

## B. TECH.

 THIRD SEMESTER EXAMINATION, 2003-2004  
 MATHEMATICS - III

Time : 3 Hours

Total Marks : 100

Note : (1) Attempt ALL questions.

(2) All questions carry equal marks.

1. Attempt any TWO of the following :— (10×2)

(a) Solve, in series, the following differential equation :—

$$xy'' + 2y' + xy = 0.$$

(b) Using Z-transform, solve the following difference equation :—

$$y_{K+2} + 4y_{K+1} + 3y_K = 3^K,$$

given  $y_0 = 0$  and  $y_1 = 1$ .(c) An uncharged condenser of capacity  $C$  is charged by applying an e.m.f. of value
 $E \sin \frac{t}{\sqrt{LC}}$  through leads of inductance  $L$ 
and of negligible resistance. The charge  $q$  on the plate of condenser satisfies the equation

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}.$$

Prove that the charge at any time  $t$  is

$$\frac{EC}{2} \left[ \sin \frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} \right].$$

2. Attempt any FOUR of the following :—

(a) Prove that  $P'_n(1) = \frac{1}{2}n(n+1)$ .

(b) Prove that

$$J_0^2(x) + 2J_1^2(x) + 2J_2^2(x) + \dots = 1,$$

where notations have their usual meanings.

(c) Find the Fourier sine transform of  $e^{-|x|}$ .  
Hence evaluate :

$$\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx.$$

(d) Find the Hankel transform of  $e^{-ax}/x^2$  and then apply the inversion formula to get the original function.

(e) Solve the equation, using Fourier transform :—

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0,$$

subject to the conditions —

(i)  $u = 0$  when  $x = 0, t > 0$

(ii)  $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$  when  $t = 0$

(iii)  $u(x, t)$  is bounded.

(f) Prove that

$$\int_{-1}^{-1} \frac{T_n(x)T_m(x)}{\sqrt{1-x^2}} dx = \begin{cases} 0 & ; m \neq n \\ \pi/2 & ; m = n \neq 0 \\ \pi & ; m = n = 0, \end{cases}$$

where  $T_n(x)$  is the Chebyshev polynomial of first kind.

Attempt any TWO of the following :— (10×2)

a) Solve the partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos 2y (\sin x + \cos x).$$

b) Solve one-dimensional wave equation for a string of length  $l$  with zero initial velocity and a profile given by  $f(x)$ , where

$$f(x) = A \sin 2\pi x.$$

c) Show that the equation  $u_{xx} - 2u_{xy} + u_{yy} = 0$  is parabolic. Reduce this equation to its normal form, using the transformation  $v = x$ ,  $z = x + y$ , and solve it.

Attempt any FOUR of the following :— (5×4)

d) If  $\phi$  and  $\psi$  are functions satisfying Laplace's equation, show that  $(s+it)$  is analytic, where

$$s = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \quad \text{and} \quad t = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}.$$

e) Show that the function  $f(z) = z|z|$  is not analytic anywhere.

f) Find the value of  $\int_C \frac{2z^2 + z}{z^2 - 1} dz$ , where  $C$  is the circle of unit radius with centre at  $z=1$ .