B.Tech.

FOURTH SEMESTER EXAMINATION, 2008-09

THEORY OF AUTOMATA & FORMAL LANGUAGES

(TCS-405)

Time : 3 Hours] *[Total Marks : 100*

Note : (1) Attempt **all** questions.

(2) All questions carry **equal marks.**

Q. 1. Attempt any four parts of the following : **5 × 4 = 20**

(a) Let $S = \{ab, bb\}$ and let $T = \{ab, bb, bbbb\}$, show that $S^* = T^*$.

Ans. $S = \{aa, bb\}$ & $T = \{ab, bb, bbbb\}$

now $S^* = \{\epsilon, aa, bb, bbaa, aabb, aabbaa, bbbb, ...\}$ all possible strings of aa & bb

& $T^* = \{\epsilon, aa, bb, bbbb, aabb, bbaa, aabbbbbb, bbbbaa, ...\}$

all the strings under T^* can be also made by S^*.

So $S^* = T^*$

Q. 1. (b) What do you mean by the Kleene closure of set A ?

Ans. Kleene Closure of a set A : If Σ is the set of alphabets, the closure is denoted by Σ^*. Notation is also known as Kleene star.

If $\Sigma = \{a\}$ then $\Sigma^* = \{\epsilon, a, aa, aaa, ...\}$

If $\Sigma = \{ab\}$ then $\Sigma^* = \{\epsilon, a, aa, b, bb, ab, ba, ...\}$

we can make an infinite language of string of letters out of *an* **albhabet.**

Q. 1. (c) Construct a grammar for each of the following languages :

(i) $\{a^m b^m | m \geq 1\} \cup \{b^n a^n | n \geq 1\}$ (ii) $\{a^l b^m c^n | l + m = n, l, m \geq 1\}$

Ans. (i) $\{a^m b^m | m \geq 1\} U \{b^n a^n | n \geq 1\}$ **(ii)** $\{a^l b^m c^n | l + m = n, l, m \geq 1\}$

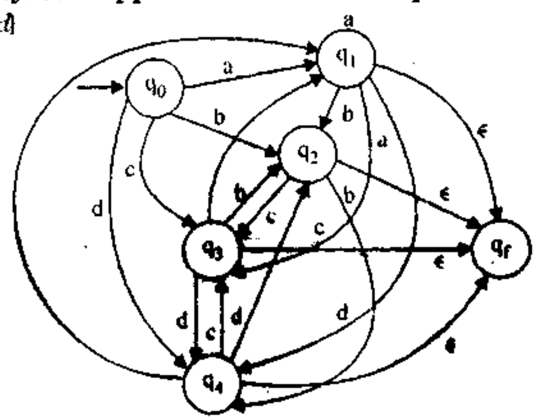
$S \rightarrow aSbbSa$ $\quad\quad$ $S \rightarrow aSbSccS$

$S \rightarrow aSb \,/\, bSa \,|\, \epsilon$ $\quad\quad$ $S \rightarrow aS | bS | cS$

$S \rightarrow \epsilon$

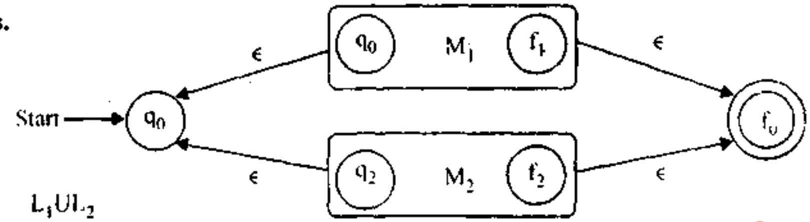
Q. 1. (d) Design a FA recognizing the language over $\{a, b, c, d\}$ which shall accept only those strings in which no symbol appears in consecutive positions.

