begin{Bmatrix} \text{MoI of two rectangle of} \\ \text{dimension } (1 \times 20) \end{Bmatrix}$

$= \left(\frac{1}{12} \times 38 \times 1^3\right) + 2\left(\frac{1}{12} \times 1 \times 20^3\right)$

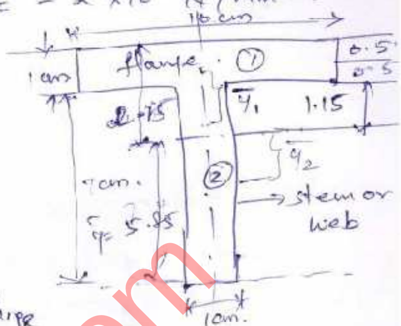
$I_{YY} = 1336.5\ cm^4.$

$\therefore$ Least ~~Moment~~ Moment of Inertia $I_{YY} = 1336.5\ cm^4$
$= 1336.5 \times 10^4\ mm^4.$

$$P_{Euler} = \frac{\pi^2 E I}{l_e^2} = \frac{\pi^2 E I}{(l/2)^2} = \frac{4\pi^2 E I}{l^2}.$$

$$= \frac{4 \times \pi^2 \times 2.1 \times 10^5 \times 1336.5 \times 10^4}{\quad} = 4.432\ MN.$$

Calculate the euler's critical load for a strut of T section the flange width being 10 cm, overall depth 8 cm and both flange and stem 1 cm thick, the strut is 3m long and is built in at both ends. Take $E = 2\times10^5\ N/mm^2$.



Given:

$l = 3m = 3000\ mm$ $\qquad l_e = \frac{l}{2}$

$E = 2\times10^5\ N/mm^2$.

End condi: Both ends fixed.

$\bar{y}$ = Dist of CG from bottom end of flange

Flange: $a_1 = 10\times1 = 10\ cm^2$. $\qquad b_1 = 10\ cm$

$y_1 = 7 + \frac{1}{2} = 7.5\ cm.$ $\qquad d_1 = 1\ cm$

Web: $a_2 = 7\times1 = 7\ cm^2$. $\qquad b_2 = 1\ cm$

$y_1 = 7/2 = 3.5\ cm.$ $\qquad d_2 = 7\ cm.$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1+a_2} = \frac{(10\times7.5)+(7\times3.5)}{10+7} = 5.85\ cm.$$

$$I_{xx} = I_{xx1} + I_{xx2}.$$

$$= \left[\frac{1}{12}b_1d_1^3 + A_1(\bar{y}_1)^2\right] + \frac{1}{12}b_2d_2^3 + a_2(\bar{y}-y_2)^2$$

(See diagram)

$$= \left[\left(\frac{1}{12}\times10\times1^3\right) + 10(2.15-0.5)^2\right] + \left[\frac{1}{12}\times1\times7^3 + 7(5.85-3.5)^2\right]$$

$$= 95.298\ cm^4.$$

From diagram $\left[\bar{y}_1 = 2.15 - 0.5 = 1.15 + 0.5,\ \bar{y}_2 = 5.85 - 3.5\right]$

$$I_{yy} = \frac{1}{12}d_1b_1^3 + \frac{1}{12}d_2b_2^3$$

$$= \left[\frac{1}{12}\times1\times10^3\right] + \left[\frac{1}{12}\times7\times1^3\right]$$

$$= 83.916\ cm^4.$$

Least Moment of Inertia = $I_{yy} = 83.916\ cm^4 = 83.916\times10^4\ mm^4$

$$P = \frac{\pi^2EI}{l_e^2} = \frac{4\pi^2EI}{l^2} = \frac{4\times\pi^2\times2\times10^5\times83.916\times10^4}{l^2} = 736190\ N$$

(6) The external and internal diameter of a hollow cast column are 5cm and 4cm respectively. If the length of column is 3m and both of its ends are fixed. Determine the crippling load using Rankine's formula. Take values of $f_c = 550\ N/mm^2$. and $a = \frac{1}{1600}$ in Rankine's formula

Given:

$D = 5\ cm = 50\ mm$

$d = 4\ cm = 40\ mm$

$l = 3\ m = 3000\ mm$

$f_c = 550\ N/mm^2$

$a = \frac{1}{1600}$

End condition: both ends fixed. $l_e = l/2$.

