B.Tech.

FIRST SEMESTER EXAMINATION, 2010-11

ENGG. MECHANICS

(FME-102)

Time : *3 Hours]* *[Total Marks : 100*

Note : (1) *Attempt **all** questions. Marks are indicated against each questions.*

(2) *Assume missing data suitably, if any.*

SECTION—A

Q. 1. Attempt all parts: — **$(10 \times 2 = 20)$**

Note: In parts (i) and (ii), choose the correct choice:

(i) A body of weight 100 N is resting on a rough horizontal table. The friction force acting on it is:

(a) 20 N (b) 10 N (c) 0

(d) The question cannot be answered without knowing the co-efficient of the friction.

Ans. (d)

(ii) First moment of area about an axis is zero. The axis:

(a) Must be an axis of symmetry **(b) Must pass through CG**

(c) Both (a) and (b) **(d) None**

Ans. (a)

Note : In parts (iii) and (iv), choose whether the statement is true or false:

(iii) When nonconcurrent coplanar forces act on a body, it is possible that the resultant force is zero even if the body is not in equilibrium.

Ans. F

(iv) A solid shaft is stronger compared to a hollow shaft if material, weight and length of the shafts are same.

Ans. F

Note : In parts (v) and (vi), fill in the blanks:

You will be awarded full marks, if all the entries in pair are correct otherwise will be awarded zero.

(v) A prefect truss has minimum _______ joints and _____ members.

Ans. two & one

(vi) The bending stress at neutral axis is _______ and at the top layer is ________

Ans. zero & maximum.

Note: In parts (vii) and (viii), matching types:

You will be awarded full marks, if all the matches are correct otherwise will be awarded zero.

(vii) Match the following columns:—

	Column I	Column 2
(a)	Moment of inertia of a circular plate of mas M and radius R about its axis is.	(1) $2MR^2/5$
(b)	Moment of inertia of a circular ring of mass M and radius R about its axis is	(2) $MR^2/2$
(c)	Moments of Inertia of a solid sphere of mass M and radius R about its diameter is	(3) $2MR^2/3$
(d)	Moment of Inertia of a thin spherical shell of mass M and radius R about its diameter is	(4) MR^2

Ans. (a) $\longrightarrow$ (2) $\frac{MR^2}{2}$

(b) $\longrightarrow$ (4) MR^2

(c) $\longrightarrow$ (1) $\frac{2}{5}MR^2$

(d) $\longrightarrow$ (3) $\frac{2}{3}MR^2$

(viii) Match the following columns : —

	Column 1	Column 2
(a)	In pure rotation of rigid body,	(1) $R\alpha$
(b)	In pure rotation of rigid body, tangential component of acceleration is	(2) $\frac{d^2s}{dt^2}$
(c)	In curvilinear motion of rigid body, the tangential component of acceleration is	(3) $\frac{1}{R}\left(\frac{ds}{dt}\right)^2$
(d)	In curvilinear motion of rigid body, normal component of acceleration is	(4) $R\omega^2$

(viii) (a) $\longrightarrow$ (4) $R\omega^2$

(b) $\longrightarrow$ (1) $R\alpha$

(c) $\longrightarrow$ (2) $\frac{d^2S}{dt^2}$

(d) $\longrightarrow$ (3) $\frac{1}{R}\left(\frac{ds}{dt}\right)^2$

Note : In questions (ix) and (x), two statements are given followed by four choices. Choice the correct choice.

(ix) **Statement 1:** A moment of 10 N-m is applied in the middle of the simply supported beam. The magnitude of bending moment in the beam at the middle is 5 Nm.

Statement 2: A cantilever beam of length 2 m carries 10 kN force at a distance of 1 m from the support. The bending moment at a distance of 1.5 m from the support is 15 kNm.

(a) Only statement 1 is true (b) Only statements 2 is true
(c) Both statements are true (d) Neither statements is true

Ans. (d)

(x) **Statement 1:** A wire of length L is bent, the CG will remain at the middle.

Statement 2: From a solid circular plate of radius R a concentric circular plate of radius R/2 is removed. CG will remain unchanged.

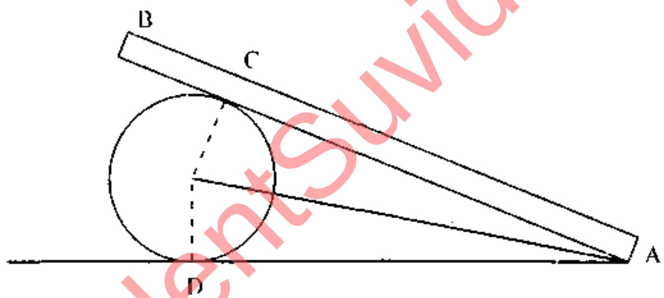
(a) Only statement 1 is true (b) Only statement 2 is true
(c) Both statement are true (d) Neither statement is true

Ans. (b).

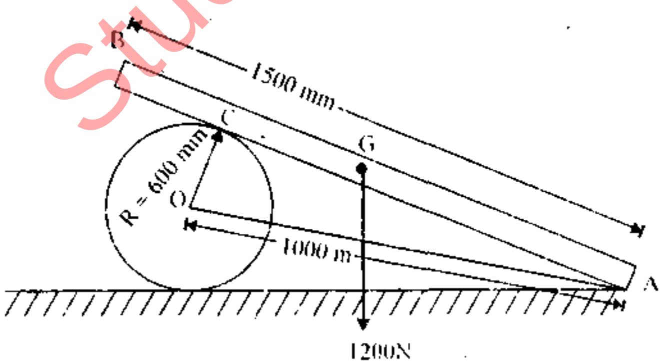
SECTION—B

Q. 2. Answer any three parts of the following :— **(10 × 3 = 30)**

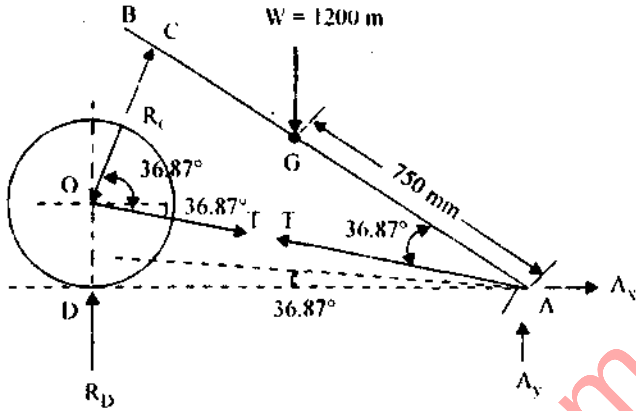
(a) A smooth weightless cylinder of radius 600 mm rests on a horizontal plane and is kept from rolling by an inclined string of length 1000 mm. A bar AB of length 1500 mm and weight 1200 N is hinged at A and placed against the cylinder of negligible weight. Determine tension in the string.



