B.TECH.

ECS-303

THIRD SEMESTER THEORY EXAMINATION 2010-11

DISCRETE MATHEMATICAL STRUCTURES

Time: 3 Hours *Total Marks: 100*

Note: *(1) Attempt all the questions.*

(2) All questions carry equal marks.

Q. 1. Attempt any four parts of the following : **($5 \times 4 = 20$)**

(a) Consider a universal se U = {$x|x$ is an even integer} and Y = {$x|x$ is a negative odd integer}. Find the following

(i) $X - Y$

(ii) $X^C - Y$, where X^C is the complement of set X.

Ans. (i) $X - Y = \{x : x$ is a positive integer$\}$

(ii) $X^C - Y = \{x : x$ is a negative integer$\}$

(b) Consider a set $S_k = \{1, 2, ... K\}$.

Find $\bigcup_{k=1}^{n} s_k$ and $\bigcup_{k=1}^{\infty} s_k$.

Ans. $\bigcup_{k=1}^{n} s_k = \{S_1, S_2, S_3, ...S_n\}$

$= \{1, 2, 3, ... n\}$

and $\bigcup_{k=1}^{\infty} = \{S_1, S_2, S_3, ...S_\infty\}$

$= \{1, 2, 3, 4, 5, 6, ...\infty\}$

(c) Let R be a relation on N, the set of natural number such that R = {(x,y): $2x + 3y$ and $x, y \in$ N}

Find (i) The domain and co domain of R.

(ii) R^{-1}.

Ans. (i) The domain of R = set of Natural Numbers

Co-domain of R = set of Natural Numbers

(ii) $R^{-1} = \{(x,y): 2x + 3y$ and $x, y \in \}$

(d) Show that the function $f(x) = x^3 + 1$ and $g(x) = (x - 1)^{1/3}$ are converse to each other.

Ans. According to given function

$\text{fog}(x) = (x^3 + 1 - 1)^{1/3} = x$

$\text{gof}(x) = ((x - 1)^{1/3})^3 + 1 = x$

Hence both functions are converse of each other.

(e) Prove that if f_n is a Fibonacci number then

$$f_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$$

For all $n \in N$, the set of natural numbers.

Ans. By mathematical Indication

Let for $n = 1$,

$$f_1 = \frac{1}{\sqrt{5}}\left[\left(\frac{(1+\sqrt{5})}{2}\right)^{1+1} - \left(\frac{(1-\sqrt{5})}{2}\right)^{1+1}\right]$$

$$= \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{2} - \left(\frac{1-\sqrt{5}}{2}\right)^{2}\right]$$

$$=\frac{1}{\sqrt{5}}\left[\left(\frac{(1+\sqrt{5})}{2}\right)-\left(\frac{(1-\sqrt{5})}{2}\right)\left(\frac{(1+\sqrt{5})}{2}\right)+\left(\frac{(1-\sqrt{5})}{2}\right)\right]$$

$$=1$$

Similar it is true for $n=2$

Now, Let us consider it is true for n also and we have to prove that $n+1$

So

$$f_{n+1}=\frac{1}{\sqrt{5}}\left[\left(\frac{(1+\sqrt{5})}{2}\right)^{(n+2)}-\left(\frac{(1-\sqrt{5})}{2}\right)^{(n+2)}\right]$$

$$=\frac{1}{\sqrt{5}}\left[\left(\frac{(1+\sqrt{5})}{2}\right)^{n}\left(1+\frac{5}{4}+\sqrt{5}\right)-\left(\frac{1-\sqrt{5}}{2}\right)^{n}\left(1+\frac{5}{4}-\sqrt{5}\right)\right]$$ **Hence proved.**

(f) Let f: $x>y$ and $x=y=R$, the set of real numbers. Find f^{-1} if

(i) $f(x)=x^2$

(ii) $f(x)=\dfrac{2x^2-1}{5}$

Ans.

(i) $y=x^2$

$$x=+y \text{ or } -y$$

$$f^{-1}=\{(y,x) \text{ belongs to } f(x)\}$$

(ii) $y=\dfrac{2x-1}{5}$

$$x=\frac{5y+1}{2}$$

$$f^{-1}=\{(y,x) \text{ belongs to } f(x)\}$$

Q. 2. Attempt any two parts of the following: (10 × 2 = 20)

(a) Let $G=\{1,-1,i,-i\}$ with the binary operation multiplication be an algebraic structure, where $i=1$.

