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(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9618 Roll No.

B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10

MATHEMATICS-III (SCIENCE BASED)

Time : 3 Hours] *[Total Marks : 100*

Note : *Attempt* ***all*** *the questions.*

1 Attempt any **four** parts of the following : 5×4=20

(a) Define a harmonic function and conjugate harmonic function. Find the harmonic conjugate function of the function $U(x, y) = 2x(1 - y)$.

(b) Evaluate $\int_C (12z^2 - 4iz)$ along the curve C joining the points (1, 1) and (2, 3).

(c) State the Cauchy's integral formula. Show that

$$\int_C \frac{e^{zt}}{z^2+1}\,dz = \sin t$$

if $t > 0$ and C is the circle $|z| = 3$.

(d) Define the Laurent series expansion of a function. Expand $f(z) = e^{z/z-2}$ in a Laurent series about the point $z = 2$.

(e) Using Residue theorem, evaluate

$$\frac{1}{2\pi i}\int_C \frac{e^{zt}\,dz}{z^2(z^2+2z+2)}$$

where C is the circle $|z|=3$.

(f) Show that $\int_0^{2\pi} \frac{\cos 3\theta\, d\theta}{5-4\cos\theta} = \frac{\pi}{12}$.

2 Attempt any **two** parts of the following : **10×2=20**

(a) The first four moments of a distribution about the value '0' are -0.20, 1.76, -2.36 and 10.88. Find the moments about the mean and measure the kurtosis.

(b) Fit a second degree parabola to the following data

x	1	2	3	4	5	6	7	8	9	10
y	124	129	140	159	228	289	315	302	263	210

(c) In a partially destroyed laboratory record of an analysis of correlation data, the following results only are legible. Variance of $x=9$.

Regression lines

$$8x-10y+66=0$$

$$40x-18y-214=0$$

What were

(i) The mean value of x and y

(ii) The standard deviation of y and

(iii) The coefficient of correlation between x and y.

3 Attempt any **two** parts of the following : **10×2=20**

(a) To test the effectiveness of inoculation against chotera, the following table was obtained.

	Attacked	*Not attacked*	*Total*
Inoculated	30	160	190
Not inoculated	140	460	600
Total	170	620	790

(The figures represent the number of persons).

Use χ^2 -test to Defend or refute the statement. The inoculation prevents attack from cholera.

(b) If there are 3 misprints in a book of 1000 pages, find the probability that a given page will contain (i) no misprint (ii) more than 2 misprints.

(c) Show that the mean deviation from the mean of the normal distribution is about $\frac{4}{5}$ of its standard deviation.

4 Attempt any **two** parts of the following : **10×2=20**

(a) Derive the Newton-Raphson formula for finding a root of a non-linear equation. Find a root of

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0$$

up to 10 iterations.

(b) Define the shift operator, forward and backward difference operators, the central difference operator and the average operators. Establish :

(i) $\left(E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right) = 2\left(1 + \frac{1}{2}\Delta\right)(1+\Delta)^{1/2}$

(ii) $\Delta(1+\Delta)^{-1/2} = \nabla(1-\nabla)^{-1/2}$

(iii) $\mu = \sqrt{\left(1+\frac{1}{4}\delta^2\right)}$

where all the above notations have usual meanings.

(c) Develop the divided-difference table from the data given below and obtain the interpolation polynomial $f(x)$:

x	1	3	5	7	11
$f(x)$	5	11	17	23	29

also, find the value of $f(19.5)$.

5 Attempt any **two** parts of the following : **10×2=20**

(a) Describe a method for solving a system of linear equations. Solve the following system of linear equations using Gauss-Seidel method :

$$23x_1 + 13x_2 + 3x_3 = 29$$
$$5x_1 + 23x_2 + 7x_3 = 37$$
$$11x_1 + x_2 + 23x_3 = 43$$

(b) Derive Simpson's $\frac{3}{8}$ – formula for numerical integration. Using this rule evaluate $\int_0^1 \frac{dx}{x^3+x+1}dx$. Choose with steplength 0.25.

(c) Solve the following initial value problem

$$\frac{dy}{dx} = -2xy^2,\ y(0) = 1$$

with $h = 0.2$ on interval [0, 0.6] using fourth order Runge-Kutta method. Compare with the exact solution.