Ans.



Draw the free body diagram of cylinder and bar AB.



Geometry :

In $\Delta ABOC = AC = \sqrt{(AO)^2 - (OC)^2} = \sqrt{(1000)^2 - (600)^2} = 800$ mm

$$\sin\theta = \frac{OC}{OA} = \frac{600}{1000} = 0.6$$

$$\theta = 36.87°$$

For Bar taking moment about A & equlibrium condition

$$\sum M_A = 0$$

$\therefore$ $1200 \times (750 \cos 73.74°) - R_c \times 800 = 0$

$\therefore$ $R_c = 315$ N

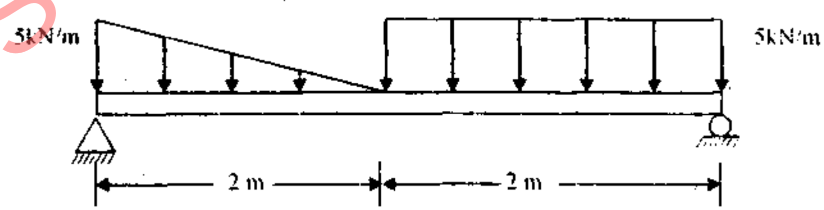
Now for cylinder use lam's theorem

$$\frac{R_D}{\sin(53.13 \times 2)} = \frac{I}{S_{in}(53.13 + 73.74)} = \frac{R_c}{S_{in}126.87}$$

$\therefore$ $T = 315$ N

$\therefore$Tension in the string is 315 N $R_D = 302.4$ N

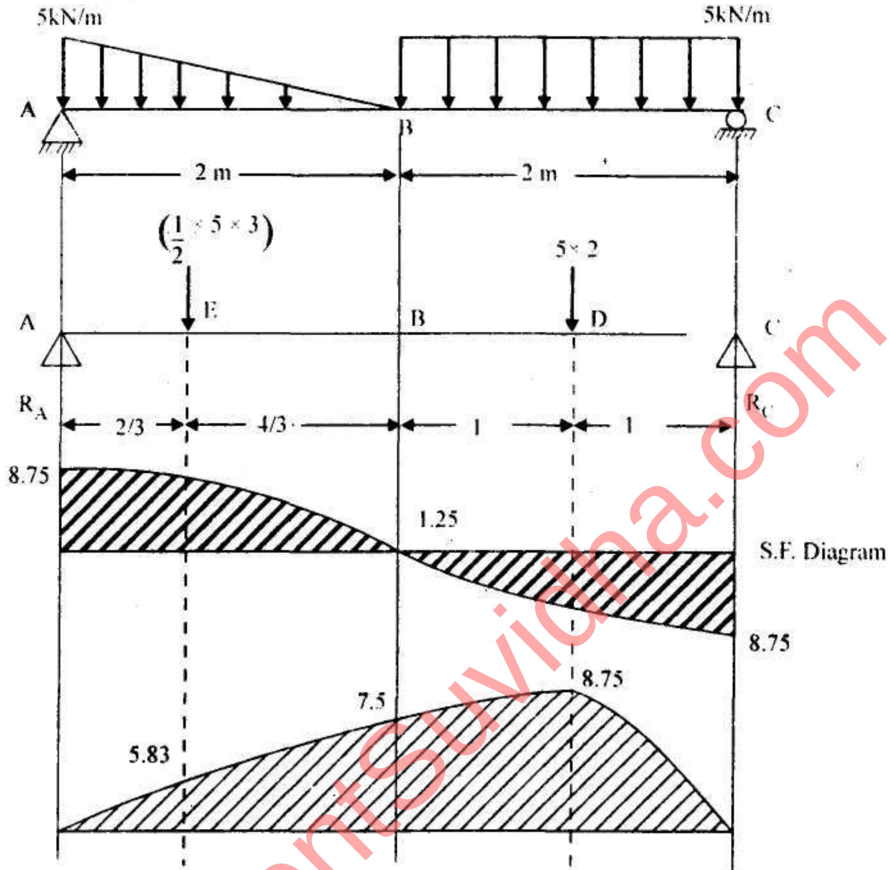
Q. 2. (b) For the simply supported beam as shown in figure, draw shear force and bending moment diagrams after finding the equations for shear force and bending moment.



Ans. To find

Reactions R_A & R_C, by equilibrium condition $\sum V = 0 \sum MA = 0$

$$R_A + R_C = \frac{15}{2} + 10 = \frac{35}{2}$$



Moment about A $M_A = \frac{1}{2}\times 5\times 3\times\frac{2}{3} + 10\times 3 - R_C\times 4 = 0$

$\therefore \quad R_C = \frac{35}{4} = 8.75\text{ kN}$

$2\, R_A = 8.75\text{ kN}$

For S.F. calculation

S. F at $A = R_A = 8.75$ kN

S.F between $A_E = +8.75$ kN

S.F between $EB = +8.75 - \frac{15}{2} = 1.25$ kN

S.F between $BD = 1.25$ kN

S.F between $CD = 1.25 - 10 = -8.75$ kN

S.F at $C = -8.75 + 8.75 = 0$

For B.M calculation

B.M at $A = 0$

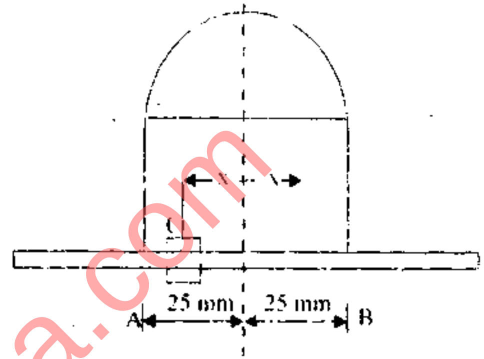
B.M at $E = 8.75\times 2/3 = 5.83$ kN

B.M at $B = 8.75\times 2 - \frac{15}{2}\times 4/3 = 7.5$ kN

$$\text{B.M at } D = 8.75 \times 3 - \frac{15}{2} \times \left(\frac{4}{3} + 1\right) = 8.75 \text{ kN}$$

$$\text{B.M at } C = 8.75 \times 4 - \frac{15}{2} \times \left(\frac{4}{3} + 2\right) - 10 \times 1 = 0 \text{ kN}$$

Q. 2. (c) Rod AB of weight 500 N is supported by a cable wrapped around a semi-cylinder having coefficient of friction of 0.2. A weight C weighing 100 N can slide without friction on rod AB. What is the maximum range x from centerline the mass C can be placed without causing slippage ?