Ans. Language over $\{a, b, c, d\}$



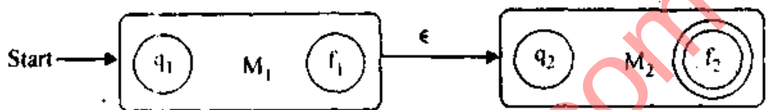
Q. 1. (e) Find two different FAs M_1 and M_2 recognizing languages L_1 and L_2 respectively, such that the languages $L_1 \cup L_2$ and L_1L_2 are the same.

Ans.



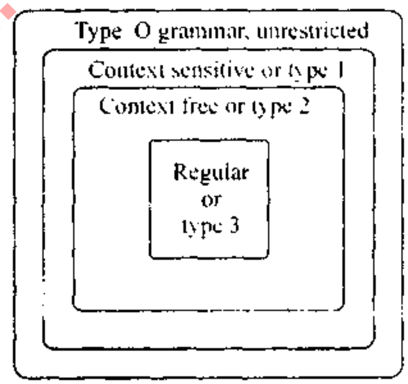
$L_1 \cup L_2$

L_1L_2



Q. 1. (f) Show that every context-free language is context-sensitive.

Ans. According to Chomsky hierarchy as shown in Chomsky hierarchy, every regular language in context free. Every context free language is context sensitive & every content sensitive language is reassively enumerable.



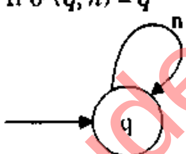
Q. 2. Attempt any four parts of the following :

5 × 4 = 20

(a) Using induction show that if for some state q ans some string n, $\delta^*(q, n) = q$, then for every $n \geq 0$, $\delta^*(q, n^n) = q$.

Ans. If $\delta^y(q, n) = q$

then

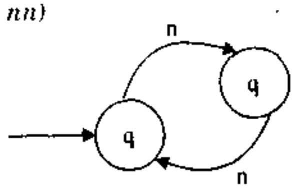


if $\delta^y(q, n^2)$

mean

$\delta^y(q, nn)$

the



So if $\delta^y(q, n^n)$

then

$\delta^y(q, nnn\ldots n!)$

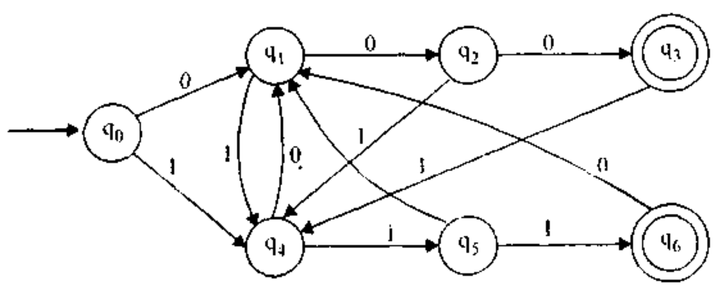
then

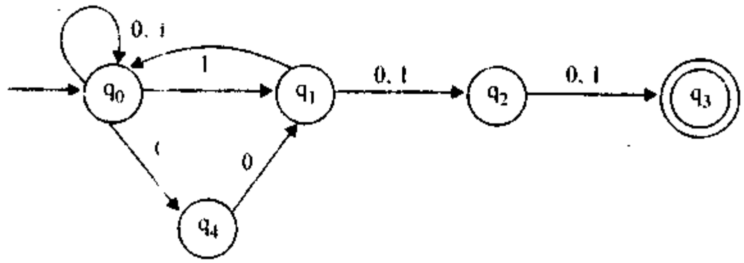
$\delta^y(q, n^n) = q$

because after processive each n it will beon the stage q.

