Printed Pages : 4 TAS-301/TMA-301

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958 Roll No.

B. Tech.

(SEM. III) EXAMINATION, 2007-08

MATHEMATICS - III

Time : 3 Hours] *[Total Marks : 100*

Note : (1) Attempt all questions.

(2) Marks are shown against each equation.

1 Attempt any **four** of the following : 5×4=20

(a) Using Fourier integral representation, show that

$$\int_0^\infty \frac{\cos(\lambda x)}{(1+\lambda^2)}\, d\lambda = \frac{\pi}{2} e^{-x}, \quad (x > 0)$$

(b) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & for \ |x| \le 1, \\ 0 & for \ |x| > 1 \end{cases}$$

Hence or otherwise evaluate $\int_0^\infty \frac{\sin x}{x}\, dx$.

(c) Find the Fourier sine transform of $\left(e^{-ax} \mid x\right)$, $a > 0$.

(d) Using Fourier transform solve

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}, \quad -\infty < x < \infty, \quad t > 0; \quad V(x, 0) = f(x).$$

(e) Find the Z – transform of

$$\cosh\left(\frac{n\pi}{2} + \theta\right).$$

(f) Solve by Z-transform :

$$y_{n+2} - 4\, y_n = 0 \quad \text{given that} \quad y_0 = 0, \; y_1 = 2.$$

2 Attempt any **four** of the following : 5×4=20

(a) If $f(z) = u(r, \theta) + iv(r, \theta)$ is analytic in a domain D, prove that

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}.$$

(b) Prove that an analytic function with constant modulus is constant.

(c) Determine an analytic function $f(z)$, in terms of z whose real part is $e^{-x}(x \sin y - y \cos y)$.

(d) Evaluate $\int_C \frac{\sin(\pi z) + \cos(\pi z)}{(z-1)(z-2)}\, dz$ where C is the circle $|z| = 4$.

(e) Prove that if $f(z)$ is analytic and bounded in the entire complex plane then $f(z)$ is a constant.

(f) Evaluate : $\int_C \frac{e^z}{\left(z^2+\pi^2\right)^2} dz$ where C is $|z|=4$.

3 Attempt any **two** of the following : **10×2=20**

(a) Expand $f(z)=1/\{(z-1)(z-2)\}$ in the region :

(1) $|z|<1$

(2) $1<|z|<2$

(3) $|z|>2$

(b) Using residue theorem, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}.$$

(c) Find the bilinear transformation which maps $z=1,\ i,\ -1$ onto the points $w=i,\ 0,\ -i$ respectively. Hence find (i) the image of $|z|<1$. (ii) the invariant points of the transformation.

4 Attempt any **two** of the following : **10×2=20**

(a) In a partially destroyed laboratory record, only the lines of regression of y on x and x on y are available as

$$4x-5y+33=0 \quad \text{and} \quad 20x-9y=107$$

respectively. Calculate $\bar{x}$, $\bar{y}$ and the coefficient of correlation between x and y.

(b) In a test on 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours, estimate the number of bulbs likely to burn for

(1) more than 2150 hours

(2) less than 1950 hours

(3) More than 1920 hours and but less than 2160 hours.

(c) Find the mean of a binomial distribution. Also find the moment generating function of the binomial distribution about its mean.

5 Attempt any **two** of the following : **10×2=20**

(a) Solve $x^3 - 3x^2 + 12x + 16 = 0$.

(b) By the method of least squares, find the straight line that best fits the following data :

x	1	2	3	4	5	6	7
y	14	27	40	55	68	77	85

(c) Find the least squares fit of the form $y = a_0 + a_1 x^2$ to the following data :

x	−1	0	1	2
y	2	5	3	0