Solution:

$$P = \frac{f_c \cdot A}{1 + a\left[\frac{l_e}{k}\right]^2} = \frac{f_c \cdot A}{1 + a\left[\frac{l_e}{K}\right]^2}$$

$$A = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}[50^2 - 40^2] = 706.5\ mm^2$$

$$k = \sqrt{\frac{I}{A}}$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{64}(50^4 - 40^4) = 181039.84\ mm^4$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{181039.84}{706.5}} = 25.625\ mm$$

$$P = \frac{550 \times 706.5}{1 + \left(\frac{1}{1600}\right)\left[\frac{\frac{3000}{2}}{25.625}\right]^2} = 123750\ N$$

(7) A hollow cylindrical cast iron column is 4m long with both ends fixed. Determine the minimum diameter of the

...lumn if it has to carry a safe load of 250 kN with a factor of safety of 5. Take the internal diameter as 0.8 times the external diameter. Take $f_c = 550\ N/mm^2$. and $a = \frac{1}{1600}$ in Rankine's formula.

Given :-

$l = 4\ m = 4000\ mm.$

End condi = Both ends fixed.

$l_e = l/2 = \frac{4000}{2} = 2000\ mm.$

Safe load = 250 kN.

FOS = 5

Ext dia = D

internal dia = $d = 0.8D$

$f_c = 550\ N/mm^2.$

$a = \frac{1}{1600} = 0.000625$

Solution

$$P = \frac{f_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

$$\text{Safe load} = \frac{P}{FOS}; \quad P \Rightarrow \text{Safe load} \times FOS.$$

$$= 250 \times 5$$

$$= 1250\ kN. = 1250000\ N.$$

$$A = \pi/4\,(D^2 - d^2)$$

$$= \pi/4\left[D^2 - (0.8D)^2\right] \Rightarrow \pi/4 \times 0.36 D^2 \Rightarrow 0.2826 D^2$$

$$K = \sqrt{\frac{I}{A}}.$$

$$I = \pi/4\,(D^4 - (0.8D)^4) = \pi/4\,(D^4 - 0.4096 D^4) = \pi/4 \times 0.5904 D^4$$

$$= 0.4634 D^4.$$

$$K = \sqrt{\frac{0.4634 D^4}{0.2826 D^2}} = \sqrt{1.6397 D^2} = 1.2805 D$$

$$P = \frac{f_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{550 \times 0.2826 D^2}{1 + \left(0.000625 \cdot \left(\frac{2000}{1.2805 D}\right)^2\right)}$$

$$1250000 = \frac{550 \times 0.2826 D^2}{1 + \left[0.000625 \left(\frac{1561^2}{D^2}\right)\right]} \Rightarrow 8038 = \frac{D^2 \times D^2}{D^2 + 2441.4}$$

$$8038D^2 + 8038 \times 24414 = D^4$$

$$D^4 - 8038\,D^2 - 8038 \times 24414 = 0$$

$x = D^2$;

$$D^2 = \frac{8038 \pm \sqrt{8038^2 + 4 \times 1 \times 196239700}}{2 \times 1}$$

$$D^2 = 18592.5 \text{ mm}^2$$

$$\boxed{D = 136.3 \text{ mm}}$$

$$d = 0.8D \Rightarrow 0.8 \times 136.3 = 109 \text{ mm}$$

$$\boxed{d = 109 \text{ mm}} \;//$$

(8) A hollow CI column whose outside diameter is 200 mm has a thickness of 200 mm. It is 4.5 m long and is fixed at both ends. Calculate the safe load by Rankine's formula using a factor of safety of 4. Calculate the slenderness ratio and ratio of Euler's and Rankine's critical loads. Take $f_c = 550\ N/mm^2$; $a = \frac{1}{1600}$ in Rankine's formula & $E = 9.4 \times 10^4\ N/mm^2$.

