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TAS – 301 / MA – 301 (N) / MA – 301 (O) / TCF – 304

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9958 9964 9909 9959 **Roll No.** ____________

060b4

B.Tech.

THIRD SEMESTER EXAMINATION, 2006-07

MATHEMATICS - III

Time : 3 Hours *Total Marks : 100*

Note : *(i)* *Attempt **ALL** questions.*

(ii) *All questions carry equal marks.*

(iii) *In case of numerical problems assume data wherever not provided.*

(iv) *Be precise in your answer.*

(v) *Question No. 4 and 5 are separate for New and Old Syllabus.*

1. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the convolution of two functions. Prove that the Fourier transform of the convolution of the two functions is the product of their Fourier transforms. Verify this statement to find the Fourier inverse transform of e^{-as} sin bs.

(b) Define the Z-transform of the sequence $\{f_k\}$. Solve the following difference equation

$$8y_{k+2} - 6y_{k+1} + y_k = 5\sin\left(\frac{k\pi}{2}\right)$$

(c) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$

Subject to $u(0, t) = 0$. and $u(x, 0) = e^{-x}$, $x>0$

Using the method of Fourier transform.

2. Attempt *any four* parts of the following : **(5x4=20)**

(a) Define a harmonic function. Show that the function $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonic. Also find the analytic function $f(z) = u(x, y) + iv(x, y)$

(b) If $f(z)$ is an analytic function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) | f(z)|^2 = 4 | f'(z)|^2$$

(c) Evaluate the line integral $\int_C z^2 \, dz$ where C is the boundary of a triangle with vertices $0, 1+i, -1+i$ clockwize.

(d) Derive Cauchy's integral formula.

Evaluate $\int_C \frac{e^{3iz}}{(z+\pi)^3} dz$

where C is the circle $|z-\pi| = 3.2$

(e) Evaluate $\int_C (z+1)^2 \, dz$ where C is the boundary of the rectangle with vertices at the points $a+ib$, $-a+ib$, $-a-ib$, $a-ib$.

(f) State and prove Liouville's theorem.

3. Attempt *any four* parts of the following : **(5x4=20)**

(a) Expand the function $\sin^{-1} z$ in powers of z.

(b) Find Laurent series expansion of $\dfrac{4z-1}{z^4-1}$ about the point $z=0$.

(c) Evaluate $\int_0^{\infty} \frac{\sin x}{x}\, dx$

(d) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^4}$

(e) Define a conformal mapping. Prove that an analytic function $f(z)$ ceases to be conformal at the points where $f'(z)=0$

(f) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta}{5+4\cos\theta}\, d\theta$

Note : **Following Q.No. 4 and 5 are for New Syllabus only (TAS–301 / MA–301 (N) / TCF–304).**

4. Attempt *any two* parts of the following : **(10x2=20)**

(a) Define the coefficients of Skewness and Kurtosis. The first four moments of a distribution about the value 4 of the variable are -1.5, 17, -30 and 108. Find the moments about the origin. State whether the distribution leptokurtic or platy kurtic.

(b) Define the lines of regression and coefficient of correlation. If θ is the acute angle between the two regression lines in case of two variables x and y, show that $\tan\theta = \frac{1-r^2}{r}\frac{\sigma_x\sigma_y}{\sigma_x^2+\sigma_y^2}$ where r, σ_x, σ_y have their ususual meanings. Explain the significance of the formula when $r=0$ and $r=\pm 1$.

(c) Define the Poisson's distribution show that for the Poisson distribution with mean m

$$\mu_{r+1}=rm\mu_{r-1}+m\frac{d\mu_r}{dm}$$

Where $\mu_r=\sum_{x=0}^{\infty}(x-m)^r\frac{e^{-m}\,m^x}{\lfloor x}$,

(the r^{th} moment of poisson distribution).

5. Attempt *any two* parts of the following : **(10x2=20)**

(a) Solve the biquadratic equation (by Ferrari method) $x^4+3x^3+x^2-2=0$

(b) Fit a parabola to the following data.

x	1	2	3	4	5
y	1090	1220	1390	1625	1915

(c) Define the normal distribution. Derive the expression for it as a limiting case of binomial distribution (when $p=q$, where $p+q=1$).

Note : **Following Q.No. 4 and 5 are for Old Syllabus only (MA–301 (O)).**

4. Attempt *any two* of the following : **(10x2=20)**

(a) Solve the following differential equation :

$$\frac{d^3 y}{dx^3} - 2\frac{d^2 y}{dx^2} - 5\frac{dy}{dx} + 6y$$
$$= 2e^x + 4e^{3x} + 7e^{-2x} + 8e^{2x} + 15$$

(b) Using power series method, solve

$$(1-x^2)\frac{d^2 y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

(c) State and prove Rodrigue's formula for legendre polynomials.

5. Attempt *any two* of the following : **(10x2=20)**

(a) Solve the following partial differential equation :

$$3\frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial y} = \cos(3y + 2x)$$

(b) Solve the following partial differential equation by separation of variables method :

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

Subject to the conditions

$y(0, t) = 0,\ y(L, t) = 0$

$y(x, 0) = f(x)$

and $\left.\frac{\partial y}{\partial t}\right|_{t=0} = g(x)$

Where L is a non-zero finite constant.

(c) Find the temperature in a thin metal rod of length L, with both ends insulated (so that there is no passage of heat through the ends) and with initial temperature in the rod $\sin(\pi x/L)$.

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