Ans. $\dfrac{T_1}{T_2} = e^{0.2 \times \pi}$

$T_1 = 1.874\, T_2$ ...(1)

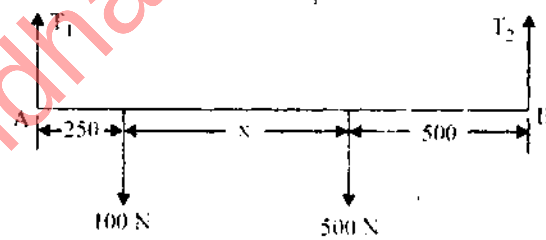
$T_1 + T_2 = 600$ ...(2)

By solving equation (1) and (2)

$T_1 = 391.23\, N$

$T_2 = 208.77\, N$

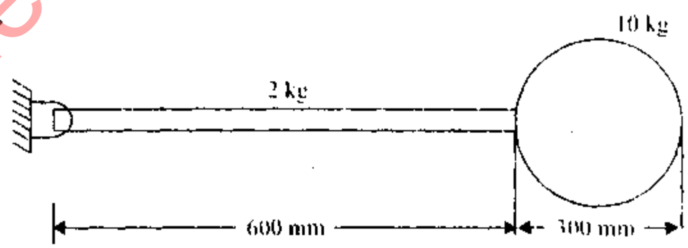
Taking moment about A $\quad \sum M_A = 0$



$100(250 - x) + 500 \times 250 - 208.77 \times 500 = 0$

$x = 456.15$ mm

Q. 2. (d) A homogeneous sphere weighing 10 kg is attached to a slender rod of mass 2 kg. If the system is released from horizontal position in rest condition, find the magnitude of angular acceleration. Also find angular velocity of system when it passes through vertical position.



Ans. Let α be the angular acceleration

I be the mass of moment of inertia of other assembly about the axis of rotation A.

Using the transfer formula

$$I = I_g + Md^2 = \frac{1}{12} \times 2000 \times \left(\frac{600}{1000}\right) + 10 \times \left(\frac{300}{1000}\right)^2 = 0.06 + 0.9 = 0.96$$

Now moment of inertia of the cylinder about A

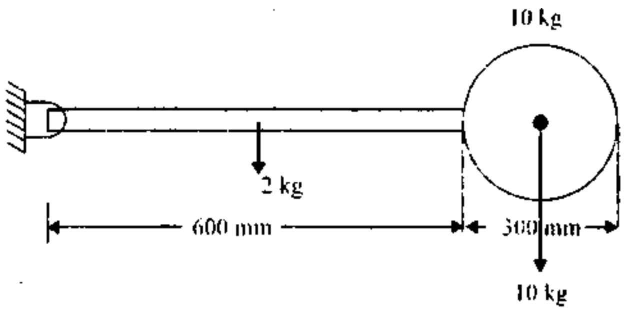
$$= \frac{1}{2} \times 10 \times \left(\frac{150}{1000}\right)^2 + (10) \times \left(\frac{750}{1000}\right)^2 = 5.625 + 0.1125 = 5.7375$$

$\therefore$ mass moment of inertia of the system $I = 5.7375 + 0.96 = 6.6975$

Rotation moment about A

$$M_t = 2 \times 9.8 \times \frac{300}{51000} \times 10 \times 9.8 \times \frac{750}{1000}$$

$= 5.88 + 73.5 = 79.38$ N-m



Equating to $I\alpha$ we get $6.6975 \times \alpha = 79.38$

$\alpha = 11.85$ rad/sec is the angular acceleration.

Angular velocity of system is $r_1\alpha$

$= \frac{300}{1000} \times 11.85 = 3.56$ m/sec

Q. 2 (e) (i) What do you understand by "Pure Bending" ? **(2)**

Ans. Pure Bending : When a beam is acted upon by end couples, then whole of the beam bends in a circular shape. There is then no shear in the beam. Such a case is then called as pure bending.

Q. 2 (e) (ii) Determine the suitable values for insider and outside diameters of hollow steel shaft whose internal diameter of 0.6 times its external diameter. The shaft transmits 220 kW at 200 rpm. The allowable shear stress is limited to 75 MPa, and angle of twist is limited to 1° per meter. The modulus of rigidity for shaft material is 80 kN/mm^2. **(8)**

Ans. Given internal diameter $di = 0.6$ of external diameter do

$\therefore\ di = 0.6\,do$

$\therefore$ Power transmitted = 220 KW

Speed $N = 200$ r.p.m

The allowable shear stress is limited to 75 $mP_a = \tau_{max}$

The angle of twist is limited to 1° per meter $\therefore\ \theta = 1°$/meter

Modules of rigidity $G = 80$ kN/m^2

Power transmitted $= \frac{2\pi NT}{60}$

$$\therefore \quad \frac{2\pi NT}{60} = 220 \times 10^3$$

$$T = \frac{220 \times 10^3 \times 60}{2\pi N} = \frac{220 \times 10 \times 60}{2 \times \pi \times 200} = 10504.23 \text{ N.m}$$

$$\frac{T}{J} = \frac{\tau_{max}}{r_0}$$

$$\therefore \quad \frac{10504.23}{\frac{\pi}{32}(d_0^4 - di_0^4)} = \frac{75 \times 10^6}{\frac{d_0}{2}}$$

$$\therefore \quad (d_0^4 - di_0^4) = 713.30 \times 10^{-6}\, do$$

$$\therefore \quad \frac{[d_0^4 - (0.6do)^4]}{do} = 713.30 \times 10^{-6}$$

$do = 93.6$ mm $\quad di = 56.16$ mm

$$\frac{T}{J} = \frac{G\theta}{L} \quad \theta = 1° = 0.0174 \text{ rad}$$

$$\therefore \quad \frac{10504.23}{\frac{\pi}{32}\left(d_0^4 - di^4\right)} = 80 \times 10^3 \times 0.0174$$

$$\therefore \quad d_0^4 - di^4 = \frac{10504.23 \times 32}{\pi \times 80 \times 10^3 \times 0.0174}$$

$$\therefore \quad d_0^4 - (0.6do)^4 = 76.864$$

$$d_0^4 = 88.31$$

$$d_0 = 306 \text{ mm}$$

$$d_i = 1.83 \text{ mm}$$

So internal diameter of shaft is 56.16 mm & outer diameter of shaft is 93.6 mm.