Given:-

$D = 200$ mm

$t = 200$ mm. $\quad d = D - 2 \times t = 160$ mm.

$l = 4.5$ m $= 4500$ mm.

End con = fixed at both ends $= l_e = \frac{l}{2}$.

FOS = 4

Safe load = ?

Slenderness ratio $= \frac{l}{k} = ?$

Ratio $= \frac{P_{Euler}}{P_{Rankine}} = ?$

$f_c = 550\ N/mm^2$

$a = \frac{1}{1600}$

$P_{Euler} = \frac{\pi^2 EI}{l_e^2} \quad \left(l_e = \frac{l}{2}\right)$

$= \frac{4\pi^2 EI}{l^2}$

$P_{Rank} = \frac{f_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2}$

Solution

$A = \pi/4 (D^2 - d^2) = \pi/4 (200^2 - 160^2) = 11310 \text{ mm}^2$.

$I = \pi/64 (D^4 - d^4) = \pi/4 (200^4 - 160^4) = 46370000 \text{ mm}^4$

$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{46370000}{11310}} = 64 \text{ mm}.$

$l = 4500 \text{ mm}.$

$l_e = 2250 \text{ mm}$

(i) slenderness ratio.

$\frac{l}{k} = \frac{4500}{64} = 70.3$

(ii) Safe load by Rankine's load.

$P = \frac{f_c \cdot A}{1 + a\left(\frac{l_e}{k}\right)^2} = \frac{550 \times 11310}{1 + \frac{1}{1600}\left(\frac{2250}{64}\right)^2}$

$P = 351100 \text{ N}.$

Safe load $= \frac{P}{FOS} = \frac{351100}{4} = 87775 \text{ N}$

(iii) Ratio of Euler's & Rankine's load.

$P_{Euler} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 \times 9.4 \times 10^4 \times 4.637 \times 10^7}{2250}$

$= 849770 \text{ N}.$

$\frac{P_{Euler}}{P_{Rankine}} : \frac{\text{Euler's critical load}}{\text{Rankine's critical load}} = \frac{849770}{351100} = 2.42$

(9) Determine the maximum and minimum hoop stress across the section of a pipe of 400mm external diameter and 100mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section.

Given :-

$d_2 = 400$ mm. $P_0 = 8$ N/mm².

$r_2 = 200$ mm.

$t = 100$ mm

$d_1 = 600$ mm.

$r_1 = 300$ mm

Radia Pressure :

$$\sigma_r = \frac{b}{r^2} - a. \Rightarrow \sigma_{r_1} = \frac{b}{r_1^2} - a \qquad \sigma_{r_1} = 0$$

$$\sigma_{r_2} = \frac{b}{r_2^2} - a. \qquad \sigma_{r_2} = 8$$

$$\sigma_{r_1} = \frac{b}{r_1^2} - a. \qquad \sigma_{r_2} = \frac{b}{r_2^2} - a.$$

$$0 = \frac{b}{300^2} - a \rightarrow (1) \qquad 8 = \frac{b}{200^2} - a. \rightarrow (2)$$

Solving (1) & (2) ; $a = 6.4$

$b = 576000$.