(i) Determine whether G is an abelian.

(ii) If G is cyclic group, then determine the generator of G.

Ans. (i) The given group G is an abelian group because it satisfies the condition of commutative law in below table.

i.e., $i*1=1*i=i$

*	1	−1	i	−i
1	1	−1	i	−i
−1	−1	1	−i	i
i	i	−i	1	−1
−i	−i	i	1	−1

As elements along the rows are same as element along columns.

(ii) $G = \{1, -1, i, -i\}$ is a cyclic group and the generator this group is i.

$$G = \{i, (i)^2, (i)^3, (i)^4\}$$

Thus $-i$ is also a generator of G.

(b) Let $G = (z^2, +)$ be a group and let H be a subgroup of G, where $H = \{(x,y): x = y\}$. Find the left cosets of H in G. Here Z is the set of integers.

Ans. left cosets of H

$a + H = \{a + h : h \in H\}$

Therefore left cosets = $\{a : a$ is a set of integers$\}$

(c) Prove that $(R, +, *)$ is a ring with zero divisors, where R is 2*2 matrix and + and * are usual addition and multiplication operations.

Ans. for R to be ring (R, +) is an abelian group

$$a = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}$$

$$b = \begin{pmatrix} 0 & 3 \\ 4 & 1 \end{pmatrix}$$

(i) Closure Property:

Since the addition of a and b is c matrix of 2*2 order and R is closed with respect to addition of 2 matrices.

(ii) Associativity: $(a + b) + c = a + (b + c)$

(iii) Existence of Identity: $a + 0 = a$ so $0 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

(iv) Existence of Inverse: $a + a^{-1} = 0$]

(v) Commutativity: $a + b = b + a$

$(R, *)$ is semigroup

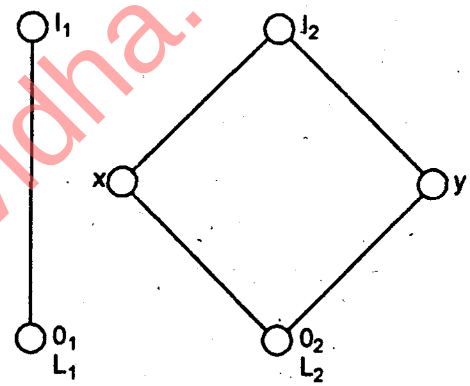
(i) Closure Property: $a * b = d$ so d is 2*2 matrix belongs to R

(ii) Associativity: $a*(b*c) = (a*b)*c$

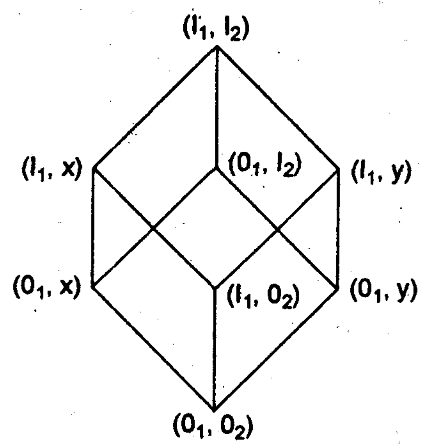
(iii) Distributive Law: Let a, b, c, belongs to R $a*(b + c) = (a*b) + (a*c)$

Q. 3. Attempt any two parts of the following:

(a) Let $(L_1, <=)$ and $(L_2, <)$ be lattices as shown below. Then draw the Hasse Diagram for the lattice $(L, <=)$ where $L = L \times L_2$.



Ans. The product of lattice L_1 and L_2 are shown in below fig.



(b) (i) Simplify the following Boolean function using K-map $f(x, y, z) = \Sigma(0, 2, 3, 7)$

Ans. The k-map of above equation is

	$x'y'$	$x'y$	xy	xy'
z'	1	1	1	
z			1	

The simplified Boolean expression for given function is : $xy + x'z'$

(ii) How are sequential circuits different from combinational circuits?

