



(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 9909**Roll No. 

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**B. Tech.****(SEM. III) EXAMINATION, 2007-08****MATHEMATICS - III****(Old Syllabus)***Time : 3 Hours]**[Total Marks : 100*

- Note :** (1) *Attempt all questions.*  
 (2) *All questions carry equal marks.*

1 Attempt any **two** of the following : **10×2=20**

(a) Solve following differential equations :

(1)  $\frac{d^2y}{dx^2} + y = 0$ , given  $y = 2$  for  $x = 0$ ,  
 $y = -2$  for  $x = \frac{\pi}{2}$ .

(2)  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^x$

(3)  $\frac{d^2y}{dx^2} + d^2y = \sin ax$ .

(b) The damped L.C.R. circuit is governed by the

equation  $L \frac{d^2\theta}{dt^2} + R \frac{d\theta}{dt} + \left(\frac{1}{C}\right)\theta = 0$

where  $L, R, C$  are positive constants. Find the conditions under which the circuit is overdamped, underdamped and critically damped. Find also the circuit resistance.

- (c) Define difference equations and  $z$ -transform. Using  $z$ -transform, solve the difference equation

$$y_{n+3} - 2y_{n+2} + y_{n+1} = 3n + 8.$$

2 Attempt any **four** of the following : 5×4=20

- (a) Find the power series solution of the differential equation  $(x^2 + 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - xy = 0$  in powers of  $x$ .

- (b) Prove that

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$$

where  $P_n(x)$  is the Legendre's polynomial.

- (c) Show that  $\frac{d}{dx} \{x^n J_n(x)\} = -x^{-n} J_{n-1}(x)$  where  $J_n(x)$  is the Bessel function of the first kind of order  $n$ .

- (d) Find the Fourier transform of  $f(x) = \begin{cases} x, & |x| \leq a \\ 0, & |x| > a \end{cases}$ .

- (e) Show that  $\int_0^{\infty} \frac{\cos \lambda x}{1 + \lambda^2} d\lambda = \frac{\pi}{2} e^{-x}$ ,  $x \geq 0$ .

- (f) Find the Hilbert transform of  $\frac{\sin ax}{x}$ .

3 Attempt any **two** of the following : 10×2=20

- (a) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + 3 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = x + y.$$

- (b) Solve that equation

$$\frac{\partial^2 u}{dt^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

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subject to the conditions :

$$u(0, t) = u(l, t) = 0 \text{ for all } t,$$

$$u(x, 0) = f(x) \text{ and } \frac{\partial u}{\partial t} = g(x) \text{ at } t = 0.$$

(c) Solve the system of partial differential equations

$$\frac{\partial^2 u}{\partial x^2} = 0, \quad \frac{\partial^2 u}{\partial y^2} = 0.$$

4 Attempt any **four** of the following : 5×4=20

(a) Is the function

$$f(z) = \frac{xy^2(x+iy)}{x^2+y^4}, \quad z \neq 0, \quad f(0) = 0$$

analytic at  $z = 0$  ?

(b) Show that  $e^x \cos y$  is a harmonic function, find the analytic function of which it is real part.

(c) Show that the function

$f(z) = e^{-z^4}$ ,  $z \neq 0$ ,  $f(0) = 0$  is not analytic at the origin, although Cauchy-Riemann equations are satisfied at origin.

(d) If  $M$  is the upper bound of  $|f(z)|$  on a curve  $C$

of length  $l$  then prove that  $\left| \int_C f(z) dz \right| \leq Ml$ .

(e) State and prove Cauchy's integral formula.

(f) If  $f(z) = u(x, y) + iv(x, y)$  is analytic function and  $u(x, y) - v(x, y) = e^x(\cos y - \sin y)$  then find  $f(z)$  in term of  $z$ .

5 Attempt any **two** of the following :

10×2=20

(a) Using contour integration, evaluate

$$\int_0^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx \quad \text{and}$$

$$\int_0^{\infty} \frac{\sin x}{(x^2 + a^2)(x^2 + b^2)} dx$$

OR

(a) Using contour-integration prove that

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}.$$

(b) Define Taylor's series and Laurent's series with suitable examples. Prove that

$$e^{\frac{1}{2}z\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(z)t^n,$$

$$\text{where } J_n(z) = \frac{1}{\pi} \int_0^{\pi} \cos(n\theta - z \sin \theta) d\theta.$$

(c) Define poles, singularities and zero of a complex function  $f(z)$  with suitable examples. What kind of singularities have the following :

$$(1) \frac{1}{\sin z - \cos z} \quad \text{at } z = \frac{\pi}{4}$$

$$(2) \frac{\cot(\pi z)}{(z-a)^2} \quad \text{at } z = a \quad \text{and } z = \infty$$

$$(3) \frac{1-e^z}{1+e^z} \quad \text{at } z = \infty.$$

OR

(c) Define conformal mapping and linear fractional transformations. Prove that if  $f(z)$  is analytic at  $z_0$  then  $w = f(z)$  is conformal at  $z_0$  provided  $f'(z_0) \neq 0$ .