Hoop stress. $= \sigma_c = \frac{b}{r^2} + a.$

$$\sigma_{c_1} = \frac{b}{r_1^2} + a.$$

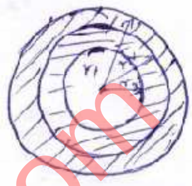
$$\sigma_{c_1 (300)} = \frac{576000}{300^2} + 6.4. = 12.8 \text{ N/mm}^2.$$

$$\sigma_{c_2} = \frac{b}{r_2^2} + a.$$

$$(200) = \frac{576000}{200^2} + 6.4 = 20.8 \text{ N/mm}^2.$$

compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250 mm over another cylinder of external diameter 250 mm and internal diameter 200 mm. The radia pressure at the junction after shrinking is 8 N/mm2. Find the final stresses set up across the section, when the compound cylinder is subjected to an external fluid pressure of 84.5 N/mm^2.

Given :-



For outer cylinder :

Ext dia = 300 mm

$r_1 = \frac{300}{2} = 150$ mm.

Int dia = 250 mm.

$r^* = 125$ mm.

For inner cylinder :

Int dia = 200 mm

$r_2 = 100$ mm.

Radial pressure due to shrinking

$P^* = 8$ N/mm^2.

Fluid pressure in compound cylinder

$P = 84.5$ N/mm2.

(i) Stresses due to shrinking in the outer & inner cylinder before the fluid pressure is admitted.

We know Lame's equation

$$\sigma_r = \frac{b_1}{r^2} - a_1 \; ; \quad \sigma_c = \frac{b_1}{r^2} + a_1$$

$$\sigma_{r_1} = \frac{b_1}{r_1^2} - a_1 \rightarrow \left[\begin{matrix} \sigma_{r_1} = 0 & r_1 = 150 \text{ mm} \\ \sigma_{r^*} = 8 & r_* = 125 \text{ mm} \end{matrix}\right]$$

$$\sigma_{r_*} = \frac{b_1}{r_*^2} - a_1.$$

$$0 = \frac{b_1}{150^2} - a_1 \rightarrow ① \qquad \text{solving ① \& ②}$$

$$a_1 = 18.18 \; ; \; b_1 = 409090.9$$

$$8 = \frac{b_1}{125^2} - a_1 \rightarrow ②$$

$$\therefore \quad \sigma_{c_1} = \frac{b_1}{r_1^2} + a_1. \qquad \sigma_{c_2} = \frac{b_1}{r^{*2}} + a_1$$

$$\sigma_{(150)} = \frac{409090.9}{150^2} + 18.18 = 36.36 \text{ N/mm}^2.$$

$$\sigma_{125} = \frac{409090}{125^2} + 18.18 = 44.36 \text{ N/mm}^2.$$

(b) Lame's eqn. for inner cylinder are.

$\sigma_r = \frac{b_2}{r^2} - a_2$; $\sigma_c = \frac{b_2}{r^2} + a_2$

$\sigma_{r_2} = \frac{b_2}{r_2^2} - a_2$

$\sigma_{r_*} = \frac{b_2}{r_*^2} - a_2$

$\begin{bmatrix} \sigma_{r_2} = 0 & \sigma_{r_*} = 8 \\ r_2 = 100 & r_* = 125 \end{bmatrix}$

($\sigma_{r_2} = 0$ There is no fluid under pressure)

$0 = \frac{b_2}{100^2} - a_2 \rightarrow (3)$

$8 = \frac{b_2}{125^2} - a_2 \rightarrow (4)$

solving (3) & (4). $a = -22.22$, $b = -222222$

sub (a) & b in σ_c.

Hoop stress $\sigma_{125} = \frac{-222222}{125^2} - 22.22$

$= -36.22$ (compressive)

$\sigma_{100} = \frac{-222222}{100^2} - 22.22$

$= -44.44$ (compressive)

(iii) stress due to fluid pressure alone :- [when fluid under pressure two cylinders are considered as one single unit]

$\sigma_r = \frac{B}{r^2} - A$ $\qquad$ $\sigma_c = \frac{B}{r^2} + A$.

$\sigma_{r_1} = \frac{B}{r_1^2} - A$.

$\sigma_{r_2} = \frac{B}{r_2^2} - A$.