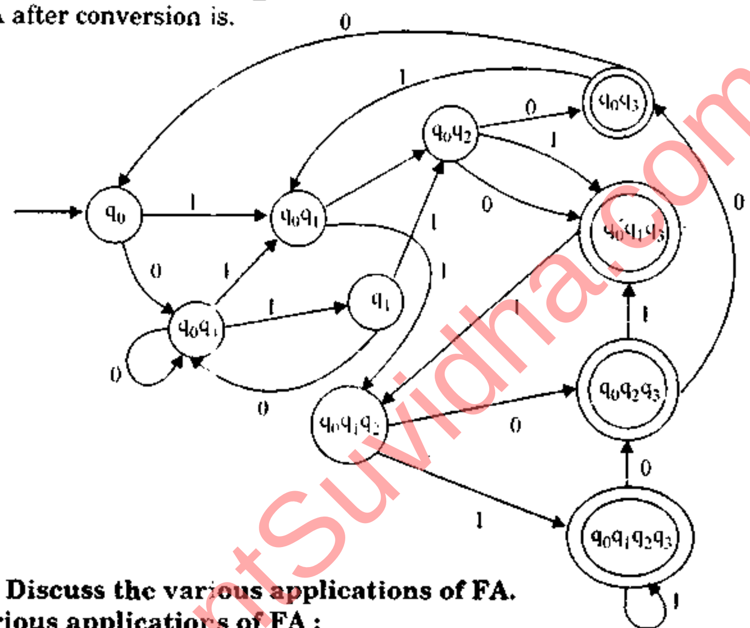
Q. 2. (b) Construct an NFA which recognizes a set of strings containing three consecutive 0's and three consecutive 1's. Also correct this NFA into an equivalent DFA.

Ans. The NFA is





The DFA after conversion is.



Q. 2. (c) Discuss the various applications of FA.

Ans. Various applications of FA :

Lexial analysis

(letter) (letter + digit)*

Where "letter" stands for A + B + ... + Z + a + b + ... + z, and "digit" stands for 0 + 1+ 1 ... + 9.

→ Text Editors.

Q. 2. (d) Construct a Moore machine that determines whether an input string contains an event or odd number of 1's. The machine should give I as output if an even number of 1's are in the string and 0 otherwise.

Ans. Moore machines

The table of moore machines

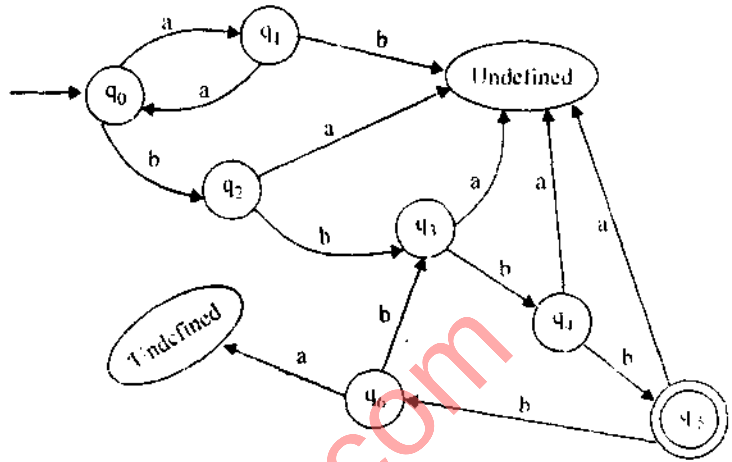
Present State	Next State		o/p
	$a = 0$ state	$a = 1$ state	
q_1	q_4	q_2	0
q_2	q_2	q_1	1
q_3	q_3	q_2	0
q_4	q_1	q_1	0

Q. 2. (e) Construct a DFA for the following language :
$\{a^m b^n | m$ is divisible by 2 and n is divisible by 4.

Ans.

$a^m b^n | m$ is divisible by 2.

n is divisible by 2f.



Q. 2. (f) Discuss the conversion of Moore to mealy machine with the help of an example.

Ans. Moore to Melay Machine.

Present state	Next state		Output
	$a=0$ state	$a=1$ state	
$\rightarrow q_1$	q_4	q_2	0
q_2	q_2	q_3	1
q_3	q_3	q_4	0
q_4	q_4	q_1	0

Without assigning the output values. We eliminate the o/p column of moore machine & insent two o/p columns in next state column corresponding to $a=0$ & $a=1$.