SECTION—C

Q. 3. Attempt any one parts of the following : **10**

(a) (i) State and prove Lami's theorem.

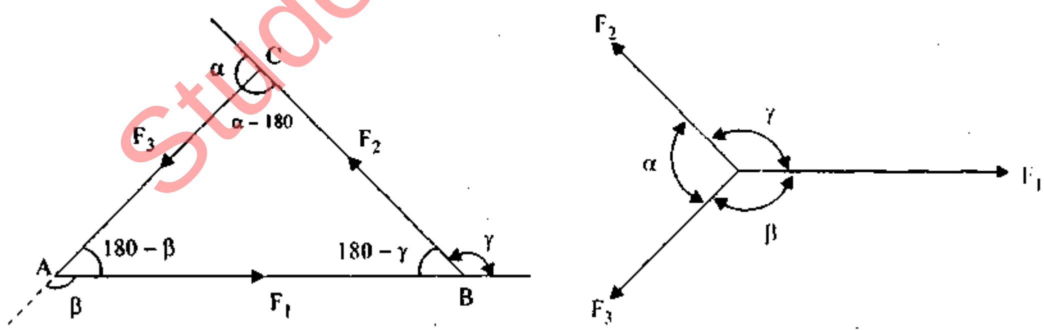
Ans. Lamis theorem : Lamis theorem states that of a body is in equilibrium under the action forces, then each force is proportional to the size of the angle between the other two forces, symbolically.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Let F_1, F_2, F_3 be three forces in equilibrium acting on a body along the directions as shown in above figure. Since the forces are in equilibrium they can be represented by the three sides of the triangle.

Now applying the sine rule for tangle, ABC

$$\frac{AB}{\sin(180-\alpha)} = \frac{BC}{S_m(180-\beta)} = \frac{AC}{\sin(180-\gamma)}$$

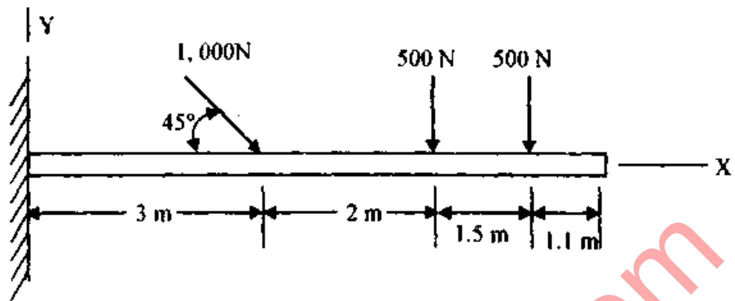


$$\frac{AB}{\sin\alpha} = \frac{BC}{\sin\beta} = \frac{AC}{\sin\gamma}$$

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

where α, β and γ are the angles between $\vec{F_2}, \vec{F_3}, \vec{F1}, \vec{F_3}, \& \vec{F_1}, \vec{F_2}$

Q. 3. (a) (ii) A system of forces acting on a cantilever beam is shown in figure. Reduce this system to a single force system and find the point of application of this force on the beam.



Ans. $\sum F_v = V_A - 1000\sin 45° + 500 - 500 = 0$

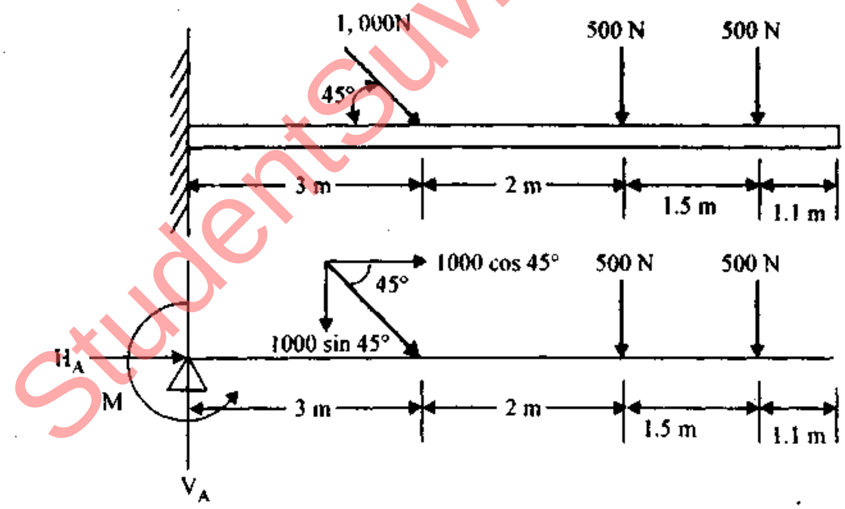
$$V_A = 1000\sin 45° = 707.11\text{ N}$$

Moment about A $\sum M_A = M - 100\sin 45° \times 3 + 500 \times 5 - 500 \times 6.5 = 0$

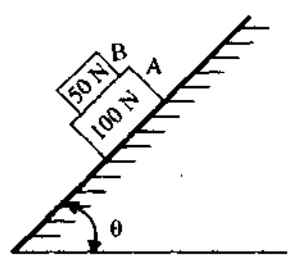
$$M = 212.13 - 2500 + 3250 = 962.3\text{ N-m}$$

$$\sum F_H = H_A + 1000\cos 45° = 0$$

$$H_A = 707.11\text{ N}$$



Q. 3. (b) Blocks A and B, of weight 150 N and 200 N, respectively rest on an inclined plane as shown in the figure. The coefficient of friction between the two blocks is 0.3 and between block A and inclined plane is 0.4. Find the value of θ for which either one or both the blocks start slipping. At that instant, what is the friction force between B and A ? Between A and inclined plane.



