**EAS401**

(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 3987**     **Roll No.** ☐☐☐☐☐☐☐☐☐☐☐

# B. Tech.

## (SEM. IV) THEORY EXAMINATION 2011-12

## MATHEMATICS—III

*Time : 3 Hours*                                   *Total Marks : 100*

**Note :** (1)  Attempt *all* questions. Marks are indicated against each question.

(2)  Provide the statistical tables which are required by the students.

1.  Attempt any *four* parts of the following :                (4×5=20)

(a)  Define analytic function. Discuss the analyticity and differentiability of $f(z) = |z|^4$ at $z = 0$.

(b)  Define conjugate harmonic function. Find the analytic function $f(z) = u + iv$ whose real part is $u(x, y) = (x - y)(x^2 + 4xy + y^2)$.

(c)  Evaluate $\oint_C \dfrac{1}{z^2 \sin z}\, dz$, where C is the triangle with vertices $(0, 1)$, $(2, -2)$ and $(7, 1)$.

(d)  Verify Cauchy's theorem for $f(z) = z^2 + 3z + 2$ where C is the perimeter of square with vertices $1 \pm i$, $-1 \pm i$.

(e)  Expand $f(z) = \dfrac{z}{(z^2 - 1)(z^2 + 4)}$ in Laurent series in $1 < |z| < 2$.

(f)  Use contour integration to evaluate :

$$\int_0^\infty \frac{x^2\, dx}{(x^2 + 1)^2}.$$

2. Attempt any *four* parts of the following : **(4×5=20)**

   (a) Define moment generating function. Find the moment generating function of a random variable X whose probability function is given by :

   $$P(X = x) = p(1 - p)^x, x = 0, 1, 2, ....., \infty.$$

   (b) In two frequency distributions the second moments about mean are 36 and 49 respectively while third moments about mean are 43.2 and 85.75. Compare the skewness in the two frequency distributions.

   (c) Fit the curve $y = ae^{bx}$ to the following data :

   | x : | 1 | 2 | 3 | 4 | 5 | 6 |
   |---|---|---|---|---|---|---|
   | y : | 15.3 | 20.5 | 27.4 | 36.6 | 49.1 | 65.6 |

   (d) Calculate Karl Pearson's coefficient of correlation for the following data :

   | x : | 11 | 10 | 09 | 08 | 07 | 06 | 05 |
   |---|---|---|---|---|---|---|---|
   | y : | 20 | 18 | 12 | 8 | 10 | 5 | 4 |

   (e) The line of regression of marks in Statistics (x) on the marks in Accountancy (y) for a class of 50 students is $3y - 5x + 180 = 0$. Average marks in Accountancy is 44 and variance of marks in Statistics is $\frac{9}{16}$ of variance of marks in Accountancy. Find (i) the average marks in Statistics and (ii) coefficient of correlation between marks of both subjects.

   (f) A candidate is called for an interview by three companies. For the first company, there are 12 candidates and for the second there are 15 candidates and for the third, there are 10 candidates. What are the chances of his getting at least one of the companies ?

3. Attempt any *two* parts of the following : **(2×10=20)**

   (a) In a Poisson distribution with unity mean, show that the mean deviation from mean is $(2/e)$ times the standard deviation.

(b) The figures of production (in Lakh tonne) of a sugar factory are given below :

| Year | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
|------|------|------|------|------|------|------|------|
| Production | 7.7 | 8.8 | 9.4 | 8.5 | 9.1 | 9.8 | 9.0 |

Fit a straight line by the method of least square and show the trend values. Also determine the monthly increase in production.

(c) The areas are of two types, urban and rural and two parties A and B participated in the election. The random sample consists of 33 individuals from rural areas of whom 18 voted in favour of party A and 27 individuals from urban areas of whom 12 voted in favour of party A. Use Chi-square test to examine whether the nature of area is related to voting preference. (Given that the value of Chi-square at 5% for 1 d.f. = 3.84).

4. Attempt any *four* parts of the following : (4×5=20)

   (a) Find the smallest positive root of $x^4 - x = 10$ correct to three decimal places using Regula-Falsi method.

   (b) Use Newton-Raphson method to find the root of $3x - \log_{10} x = 6$, correct to four decimal places.

   (c) Define central difference operator, shift operator and the average operators. Establish $\mu\delta = \frac{1}{2}(E - E^{-1})$.

   (d) Fit a polynomial of degree 3 and hence determine $y(3.5)$ for the following data :

   | x : | 3 | 4 | 5 | 6 |
   |-----|---|---|---|---|
   | y : | 6 | 24 | 60 | 120 |

   (e) Use Newton's Divided difference formula to compute $f(8)$ for the following data :

   | x : | 3 | 7 | 9 | 10 |
   |-----|---|---|---|----|
   | f(x): | 168 | 120 | 72 | 63 |

(f) Use Lagrange's interpolation formula to evaluate y(9.5) from the following data :

| x : | 7 | 8 | 9 | 10 |
|-----|---|---|---|----|
| y : | 3 | 1 | 1 | 9  |

5. Attempt any *two* parts of the following : **(2×10=20)**

(a) Factorize the matrix :

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 7 & 1 & -5 \\ 3 & 7 & 4 \end{bmatrix}$$

in the form LU, where L is the lower triangular matrix and U is the unit upper triangular matrix and hence solve the system of equations :

$$5x - 2y + z = 4;\ 7x + y - 5z = 8;\ 3x + 7y + 4z = 10.$$

(b) (i) Find f'(5) from the following table :

| x : | 0 | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|---|
| f(x) : | 4930 | 5026 | 5122 | 5217 | 5312 | 5407 |

(ii) The acceleration of a rocket which is launched from the ground is given according to time t as follows :

| t(sec.) | 0 | 10 | 20 | 30 | 40 |
|---------|---|----|----|----|----|
| f(m/sec²) | 30 | 31.63 | 33.34 | 35.47 | 37.75 |

| t(sec.) | 50 | 60 | 70 | 80 |
|---------|----|----|----|----|
| f(m/sec²) | 40.33 | 43.25 | 46.69 | 50.67 |

Find the velocity of the rocket at t = 80 sec using Simpson's 1/3 rule.

(c) Find the value of y(1.1) using Runge-Kutta method of fourth order for the differential equation :

$$\frac{dy}{dx} = y^2 + xy,\ y(1) = 1.0$$

Take h = 0.05.