(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 3987 Roll No.

B.Tech.

(SEM IV) EVEN SEMESTER THEORY EXAMINATION, 2009-2010

MATHEMATICS - III

Time : 3 Hours *Total Marks : 100*

Note : *(i) Attempt **ALL** questions.*

*(ii) Each question carries **equal** marks.*

1. Attempt **any four** parts of the following : **(4x5=20)**

(a) Show that the function $u = \frac{1}{2}\log(x^2+y^2)$ is harmonic and find its harmonic.

(b) Using Cauchy's integral formula, evaluate $\int_c \frac{\sin\pi z^2 + \cos\pi z^2}{(z-1)(z-2)}\,dz$ where c is $|z|=3$.

(c) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in Laurent's series valid for the regions :

(i) $1<|z|<2$

(ii) $0<|z-1|<1$

(d) Using complex integration method, evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta}\,d\theta$.

(e) Use contour integration method to evaluate $\int_0^{\infty} \frac{x\sin x}{x^2+a^2}\,dx$, $a>0$.

(f) Evaluate $\int_0^{1+i} (x^2 - iy)\,dz$ along the path $y=x$ and $y=x^2$.

2. Attempt **any four** parts of the following : **(4x5=20)**

(a) The equation $f(x)=(x-1)^2\,(x-3)^2$ has roots at $x=1$ and $x=3$. Which of the following methods can be applied to find all the roots ?

(i) Bisection method

(ii) False - position method

(iii) Newton - Raphson method

Justify your answer.

(b) Prove that the Newton Raphson method is second order convergent.

(c) Perform five iteration of false position method to compute the smallest positive root of the equation $3x+\sin x-e^x=0$.

(d) Obtain the value of $f(3.5)$ from the following data :

x	3	4	5	6	7
$f(x)$	3	6.6	15	22	35

(e) Use Newton's divided difference method to compute $f(5.5)$ from the following data :

x	0	1	4	5	6
$f(x)$	1	14	15	6	3

(f) Find the missing terms of the following data :

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	6	?	10	20	?	15	5

3. Attempt **any two** parts of the following : **(2x10=20)**

(a) Find $f'(1.1)$ and $f''(1.1)$ from the following table :

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.0	0.128	0.554	1.296	2.432	4.000

(b) Derive the formula for Simpson's $3/8$ rule and find the value of the integral

$$\int_0^1 \frac{dx}{1+x^2}.$$

Taking 12 intervals.

(c) Using Runge - Kutta fourth order method to solve the following differential equation $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1.0$ at $x=0.2,\ 0.4$.

4. Attempt **any two** parts of the following : **(2x10=20)**

(a) Fit a relation $y = ax + \frac{b}{x}$ which satisfies the following data, using method of least squares :

x	1	2	3	4	5	6	7	8
y	5.4	6.2	8.2	10.3	12.6	14.8	17.2	19.5

(b) What do you mean by regression analysis, explain ? If for two random variables, x and y with the same mean, the two regression equations are $y = ax + b$ and $x = \alpha y + \beta$

show that $\frac{b}{\beta} = \frac{1 - a}{1 - \alpha}$.

Also find the common mean.

(c) Let the random variable X assume the value 'n' with the probability law $p(X = n) = pq^{n-1}$, $n = 1, 2, 3...$

Find the moment generating function and hence mean and variance.

5. Attempt **any two** parts of the following : **(2x10=20)**

(a) What is chi-square test ? A survey of 320 families with 5 children show the following distribution :

Number of boys & girls	5 boys 0 girls	4 boys 1 girl	3 boys 2 girls	2 boys 3 girls	1 boy 4 girls	0 boys 5 girls
Number of families	18	56	110	88	40	8

(Given $\chi^2_{0.05} = 11.1$ for 5 d.f.)

Test the hypothesis that male and female births are equally probable.

(b) Distinguish between the np-chart, p-chart and e-chart of quality control analysis. The average percentage of defectives in 27 samples of size 1500 each was found to be 13.7%. Construct p-chart and give your conclusion regarding quality control.

(c) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of this distribution ?

(Note : Ask for the table of area under normal curve from c.s.)

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