Ans. Consider the block B

$\sum$ Force normal to plane $= 0$

$\therefore N_1 - 50\cos\theta = 0$

$N_1 = 50\cos\theta$

From low of friction $F_1 = 0.3\,N_1 = 0.3 \times 50\cos\theta = 15\cos\theta$

Consider the block A

$\sum$ Force normal to plan $= 0$

$N_2 - N_1 - 100\cos\theta = 0$

$N_2 = N_1 + 100\cos\theta$

$N_2 = (50 + 100)\cos\theta = 150\cos\theta$

From low of friction

$F_2 = 0.4\,N_2 = 0.4 \times 150\cos\theta = 60\cos\theta$

Now sum of all the forces parallel to the plane $= 0$

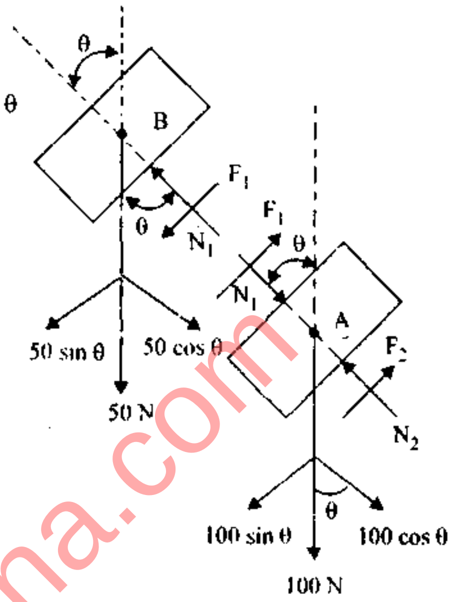
$$F_1 + F_2 - 100\sin\theta = 0$$

$$15\cos\theta + 60\cos\theta - 100\sin\theta = 0$$

$$\therefore \quad 75\cos\theta = 100\sin\theta$$

$$\tan\theta = \frac{75}{100} = \frac{3}{4}$$

$$\theta = 36.87°$$



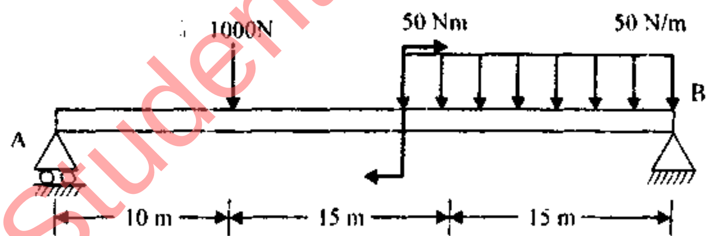
Frictional force between B and A is $F_1 = 15\cos\theta = 15\cos 36.87° = 12\text{ N}$

Frictional force between A and inclined plane is

$$F_2 = 60\cos\theta = 60\cos 36.87 = 48\text{ N}$$

Q. 4. Attempt any four parts of the following : **5 × 4 = 20**

Q. 4.(a) For the beam shown in figure, draw the shear force and bending moment diagrams.



Ans. $\sum V = V_A + V_B - 1000 - 750 = 0 \quad \therefore V_A + V_B = 1750\text{ N}$

$$\sum M_A = 1000 \times 10 + 50 + 750 \times 32.5 - V_B \times 40 = 0 \quad \therefore V_B = 859.38\text{ N}$$

$$V_A = 890.62\ N$$

S.F. Calculation :

S.F at A = 0

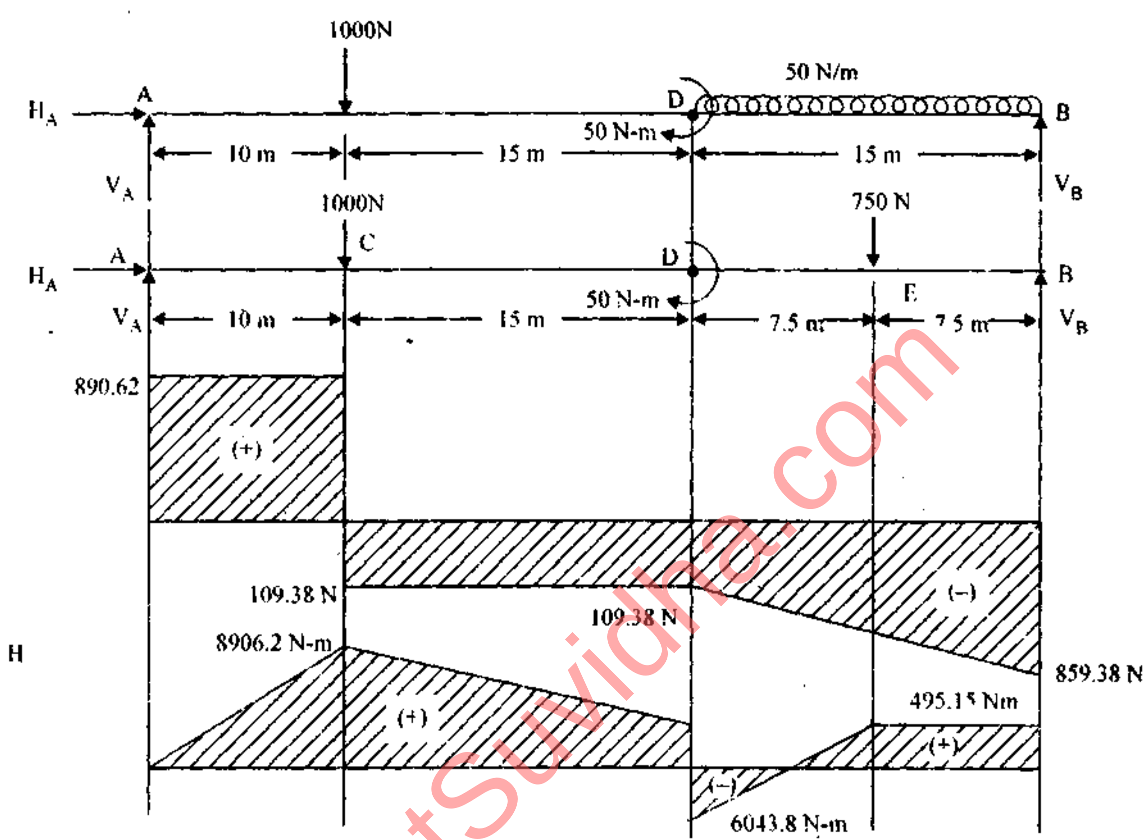
S.F between $CA = V_A = 890.62$ N

S.F between $CD = 890.62 - 1000 = -109.38$ N

S.F between $DE = -109.38$ N

S.F between $EB = -109.38 - 750 = -859.38$ N

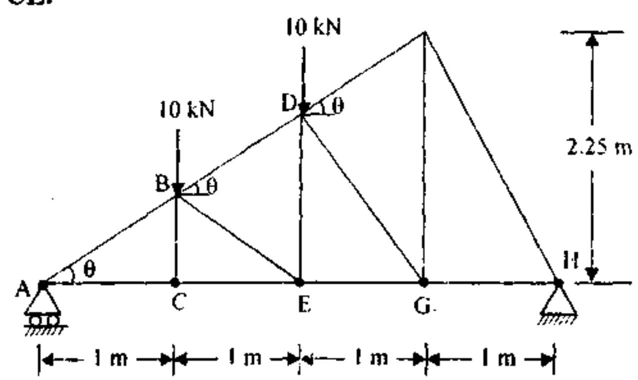
$\therefore$ S.F at $B = -859.38 + 859.38 = 0$



