Number of Printed Pages — 4

MA-301

B.TECH.

THIRD SEMESTER EXAMINATION, 2001-[illegible]

MATHEMATICS — III

Time—3 Hours | *Total Marks—100*

Note : (1) Attempt *ALL* questions.

(2) The choice of questions is internal as indicated in each question.

(3) All questions carry equal marks.

1. Attempt any *Two* parts of the following :— 10×2

(*a*) A mass M suspended from the end of a helical spring is subjected to a periodic force $f = F \sin \omega t$ in the direction of its length. The force f is measured positive vertically downwards and at zero time M is at rest. If the spring stiffness is S, prove that the displacement of M at time t from the commencement of motion is given by

$$x = \frac{F}{M\left(p^2 - \omega^2\right)}\left[\sin \omega t - \frac{\omega}{p} \sin pt\right]$$

where $p^2 = \dfrac{S}{M}$ and damping effects are neglected.

(*b*) Solve by Z-transform the difference equation

$$y_{k+2} + 6y_{k+1} + 9y_k = 2^k, \quad (y_0 = y_1 = 0)$$

(*c*) Solve in series the differential equation

$$x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + x^2 y = 0$$

2. Attempt any *Four* parts of the following :— 5 × 4

(a) Show that –

$$4J_0'''(x) + 3J_0'(x) + J_3(x) = 0$$

(b) Obtain in terms of Bessel functions the solution of differential equation

$$\frac{d^2y}{dx^2} + \left(9x - \frac{20}{x^2}\right)y = 0$$

(c) Prove that –

$$\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}$$

(d) Show that –

$$\left(1-x^2\right)U_n' = n\left(U_{n-1} - xU_n\right)$$

(e) Find the Fourier transform of the function

$$f(x) = \begin{cases} 1+\dfrac{x}{a} & \text{, for} \quad (-a < x < 0) \\[2ex] 1-\dfrac{x}{a} & \text{, for} \quad (0 < x < a) \\[2ex] 0 & \text{, otherwise} \end{cases}$$

(f) Show that if $n = 0$, the Hankel transform

$$H\left\{\frac{\sin ax}{x}\right\} = \begin{cases} 0 & \text{, if } s > a \\ \dfrac{1}{\sqrt{a^2 - s^2}} & \text{, if } 0 < s < a \end{cases}$$

3. Attempt any *Four* parts of the following :— 5 × 4

(*a*) Show that the function defined by

$$f(z) = \sqrt{|xy|}$$

satisfies Cauchy's Riemann equation at the origin but is not analytic at that point.

(*b*) Find the values of C_1 and C_2 such that the function

$$f(z) = x^2 + C_1 y^2 - 2xy + i\left(C_2 x^2 - y^2 + 2xy\right)$$

is analytic. Also find $f'(z)$.

(*c*) Determine the region of the w-plane into which the region $\frac{1}{2} \le x \le 1$ *and* $\frac{1}{2} \le y \le 1$ is mapped by the transformation $w = z^2$.

(*d*) Find the bilinear transformation which maps the point $z = 1, i, -1$ into the point $w = i, 0, -i$. Hence find the image of $|z| < 1$.

(*e*) Evaluate

$$\int_0^{2+i} \left(\bar{z}\right)^2 dz$$

along the real axis from $z = 0$ to $z = 2$ and then along a line parallel to y-axis from $z = 2$ to $z = 2 + i$.

(*f*) Use Cauchy integral formula to evaluate

$$\oint_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$$

where c is the circle $|z| = 3$.

4. Attempt any *Two* parts of the following :— 10 × 2

(*a*) State and prove Laurent's theorem.

(*b*) Use contour integration method to evaluate the following integral :—

$$\int_0^{\pi} \frac{a\,d\theta}{a^2+\sin^2\theta}, (a>0)$$

(*c*) Apply calculus of residues to evaluate

$$\int_{-\infty}^{\infty} \frac{\cos x}{\left(x^2+a^2\right)\left(x^2+b^2\right)} dx, \ (a>b>0).$$

5. Attempt any *Two* parts of the following :— 10 × 2

(*a*) Solve the partial differential equation

$$\left(D^2-D'^2-3D+3D'\right)z = xy+e^{x+2y}$$

(*b*) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving each point an initial velocity $\lambda x(l-x)$, find the displacement of the string at any distance x from one end at any time t.

(*c*) A thin rectangular plate whose surface is impervious to heat flow has at $t=0$ an arbitrary distribution of temperature $f(x, y)$. Its four edges $x=0$, $x=a$, $y=0$, $y=b$ are kept at zero temperature. Determine the temperature at a point of the plate as t increases.