In table of moore machine, the state $q_1 - q_4$ has following o/p

$q_1 \rightarrow 0$
$q_2 \rightarrow 0$
$q_3 \rightarrow 0$
$q_4 \rightarrow 0$

than o/p values at state. $q_1 - q_4$ will be in table. Corresponding to the current state in next state column (the o/p column). We write 1 in front of all occurrence of of q_2, we write **0** in front of all occurrence of state, q_1, q_3 and q_4 because o/p for there states is 0.

The constructed melay machine is ,

Present state	Next state			
	$a=0$		$a=1$	
	State	o/p	state	o/p
$\rightarrow q_1$	q_4	0	q_2	1
q_2	q_2	1	q_3	0
q_3	q_3	0	q_4	0
q_4	q_4	0	q_1	0

Q. 3. Attempt any two parts of the following : **10 × 2 = 20**

(a) Using pumping lemma, prove that the following languages are not regular :

(i) $\{wo^n | w \in \{0, 1\}^* \wedge |w| = n\}$

(ii) $\{ww | w \in \{a, b\}^*\}$

Ans. $\{ww | w$ is in $\{a_1 b\}^*\}$

That is L consist of the word whose list & but halves are the same. Suppose L were context free.

Then by theorem $L_1 = 1 \cap a^+b^+ a^+b^+$ would at = bet a CFL.

But $L_1 = \left\{a^i b^j a^i b^j | i \geq 1, j \geq 1\right\} L_1$ is not a CFL.

as we know that every regular language is also a CFL.

So that Li is also not regular.

So the language $\{ww | w$ is in $\{a, b\}^y\}$ is not regular.

Q. 3. (b) Simplify the following grammar by eliminating useless symbols and useless production :

$S \to a | aA | B | C, A \to aB | \varepsilon$

$B \to Aa, C \to cCD, D \to dd$

Also find the Chomsky Normal form of the simplified grammar.

Ans. $S \to a | aA | B | C$

$A \to aB | \epsilon$ $\quad C \to cCD$

$B \to Aa$ $\quad D \to dd$

$A \to \epsilon$

so A is nullable.

$S \to$ is also nullable.

$S \to a | aA | B | C$

$A \to aB$ $\quad C \to cCD$

$B \to Aa$ $\quad D \to dd$

Useless production we have.

$S \to aA | a | B | C$

$A \to aB$ $\quad C \to cCD$

$B \to Aa$ $\quad D \to dd$

No useless production in it.

So now convert it into CFL.

$$\boxed{\begin{array}{l} S \to aA | a | B | c \\ A \to aB \\ C \to C_1 D \\ C_1 \to cC \\ B \to cC \\ B \to Aa \\ D \to dd \end{array}}$$

Q. 3. (c) (i) Show that the CFG with productions

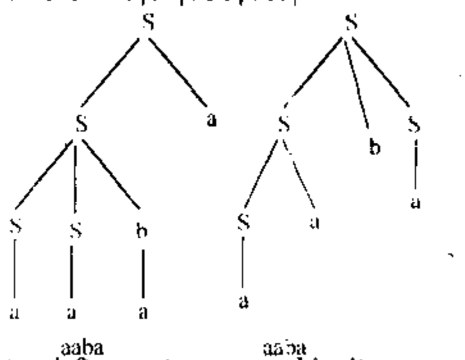
$S \to a | Sa | bSS | SSb | SbS$

is ambiguous.

Use pumping lemma to prove that the following is not CFL :

$\{a^n b^m a^n b^{n+m} | m, n \geq 0\}$

Ans. $S \rightarrow a|Sa|bSS|SSb|SbS$



two left most trees so ambiguity.

(ii) $\{a^n b^m a^n b^{n+m} | mn \geq 0\}$

for proving that it CFL or not,

use the conditions of pumping lemma

$\begin{cases} n + m \geq n \\ m \leq n \end{cases}$

so the not a regular language

Every CFL is regular also.

So it is not a CFL also.