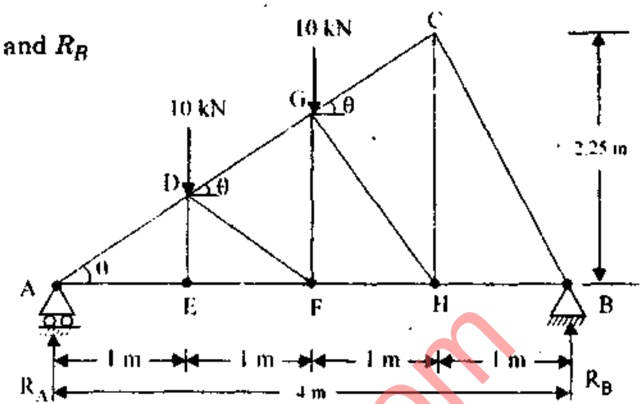
BM Calculation :

$$\text{B.M at } A = 0$$
$$\text{B.M at } C = 890.62 \times 10 = 8906.2 \text{ Nm}$$
$$\text{B.M at } D = 890.62 \times 25 - 1000 \times 15 + 50 = -6043.8 \text{ N-m}$$
$$\text{B.M at } E = 890.62 \times 32.5 - 1000 \times 22.5 + 50 = 6495.15 \text{ N-m}$$
$$\text{B.M at } B = 890.62 \times 40 - 1000 \times 30 + 50 - 750 \times 7.5 = 0 \text{ N-m.}$$

Q. 4. (b) For the simply supported truss shown in figure, find the forces in the members BD, DE, FG, and CE.



Ans. First Calculate the reaction R_A and R_B



Taking moment about A, we get

$R_B \times 4 = 1 \times 10 + 2 \times 20$

$R_B = 12.5$ N

$R_A + R_B = 20$

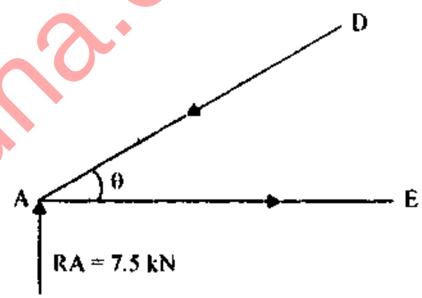
$R_A = 20 - 12.5 = 7.5$ kN

From figure, we know that

$$\tan\theta = \frac{CH}{AH} = \frac{2.25}{3} = 0.75$$

$$\cos\theta = \frac{AH}{AC} = \frac{3}{3.75} = 0.8 \text{ as } AC = \sqrt{3^2 + 2.25^2} = 3.75$$

$$\sin\theta = \frac{CH}{AC} = \frac{2.25}{3.75} = 0.6$$



Consider the equilibrium of joint A

Resolving the forces vertically $F_{AD}\sin\theta = R_A$

$$F_{AD} = \frac{7.5}{\sin\theta} = \frac{7.5}{0.6} = 12.5 \text{ kN (comp)}$$

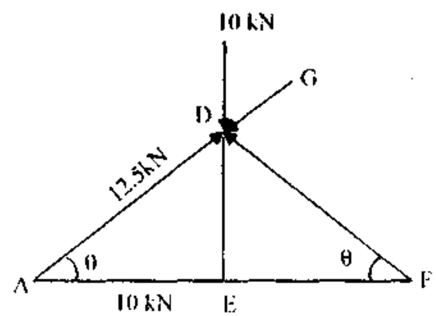
Resolving the force horizontally

$F_{AE} = F_{AD}\cos\theta = 12.5 \times 0.8 = 10$ kN (Tensile)

Joint E

F_{AE}, F_{EF}, & F_{ED} are acting at joint E Two of the forces i.e F_{AE} and F_{EF} are in the same straight line. Hence the third force *i.e.* F_{ED} should be zero.

$F_{EF} = F_{AF} = 10$ kN (Tensile)



Joint D

Let $\quad F_1$ = Force in member DG

$\quad\quad F_{DF}$ = Force in member DF

Resolving the forces vertically

$$12.5\sin\theta - 10 + F_{DG}\sin\theta + F_{DF}\sin\theta = 0$$

$$12.5 \times 0.6 - 10 + F_{DG} \times 0.6 + F_{DF} \times 0.6 = 0$$

$$F_{DG} + F_{DF} = 4.17 \text{ kN}$$

Resolving the forces horizontally

$$12.5\cos\theta + F_{DG}\cos\theta = F_{DF}\cos\theta$$

$$12.5 \times 0.8 + F_{DG} \times 0.8 = F_{DF} \times 0.8$$

$$12.5 = F_{DF} - F_{DG}$$

$$2F_{DF} = 16.67$$

$$F_{DF} = 8.335 \text{ kN (compressive)}$$

$$F_{DG} = -4.165 \text{ kN}$$

$$\therefore \quad F_{DG} = 4.165 \text{ N (compressive)}$$

Joint F

The forces in the member DF and and EF are already known.

$F_{DF} = 8.33$ N (comp)

$F_{EF'} = 10$ kN(T)

The forces are acting at the joint F as shown in figure

Let F_2 = Force in member FG

F_3 = Force in member FH

Resolving forces vertically we get

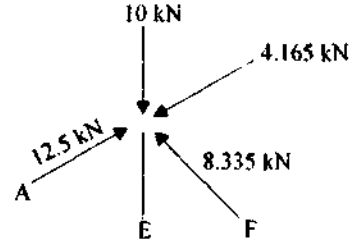
$$8.33 \sin\theta = F_2$$

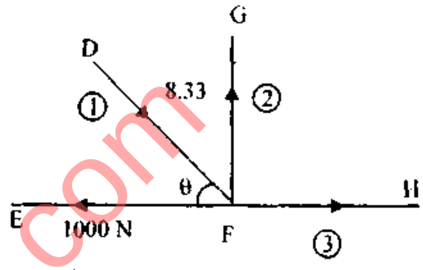
$$F_2 = 8.33 \times 0.6 = 5 \text{ kN}$$