Ans. Sequential Circuits: A sequential circuit is an interconnection of flip-flops and gates. The gates by themselves constitute a combinational circuit, but when included with the flip-flips, the overall circuit is classified as a sequential circuit. The block diagram of a clocked sequential circuit is shown in Fig.1. It consists of a combinational circuit and number of clocked flip-flops. In general, any number or type of flip-flops may be included. As shown in the diagram, the combinational circuit block receives binary signals form external inputs and from the output of the combinational circuit go to external outputs and to inputs of flip-flops. The gates in the combinational circuit determine the binary value to be stored in the flip-flops after each block transition. The output of flip-flops, in turn, are applied to the combinational circuit inputs and determince the circuit's behavior and the present state of the flip-flops. Moreover, the next state of flip-flops is also a function of their present state and external inputs. Thus a sequential circuit is specified by a time sequence of external inputs, external outputs, and internal flip-flop binary states.

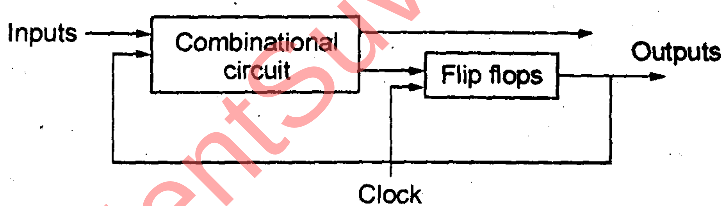


Fig. Block diagram of a synchronous sequential circuit

(c) Derive the Boolean duality principle. Write the dual of each of Boolean equations:

(i) $x + x' = x + y$

(ii) $(x.1)(0 + x') = 0$

Ans. Duality Principle: For duality replace 0 with 1 and vice-versa and replace + with. and vice-versa

(i) The duality of $x + x'y = x + y$ is $x(x' + y) = x.y$

(ii) The duality of $(x.1)(0 + x') = 0$ is $(x + 0) + (1.x') = 1$

Q. 4. Attempt any two parts of the following:

(a) (i) Show that the statements:

$P \to Q$ and $\neg Q \to \neg P$ are Equivalent.

Ans. $P \to Q = \neg P \vee Q$

$\neg Q \to \neg P = Q \vee \neg P = \neg P \vee Q$

Hence both are equivalent to each other.

By truth table we can also prove it:

$P \to Q$

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

$\neg Q \to \neg P$

P	Q	$\neg Q$	$\neg P$	$\neg Q \to \neg P$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

Hence by the truth table $P \to Q$ and $\neg Q \neg \to P$ are equivalent.

(ii) State the contra positive and converse of the statement of the following statement:

"If the triangle is equilateral, then it is equiangular."

Ans. Converse: "If it is not an Equiangular then the triangle is not euilateral."

Contra Positive: " If it is equiangular then the triangle is equilateral."

(b) Show the premises:

$R \to S, \neg Q \to S, \neg\neg P$. **and** $(T \cap U) \to R$ **imply the conclusion** $\neg (T$ **and** $U)$.

Ans. $(P \to Q) \cap (R \to S) \cap (\neg Q \to \neg S) \cap (\neg\neg P) \cap (T \cap U) R \to \neg (T \cap U)$ should be a tautology

$(\neg PVQ) \cap (\neg R V S) \cap (Q V \neg S) \cap (P) \cap (\neg (T \cap U) V R) \to \neg (T \cap U)$

$(\neg P V Q) \cap ((\neg R \cap Q) V (\neg R \cap \neg S) V (S \cap Q) V F) \cap (P) \cap (\neg T \cap U) V R \to \neg (T \cap U)$

(c) What do the following expressions mean?

(i) (for all x) $(x^2 >= x)$

(ii) (for all x) < 0 $(x^2 > 0)$

(iii) (there exists x) $\neq 0$ $(x^2 \neq 0)$

Here the domain in each case consists of the real numbers.

Ans. (i) (for all x) $(x^2 > x)$

This statement means that the square of a real number x cannot be less then the number x.

(ii) (for all x) < 0 $(x^2 > 0)$

This statement means that the square of a negative real number will always be greater then 0.

(iii) (there exists x) $\neq 0$ $(x^2 \neq 0)$

This statement means that there does not exist any real number other than 0 whose square is equal to 0.