Q. 4. Attempt any two parts of the following : **10 × 2 = 20**

(a) (i) Non-deterministic PDA is not equivalent deterministic PDA in terms of language recognition. Explain.

Ans. Non Deterministic PDA is not equivalent to deterministic PDA in terms of language recognition.

In Non deterministic PDA there are two same more from the same state which results in different outputs.

Where in Deterministic PDA these is a single move for the single input, in it is we have no two moves for lb same input on the same state.

In non deterministic

$(q_0, a, z_0) \longrightarrow (q_0, az_0)$

$(q_0, a, z_0) \dashrightarrow (q_0, \epsilon)$

In deterministic

$(q_0, a_1, z_0) \longrightarrow (q_1, az_0)$

$(q_1, a_1, z_0) \longrightarrow (q_f, \epsilon)$

Q. 4. (a) (ii) Convert the following grammar to a PDA that accepts the same language.

$S \rightarrow OSI | A$

$A \rightarrow IAO | S | \varepsilon$

Ans. gramma to PDA

$S \rightarrow OSI | A$

$A \rightarrow IAO | S | \epsilon$

$S(q_0 \cap S) = \{(q_0, OSI), (q_0, A)\}$
$S(q \cap A) = \{(q_0, IA_0), (q_0, S), (q_0, F)\}$
$\delta(q_0, \varepsilon, \in) = \{q_f, \in\}$

Q. 4. (b) Construct a PDA by empty stack which accepts the following : $\{a^m b^m c^n \mid m, n \geq 1\}$

Also convert this PDA into an equivalent CFG.

Ans. $\{a^m b^m c^n \mid m, n \geq 1\}$

$\delta(q_0, a_1, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_1, \in)$
$\delta(q, b, a) = (q, \in)$
$\delta(q_1, c, z_0) = (q_1, z_0)$
$\delta(q_0, \in, z_0) = (q_f, \in)$

Q. 4. (c) Construct a two-stack PDA for recognizing the following $\{a^n b^n c^n d^n \mid n \geq 1\}$

Ans. $\{a^n b^n c^n d^n \mid n \geq 1\}$

In stack one
$\delta(q_0, a_1, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, c, a) = (q, \in)$
$\delta(q_0, \in, z_0) = (q_f, \in)$

In stack second
$\delta(q_0, b, z_0) = (q_0, bz_0)$
$\delta(q_0, b, b) = \{q, bb\}$
$\delta(q, d, b) = (q_0, \in)$
$\delta(q_0, \in, z_0) = (q_1, \in)$

Q. 5. Attempt any two parts of the following : **$10 \times 2 = 20$**

(a) What do you mean by unsolvable problem ? Explain

Ans. Unsolvable problem : A decision problem is decidable if it is possible to build a turning machine that will always halt in a finite amount of time, producing a "yes" or "no" ("true" or "false") answer.

A decision problem is undecidable if a turing machine may run forever without producing an answer.

A decidable problem is a problem with a "ja" a "no" answer. This special class of problems is called decision problem.

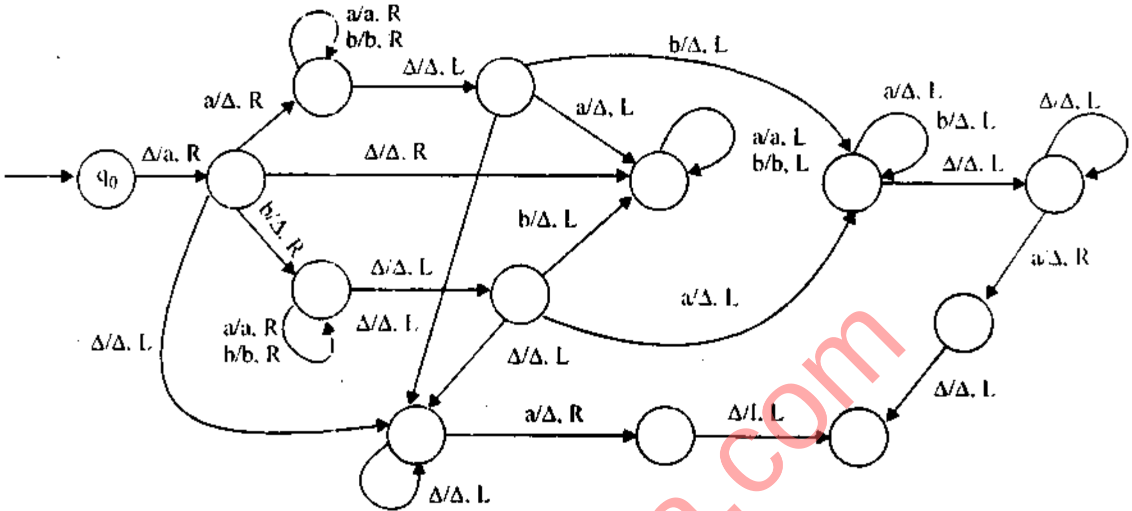
Some examples of unsolvable problem are :