Resolving forces horizontally we get

$$F_3 + 8.33 \cos\theta = 10$$

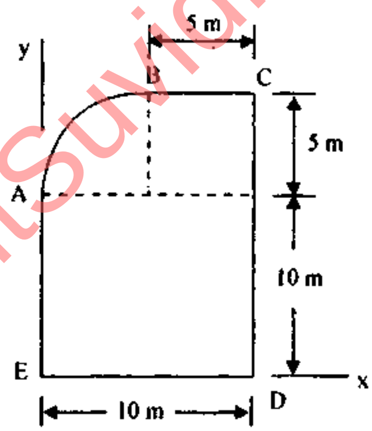
$$F_3 = 3.34 \text{ kN}$$





Q. 5. Attempt any two parts of the following : **(5 × 2 = 10)**

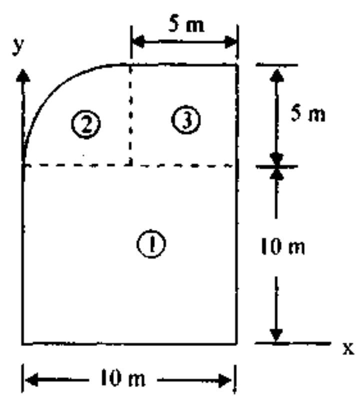
Q. 5. (a) A wire in bent into closed loop A-B-C-D-E-A as shown in figure in which portion AB is circular arc. Determine the centroid of the wire.



Ans. $A_1 = 10 \times 10 = 100 \text{ m}^2$

$$A_3 = 5 \times 5 = 25 \text{ m}^2$$

$$A_2 = \frac{\pi r^2}{4} = \frac{\pi \times 5^2}{4} = 19.63 \text{ m}^2$$



Calculation of Centroidal distances:

$$x_1 = \frac{10}{2} = 5$$

$$y_1 = \frac{10}{2} = 5$$

$$x_2 = 5 - \frac{4r}{3\pi} = 5 - \frac{4 \times 5}{3\pi} = 2.87 \text{ m}$$

$$y_2 = 10 + \frac{4r}{3\pi} = 10 + \frac{4 \times 5}{3\pi} = 12.12 \text{ m}$$

$$x_3 = 5 + \frac{5}{2} = 7.5 \text{ m}$$

$$y_3 = 10 + \frac{5}{2} = 12.5 \text{ m}$$

Calculation for $\bar{x}$ & $\bar{y}$

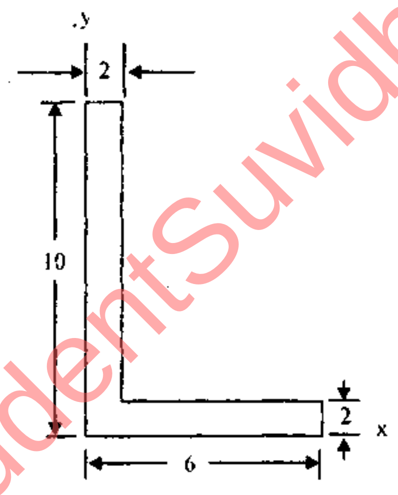
$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = \frac{100 \times 5 + 19.6 \times 2.88 + 25 \times 7.5}{100 + 19.63 + 25}$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{100 \times 5 + 19.63 \times 12.12 + 25 \times 12.5}{100 + 19.63 + 25} = 7.26$$

$\therefore$ Coordinates of centroid (5.14, 7.26)

Q. 5. (b) Find the principal moment of inertia about the origin of the area shown in figure.

All dimensions are in mm.



Ans. The given section divided into two rectangles (1) & (2)

Area of (1) $\rightarrow A_1 = 2 \times 10 = 20$

Area of (2) $\rightarrow A_2 = 2 \times 4 = 8$

Total Area $A_1 + A_2 = 20 + 8 = 28$

Two reference axis $y - y$ and $x - x$ are chooses.

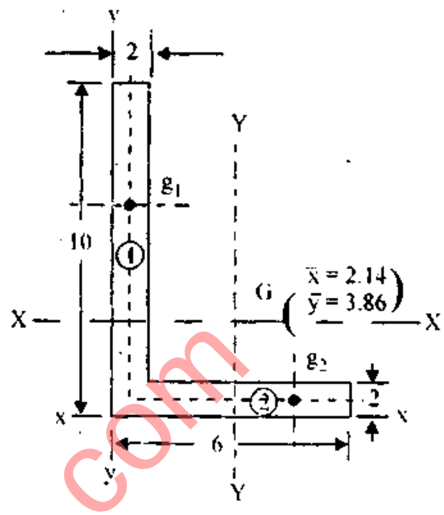
The distance of centroid from the axis $y - y$

$$= \frac{\text{Sum of moment area } A_1 \text{ \& } A_2 \text{ about } y - y}{\text{Total area}}$$

$$\bar{x} = \frac{A_1 \times 1 + A_2 \times (2 + 6/2)}{28}$$

$$= \frac{20 \times 1 + 8 \times 5}{28} = \frac{20 + 40}{28} = 2.14$$

Similarly, the distance of the centroid from the axis $x - x$



$$\bar{y} = \frac{20 \times 5 + 8 \times 1}{28} = 3.86$$

with respect to centroidal axis $x - x$ and $y - y$ the centroid A_1 is g_1 and that of A_2 is g_2

I_{xx} = Moment of inertia of A_1 about $X - X$ axis and moment of inertia of A_2 about $X - X$ axis

$$I_{xx} = \frac{2 \times 10^3}{12} + 20 \times (1.14)^2 + \frac{4 \times 2^3}{12} + 8 \times (2.86)$$

$$= 192.66 + 68.103 = 260.76$$

$$I_{yy} = \frac{10 \times 2^3}{12} + 20 \times (1.14)^2 + \frac{2 \times 4^3}{12} + 8 \times (0.86)$$

$$= 32.66 + 16.58 = 49.24$$

Q. 5. (c) Derive an expression for moment of inertia of a solid sphere about its diameter.

Ans. Consider an elemental plate of thickness dy at distance dy from the diamentral axis. Radius of this elemental circular plate x is given by the relation.

$$x^2 = R^2 - y^2$$

$\therefore$ Mass of elemental plate $dm = \rho \pi x^2 dy = \rho \pi (R^2 - y^2)\, dy$

