(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618 Roll No. | | | | | | | | | | |

B. Tech.

(SEM. III) THEORY EXAMINATION 2011-12

MATHEMATICS—III

Time : 3 Hours *Total Marks : 100*

Note :– Attempt **all** questions. Provide the statistical tables which are required by students.

1. Attempt any **four** parts of the following : **(5×4=20)**

(a) Define harmonic function. Show that the function $v = \log(x^2 + y^2) + x - 2y$ is harmonic. Also find the analytic function $f(z) = u + iv$.

(b) If $f(z)$ is an analytic function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left|\operatorname{Re} f(z)\right|^2 = 2\left|f'(z)\right|^2.$$

(c) Find the value of the integral $\oint_C \frac{\exp(i\pi z)}{2z^2 - 5z + 2}\,dz$, where C is the unit circle with centre at the origin.

(d) Verify Cauchy's Theorem for the function $f(z) = e^{iz}$ along the boundary of the triangle with vertices at the points $1 + i$, $-1 + i$ and $-1 - i$.

(e) Find the Laurent series expansion of $f(z) = \dfrac{7z - 2}{z(z+1)(z+2)}$ in the region $1 < |z+1| < 3$.

(f) Using complex integration, evaluate :

$$\int_0^{2\pi} \frac{1}{a + b\sin\theta} d\theta,\ a > |b|$$

2. Attempt any **two** parts of the following : **(10×2=20)**

(a) Find the mean of the binomial distribution. Also find the moment generating function of the binomial distribution about its mean.

(b) Obtain the normal equations to fit the curve $y = ax^2 + b$ to the given data (x_i, y_i), $i = 1, 2, \ldots, n$. Hence fit the above curve to the following data :

x	10	20	30	40	50
y	8	10	15	21	30

(c) Calculate the correlation coefficient between x and y for the following data :

x	21	23	30	54	57	58	72	78	87	90
y	60	71	72	83	110	84	100	92	113	135

3. Attempt any **two** parts of the following : **(10×2=20)**

(a) The daily wages of 1000 workers are distributed around a mean of Rs. 140 and with a standard deviation of Rs. 10. Estimate the number of workers whose daily wages will be (i) between Rs. 140 and Rs. 144, (ii) less than Rs. 126, (iii) more than Rs. 160.

(b) Out of 8000 graduates in a town, 800 are females, out of 1600 graduate employees, 120 are females. Use χ^2-test to determine if any distinction is made in appointment on the basis of sex. The value of χ^2 for 1 degree of freedom at 5% level is 3.841.

(c) A quality control engineer of a tyre company collected the following data. For each of 12 batches of 1000 tyres, he tested 5 tyres and recorded the following results with $\bar{x}$ and R meaured in thousands of kilometer :

Batch	1	2	3	4	5	6	7	8	9	10	11	12
$\bar{x}_i$	51.5	49.7	50.0	50.7	50.7	50.6	49.8	51.1	50.2	50.4	50.6	50.7
R	1.5	1.6	1.8	0.1	0.9	2.1	0.3	0.8	2.3	1.3	2.0	2.1

Draw the control chart. Is the production process in control ? Explain.

4. Attempt any **four** parts of the following : **(5×4=20)**

(a) Find a real root of $\cos x - xe^x = 0$ correct to three decimal places using Bisection method.

(b) Define rate of convergence of Newton-Raphson method. Find a positive root of $x^4 = 10 + x$ correct to four decimal places using this method.

(c) Prove that (i) $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$, (ii) $\delta^2 E = \Delta^2$, where symbols have their usual meaning for finite differences.

(d) Obtain Newton's divided difference interpolating polynomial and hence find f(5) from the given data :

x :	3	7	9	10
f :	168	120	72	63

(e) Given the data f(1) = 4, f(2) = 5, f(7) = 5, f(8) = 4. Compute f(6) using Lagrange' interpolation formula.

(f) Find a polynomial of degree three using Newton-Gregory backward difference formula which takes the following values. Hence find y(7) :

x :	3	4	5	6
y :	6	24	60	120

5. Attempt any **two** parts of the following : **(10×2=20)**

(a) Define diagonally dominant system of equations. Solve the following system of equations using Gauss-Seidal method :

$$10x + 15y + 3z = 14$$

$$-30x + y + 5z = 17$$

$$x + y + 4z = 3.$$

(b) Apply Runge-Kutta method of fourth order to solve the initial value problem:

$$5\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$$

and find y in the interval $0 \le x \le 0.2$, taking $h = 0.1$.

(c) (i) Compute y'(1.1) from the following table :

x	1.0	1.1	1.2	1.3	1.4	1.5
y(x)	7.989	8.403	8.781	9.129	9.451	9.750

(ii) Find the distance between two stations from the following data consisting of the speeds v(t) of an electric train at various times t after leaving one station until it stops at the next station. Apply Simpson's rule :

v (miles/hr)	0	13	33	39.5	40	40	36	15	0
t (min.)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0