(i) Does a given Turning machine A half on all inputs?
(ii) Does a turning machine A half for any input ?
(iii) Does a TM A half while given a blank i/p tape?
(iv) Do two turning machine A half while given a blank i/p take
(v) Is the language accepted by a turning machine finite ?
(vi) Does the language accepted by a turning machine contain any two strings of the same length.

Q. 5. (b) Design a TM recognizing the following language : $\{a^m ba^n ba^p ba^{m+n+p} \mid m, n, p > 1\}$

Ans. $\{a^m ba^n ba^p ba^{m+n+p} \mid m, n, p \geq 1\}$

$m = 2$
aa baa baa baaaaaa



Q. 5. (c) Design a 2-track TM that takes as input on track-1 a^n and leaves on track-2 the binary representation of n.

Ans. a^n

binary representation of n.

(i) Searching right for ao, M encounters a blank. Then, the no's in o^m10^n have all been changed to 1's and $n+1$ of the mo's have been changed to B. M replaces the $n+1$ is by a o & n B's, leaving $m-n$ o's on its tape.

(ii) Beginning the cycle, M can not find a o to a blank, because the first m o's already have been changed .The $n \geq m$ to $m - n = 0$.

M replaces remaining 1's & o's by B.

The function δ is —

1. $\delta(q_0, 0) = (q_1, B, R)$ begin cycle, Replace lading o by B.
2. $\delta(q_1, 0) = (q_1, 0, R)$
   $\delta(q_1, 1) = (q_2, 1, R)$ search right, looking for first 1.
3. $\delta(q_2, 1) = (q_2, 1, R)$
   $\delta(q_2, 0) = (q_3, 1, L)$ search right past 1's, unit encountering a o change that o to 1.
4. $\delta(q_3, 0) = (q_3, 0, L)$
   $\delta(q_3, 1) = (q_3, 1, L)$ move left of blank. Enter state q_0 to repeat the cycle.
   $\delta(q_3, 0) = (q_0, B, R)$
5. $\delta(q_2, B) = (q_4, B, L)$
   $\delta(q_4, 1) = (q_4, B, L)$
   $\delta(q_4, 0) = (q_4, 0, L)$
   $\delta(q_4, B) = (q_6, 0, R)$
6. $\delta(q_0, 1) = (q_5, B, R)$
   $\delta(q_5, 0) = (q_5, B, R)$
   $\delta(q_5, B) = (q_5, B, R)$
   $\delta(q_5, B) = (q_6, B, L)$

sample computation of mon input 0010 B :-

$q_0 0010 \vdash B_q, 010 \vdash BOq, 10 \vdash B01q_2, 0 \vdash BOq_3, 11 \vdash Bq_3, 011 \vdash$
$q_3 B011 \vdash Bq_0, 011 \vdash BBq, 10 \vdash BB1q_2 1 \vdash BB11q_2 \vdash BB1q_4 1 \vdash BBq_4 1 \vdash Bq_4 \vdash BOq_6$