$\begin{bmatrix} \sigma_{r_1} = 0 & \sigma_{r_2} = 84.5 \\ r_1 = 150 & r_2 = 100 \end{bmatrix}$

$0 = \frac{B}{150^2} - A \rightarrow (5)$

$84.5 = \frac{B}{100^2} - A \rightarrow (6)$

solving (5) & (6)

$A = 67.6$

$B = 1521000$

sub A & (2) in σ_c.

Hence Hoop stresses due to internal fluid pressure alone is given by,

$$f_{100} = \frac{1542000}{100^2} + 67.6.$$

$$= 219.7 \ N/mm^2 \ (tensile)$$

$$f_{125} = \frac{1521000}{125^2} + 67.6$$

$$= 164.94 \ N/mm^2$$

$$f_{150} = \frac{1521000}{150^2} + 67.6$$

$$= 135.2 \ N/mm^2.$$

Resultant stresses will be the algebric sum of the initial stresses due to shrinkage and those due to internal fluid pressure.

Inner cylinder:-

$f_{100} = f_{100}$ due to shrinkage $+ f_{100}$ due to internal fluid pressure.

$$= -44.44 + 219.7$$

$$= 175.26 \ N/mm^2 \ (tensile)$$

$$f_{125} = -36.22 + 164.94.$$

$$= 128.72 \ N/mm^2 \ (tensile)$$

Outer cylinder.

$$F_{125} = 44.36 + 164.94.$$

$$= 209.3 \ N/mm^2. \ (tensile)$$

$$F_{150} = 36.36 + 135.2.$$

$$= 171.56 \ N/mm^2 \ (tensile).$$

(11) A steel cylinder of 300mm external diameter is to be a sh to another steel cylinder of 150 mm external diameter. After shrinking the diameter at the junction is 250mm and radial pressure at the common junction is 28 N/mm². Find the original difference in radii at the junction. Take $E = 2\times10^5$ N/mm²

Solution:

Given: ext dia of outer cylinder = 300 mm.

$r_1 = 150$ mm

Int dia of inner cylinder = 150 mm

$r_2 = 75$ mm

Diameter at the junction = 250 mm

$r^* = 125$ mm.

$p^* = 28$ N/mm².

$E = 2\times10^5$

Original difference of radii at junction:

$$\boxed{= \frac{2r^*}{E}(a_1 - a_2)}$$

For outer cylinder.

$$\sigma_r = \frac{b_1}{r^2} - a_1 \qquad 0 = \frac{b_1}{150^2} - a_1 \rightarrow (1)$$

$$\sigma_{r_1} = \frac{b_1}{r_1^2} - a_1 \quad \begin{bmatrix} \sigma_{r_1} = 0 \\ \sigma_{r_*} = 28 \\ r_1 = 150 \\ r_* = 125 \end{bmatrix} \qquad 28 = \frac{b_1}{125^2} - a_1 \rightarrow (2)$$

$$\sigma_{r_*} = \frac{b_1}{r_*^2} - a_1$$

Solving (1) & (2), $a = 63.6$, $b_1 = 1432000$.

For inner cylinder:

$$\sigma_{r_*} = \frac{b_2}{r_*^2} - a_2 \quad \begin{bmatrix} r_2 = 75 & \sigma_{r_2} = 0 \\ r_* = 125 & \sigma_{r_*} = 28 \end{bmatrix} \qquad 28 = \frac{b_2}{125^2} - a_2 \rightarrow (3)$$

$$\sigma_{r_2} = \frac{b_2}{r_2^2} - a_2 \qquad 0 = \frac{b_2}{75^2} - a_2 \rightarrow (4)$$

Solving (3) & (4) $a_2 = -43.75$, $b_2 = +246100$

Difference of radii at junction.

$$= \frac{2r^*}{E}(a_1 - a_2) \Rightarrow \frac{2\times125}{2\times10^5}\left[63.6 - (-43.75)\right]$$

$$= 0.13 \text{ mm.}$$