B.E. 2nd Semester Examination, May–2012

MATHEMATICS–II

Paper–MATH–102-E

Time allowed : 3 hours] *[Maximum marks : 100*

Note : *Attempt five questions in all; taking at least **one** question from each part. All questions carry equal marks.*

Part–A

1. (a) Find the rank of the matrix

$$\begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$$ 6

(b) Find non-singular matrices P and Q such that PAQ is in the normal form for the matrices

$$\begin{bmatrix} 1 & 3 & 3 & -2 \\ 2 & -2 & 1 & 3 \\ 3 & 0 & 4 & 1 \end{bmatrix}$$ 7

(c) Test for consistency and solve the equations

$$2x - 3x + 7z = 5$$

$$3x + y - 3z = 13$$

$$2x + 19y - 47z = 32 \qquad 7$$

2. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem and find the inverse for the matrix

$$A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

Part–B

3. (a) Solve

$$(x^2y^2 + xy + 1)\, y\, dx + (x^2y^2 - xy + 1)\, x\, dy = 0$$

(b) When a resistance of R ohms is connected in series with an inductance of L henries with an emf of E volts, the current i amperes at time t is given by $L \frac{di}{dt} + Ri = E$. If $E = 10 \sin t$ volts and $i = 0$, when $t = 0$, find i as a function of t.

4. (a) Solve :

$$(D-2)^2 y = 8 (e^2x + \sin^2 x + x^2)$$

(b) Solve the equation

$$(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log (1+x)$$

5. (a) Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$, by using the method of variation of parameters.

(b) A simple pendulum of length L is oscillating through a small angle θ in a medium in which the resistance is proportional to velocity. Find the differential equation of its motion. Discuss the motion and find the period of oscillation.

Part–C

6. (a) (i) Evaluate : $\int_0^{\infty} t^2 e^{-t} \sin t \, dt$

(ii) Find inverse Laplace transformation of

$$(se^{-as})/(s^2 + \pi^2)$$

(b) Find the inverse Laplace transform of $\frac{s}{s^4 + s^2 + 1}$

7. (a) Solve :

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \; x(0) = 1, \; x\left(\frac{\pi}{2}\right) = -1$$

(b) Solve $x^2 (y - z) p + y^2 (z - x) q = z^2 (x - y)$

8. (a) Find the partial differential equation from

$$F(x^2 + y^2, z - xy) = 0$$

(b) Solve the equation, by using Charpit's method

$$z = p^2x + q^2y$$