(Following Paper ID and Roll No. to be filled in your Answer Book)

**PAPER ID : 9909** Roll No. | | | | | | | | | | |

B.Tech.

THIRD SEMESTER EXAMINATION, 2004-2005

**MATHEMATICS - III**

*Time : 3 Hours* *Total Marks : 100*

Note : (i) *Attempt ALL questions.*

(ii) *Marks are shown against each question.*

1. Attempt *any four* of the following : – (5x4=20)

(a) Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{for } |x| > 1 \end{cases}$

Rs Fourier Integral. Hence evaluate the value

$$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$$

(b) Find the complex form of the Fourier Integral representation of $f(x) = \begin{cases} e^{-kx} & x > 0 \text{ \& } k > 0 \\ 0 & \text{otherwise} \end{cases}$

(c) Find the Fourier Cosine transform of $f(x) = \left(\frac{T}{1+x^2}\right)$ and hence derive the Fourier Sine Transform of $\phi(x) = \left(\frac{x}{1+x^2}\right)$.

(d) If the inhal temperature of an infinite bar is given by

$$\mu(x,0)=\begin{cases}1 & \text{for} -C<xCC\\ 0 & \text{otherwise}\end{cases}$$

determine the temperature of an infinite bar at any point $x$ and at any time $t > 0$.

(e) Find the Z-transform of $c^k \cos(\propto k),\ k>0$.

(f) Solve the difference equation using Z-transform $y\ k\ +1-2\ y\ k-1=0,\quad k\geq 1,$ $y\ (0)=1$.

2. Attempt *any four* of the following :– **(5x4=20)**

(a) Define an Analytic function. Show that $f\ (z)=\log\ z$ is analytic everywhere in the complex plane except at the origin and that its derivative is $\left(\frac{1}{z}\right)$.

(b) If $f\ (z)$ is a regular function of $z$, show that

$$\left(\frac{\partial^2}{\partial^2 x}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\quad |f^1(z)|^2.$$

(c) Given that $u(xy) = x^2-y^2$ and $v(xy)=-[y|x^2+y^2]$. Prove that both $u$ and $v$ are harmonic functions but $u+iv$ is not analytic function of $z$.

(d) State and prove fundamental theorem of algebra.

(e) If $f\ (z)$ is analytic within a circle C, given by $|z-a|=R$ and if $|f(z)|\leq M \text{ on } C$, then

$$|\ f^n\ (a)\ |\leq \frac{M\underline{|n}}{R^n}$$

(f) If $f(z)$ is analytic within and on a closed contour C and 'a' is any point within C, then

$$f(a)=\frac{1}{2\pi i}\int_{c}\frac{f(z)}{(z-a)}\,dz.$$

3. Attempt *any two* of the following :– (10x2=20)

(a) Find the Laurent's expansion for

$f(z)=[\,(7z-2)\,/\,(z+1)\,(z)\,(z-2)\,]$.

in the regions given by

(i) $0<|z+1|<1$

(ii) $1<|z+1|<3$

(iii) $|z+1|>3$

(b) Evaluate the following complex integrations

(i) $$\int_{|z|=3}\left[\frac{\cos\pi z^2+\sin\pi z^2}{(z+1)(z+2)}\right]dz$$

(ii) $$\int_{c}\left[\frac{3z^2+2+1}{(z^2-1)\,(2+3)}\right]dz$$

where c is the circle $|z| = 2$

(c) Determine the region of the w-plane into which the region of the z plane bounded by straight lines $x=1$, $y=1$ and $x+y=1$ is mapped by transformation $w=z2$.

Attempt *any two* of the following :– (10x2=20)

(a) The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 p.m. and standard deviation of Rs. 50. Show that, of this group, about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.

(b) Fit a binomial distribution to the following frequency data :

| | | | | |
|---|---|---|---|---|
| $x$ | : | 0 | 1 | 3 | 4 |
| $f$ | : | 28 | 62 | 10 | 4 |

(c) The distribution of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be :

(i) no accident

(ii) at least 2 accidents

(iii) at most 3 accidents

(iv) between 2 and 5 accidents

5. Attempt *any two* of the following :– **(10x2=20)**

(a) Solve the cubic $x^3 - 6x^2 + 6x - 5 = 0$.

(b) If F is the pull required to lift a load W by means of a pulley, fit a linear law $F = a - bw$ connecting F and W against the following data:

| | | | | |
|---|---|---|---|---|
| W | 50 | 70 | 100 | 120 |
| F | 12 | 15 | 21 | 25 |

(c) Employ the method of least square to fit a parabola $y = a + bx + cx^2$ in the following data $(x\ y)$ :

$(-1, 2)\ (0, 0)\ (0, 1)\ (1, 2)$

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