



(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958

Roll No.

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B.Tech

(SEM III) ODD SEMESTER THEORY EXAMINATION 2009-10
MATHEMATICS -III

Time : 3 Hours]

[Total Marks : 100

Note : Attempt *all* questions. Every question carries *equal* marks.

1 Attempt any **two** of the following : **10×2=20**

(a) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$$

Using this result evaluate

$$\int_{-\infty}^{\infty} \frac{\sin at \cos at}{t} dt.$$

(b) State and prove the convolution theorem for the Fourier transform. Verify this theorem for the functions

$$f(t) = e^{-t} \text{ and } g(t) = \sin t.$$

(c) Define the Z-transform. Solve the difference equation

$$y_{n+2} + 6y_{n+1} + 9y_n = 2^n \text{ with } y_0 = y_1 = 0.$$



2 Attempt any **two** of the following : 10×2=20

(a) If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^{-x} [(x - y)\sin y - (x + y)\cos y]$

then find u , v and the analytic function $f(z)$.

(b) State and prove the Cauchy's integral theorem for the derivative of analytic function.

(c) State and prove Liouville's theorem and using this theorem prove that every polynomial equation of degree n has n roots.

3 Attempt any **four** parts of the following : 5×4=20

(a) Expand the function $f(z) = \tan^{-1} z$ in powers of z .

(b) Define the singularity of a function. Find the singularity (ties) of the functions

(i) $f(z) = \sin \frac{1}{z}$

(ii) $g(z) = \frac{e^z}{z^2}$

(c) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2 + 1)^2 (x^2 + 2x + 2)}$.

(d) Evaluate $\int_0^{2\pi} \frac{d\theta}{a + b \sin \theta}$ if $a > |b|$.



(e) Evaluate $\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx$.

(f) Define the conformal mapping. Prove that an analytic function $f(z)$ ceases to be conformal at the points z_0 , where $f'(z_0) = 0$.

Attempt any **two** of the following :

10×2=20

(a) Define the coefficients of skewness and Kurtosis. Compute the coefficient of skewness from the following data :

x :	6	7	8	9	10	11	12
f :	3	6	9	13	8	5	4

(b) Define the coefficients of regression and correlation. Calculate the coefficient of correlation between the marks obtained by 8 students in Mathematics and Statistics :

Students :	A	B	C	D	E	F	G	H
Mathematics :	25	30	32	35	37	40	42	45
Statistics :	08	10	15	17	20	23	24	25

(c) Define the binomial distribution and obtain the expression for the Poisson distribution as a limiting case of binomial distribution.



5

Attempt any **two** of the following :

10×2=20

- (a) Find the roots of the cubic equation

$$ax^3 + 3bx^2 + 3cx + d = 0$$

using Cardan's method.

- (b) Fit a parabola
- $y = ax^2 + bx + c$
- to the following data taking
- x
- as independent variable :

$x :$	1	2	3	5	7	11	13	17	19	23
$y :$	2	3	5	7	11	13	17	19	23	29

- (c) The regression lines of
- y
- on
- x
- and of
- x
- on
- y
- are respectively
- $y = ax + b$
- and
- $x = cy + d$
- .

Show that the means are $\bar{x} = (bc + d)(1 - ac)$ and $\bar{y} = (ad + b)(1 - ac)$ and correlationcoefficient between x and y is \sqrt{ac} . Also, show that the ratio of the standard deviationsof y and x is $\sqrt{a/c}$.