Q. 5. Attempt any four parts of the following: **(5 × 4 = 20)**

(a) Determine the value of each of these prefix expressions:

(i) – *2/933

Ans. (i) A given expression is – * 2/933 and its value is 3.

(ii) + – *335/^2 3 2

A given expression is + – *335/^2 3 2 and its value is 8.

(b) For each value of n do these graphs have an Euler Cycle?

(i) K_n a complete graph of n vertices.

Ans. (a) For odd values of n, each vertex of K_n has even degree so K_n has an Euler circuit.

(b) The graph K_2 has an Euler path but not an Euler. For even values of n greater than 2, K_n has more than 2 vertices of odd degree, so K_n has neither an Euler path nor an Euler circuit.

(ii) C_x, a cycle of n vertices.

Ans. (a) For odd values of n, each vertex of C_x has even degree so C_x has an Euler circuit.

(b) The graph C_x has an Euler path but not an Euler circuit. For even values of n greater than 2, C_x has more than 2 vertices of odd degree, so C_x has neither an Euler path nor an Euler circuit.

(c) Solve the Recurrence relation:

$$T(n) = 64\,T(n/4) + n^6 \; n \geq \text{ and a power of } 4.$$

Ans. A given recurrence relation $T(n) = 64\,T(n/4) + n^6$ $n \geq 4$ and a power of 4.

Using master method Compare it with $T(n) = aT(n/b) + F(n)$

We get $a = 64$, $b = 4$ and $f(n) = n^6$

Now apply master method

$$n \log_b a = n \log_4 64 = n^3.$$

So $$n^{3+\varepsilon} = F(n) = n^{3+3} \text{ i.e., } \varepsilon = 3$$

and $$64 f\left(\frac{n}{4}\right) \leq cf(n)$$

$$64\left(\frac{n}{4}\right)^6 \le c.n^6 \Rightarrow 4^3\frac{n^6}{4^6} \le c.n^6$$

$\Rightarrow$ $$\frac{1}{4^3} \le c$$

i.e., $$T(n) = \theta(n^6) \text{ for } c > \frac{1}{64}$$

(d) Solve the recurrence relation:

$$a_n = 3a_{n-1} + 4_{n-1}$$

For $n \ge 0$ and $a_0 = 1$

Ans.
$$a_n = 3a_{n-1} + 4^{n-1}$$
$$n \ge 0 \text{ and } a_0 = 1 \quad \text{...(1)}$$

Characterstic equation of equation 1

$$a - 3 = 0$$
$$a = 3$$

The homogeneous solution

$$a_n = A_1(3)^n$$

General form of particular solution of (1) is

$$a_n = A\,4^{n-1}$$
$$A4^{n-1} - 3A\,4^{n-2} = A\,4^{n-1}$$
$$4A - 12A = 4$$
$$-8A = 4$$
$$A = -4/8$$
$$a_n = -1/2\,4^{n-1}$$
$$a_n = A(3)^n - 1/2\,4^{n-1}$$

Applying initial condition

$$a_0 = 1$$

put $n = 0$

$$a_0 = A - \frac{1}{8}$$

$$A = 1 + \frac{1}{8}$$

$$A = \frac{9}{8}$$

$$a_n = \frac{9}{8}(3)^n - \frac{1}{2}(4)^{n-1}$$

(e) Determine the number of bit strings of length 10 that either begin with three 0's or end with two 1's.

Ans. If we fix the first three, are 2^7 possibilities for the entire string.

If we fix the last 2, you have 2^8 possibilities for the entire string.

If we add these together, we get all the possibilities, but there are some repetitions. How many repetitions are there? There are 2^5 because 000 xxx xx11 has only 2^5 possibilities. So you subtract this from the answer.

$$2^7 + 2^8 - 2^5 = 256 - 32 = 352$$

There are 352 strings of length 10 (base 10) that start with 000 or end with 11.

(f) How many different rooms are needed to assign 500 classes, if there are 45 different time periods during in the university time table that are available?

Ans. No. of classes = 500

No. of time period in university = 45

$$\text{No. of required room} = \left\lceil \frac{500}{45} \right\rceil = \lceil 11.11 \rceil = 12$$

So 12 rooms are required to assign 500 classes.