Moment of inertia of circular plate element about y axis is given by

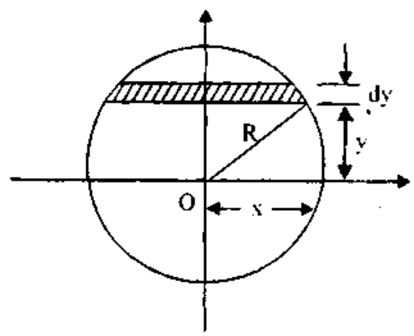
$$= \frac{1}{2} \times \text{mass} \times \text{square of radious} = \frac{1}{2} \times \rho \pi x^2 dy \times x^2 = \rho \frac{\pi}{2} x^4 dy = \rho \frac{\pi}{2} (R^2 - y^2)\, dy$$

$$= \rho \frac{\pi}{2} (R^4 - 2R^2 y^2 + y^4)\, dy$$

$$I_{yy} = 2 \int_0^R \rho \frac{\pi}{2} (R^4 - 2R^2 y^2 + y^4)\, dy$$

$$= \rho\, \pi \left[R^4 y - \frac{2R^2 y^3}{3} + \frac{y^5}{5} \right]_0^R$$

$$= \rho\, \pi\, R^5 \left[1 - \frac{2}{3} + \frac{1}{5} \right] = 8/15\, \rho \pi R^5$$



But mass of sphere $M = 2 \int_0^R dm = 2 \int_0^R \rho \pi x^2 dy$

$$= 2 \int_0^R \rho \pi (R^2 - y^2)\, dy = 2\rho \pi \left(R^2 y - \frac{y^3}{3} \right)_0^R$$

$$M = \frac{4\pi R^3}{3}$$

$\therefore$ $$I_{yy} = \frac{2}{5}MR^2$$

Q. 6. Attempt any two parts of the following : **(5 × 2 = 10)**

Q. 6. (a) The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t - 3t + 3$. Determine the time, position, velocity and distance traveled when acceleration is zero.

Ans.
$$x = 6t^4 - 2t^3 - 12t^2 - 3t + 3$$
$$\frac{dx}{dt} = 24t^3 - 6t^2 - 24t - 3$$
$$\frac{d^2x}{dt^2} = 72t^2 - 12t - 24$$

when acceleration is zero $\frac{d^2x}{dt^2} = 0$

$\therefore$
$$72t^2 - 12t - 24 = 0$$
$$6t^2 - t - 2 = 0$$
$$6t^2 - 4t + 3t - 2 = 0$$
$$2t(3t - 2) + 1(3t - 2) = 0$$
$$(3t - 2)(2t + 1) = 0$$

$\therefore$ $3t = 2$ as t cannot be – ve so we can't take $2t + 1 = 0$

$\therefore$ $t = \frac{2}{3}$

$\therefore$ time $= \frac{2}{3}$

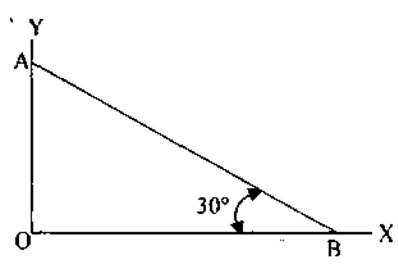
Velocity $= \frac{dx}{dt} = 24t^3 - 6t^2 - 24t - 3$

at $t = 2/3$, Velocity $= 24 \times \left(\frac{2}{3}\right)^3 - 6 \times \left(\frac{2}{3}\right)^2 - 24 \times \left(\frac{2}{3}\right) - 3 = 7.11 - 2.67 - 16 - 3 = 14.56$ m/sec

Displacement or position at $t = 2/3$

$$x = 6 \times \left(\frac{2}{3}\right)^4 - 2 \times \left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 - 3 \times \left(\frac{2}{3}\right) + 3 = 1.21 - 0.59 - 5.33 - 2 + 3 = 3.71 \text{ m}$$

Q. 6. (b) A straight link AB of length 50 cm is shown in the figure. The end B of the link moves along x-axis with a velocity of 4 m/s and accelerates with an acceleration of 10 m/s^2. The end A is constrained to move along y-axis. Find the velocity and acceleration of the end A at the given instant.



Ans. Given:

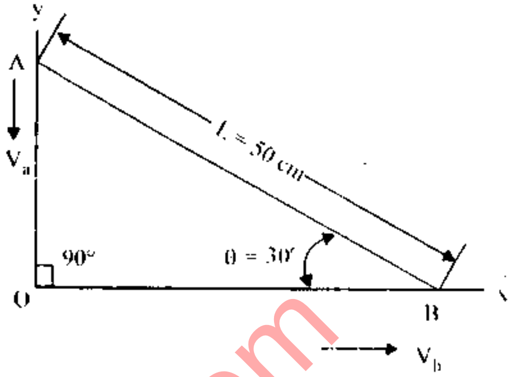
Velocity of end of B link AB 4 m/sec

$\therefore \quad v_B = 4$ m/sec

acceleration $a_B = 10\ m/\sec^2$

From figure we can write $x^2 + y^2 = L^2$

Differential w.r.t to time t



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\left(\frac{x}{y}\right)\left(\frac{dx}{dt}\right)$$

We know that $\frac{dy}{dt}$ = Velocity of end point $A = v_A$

$\frac{dx}{dt}$ = Velocity of end point $B = v_B$

$$\tan\theta = x/y$$

$v_A = -(x/y)\, v_B = -\tan\theta\,(v_B) = -\tan 30°(4)$

$= -2.31$ m/sec is the velocity of end A.

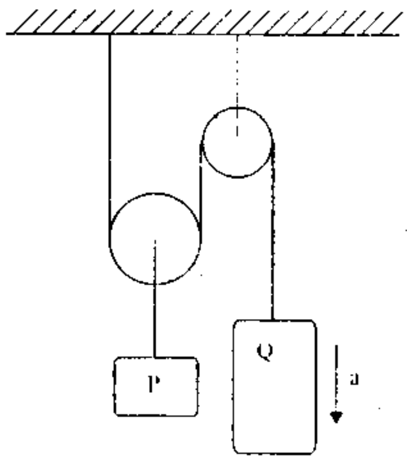
$\therefore$ acceleration of end point A

$$\frac{dv_A}{dt} = -\left[\frac{dv_B}{dt}\tan\theta + v_B\frac{d(\tan\theta)}{dt}\right]$$

acceleration of end point $A\left(\frac{dv_A}{dt}\right) = -[a_B\tan\theta - v_B\sec^2\theta]$

$= -[10\tan 30° - 4\sec^2 30°] = -[5.77 - 12] = 6.23$ m/sec^2

Q. 6. (c) Two weights P and Q are connected by the arrangement shown in figure. Neglecting friction and the inertia of the pulleys and cord, find the acceleration a of the weight Q. Assume that $P = 178\ N$ and $Q = 133.5\ N$.



Ans. The tension in the string is same as the pulley is smooth. Let T = Tension of the spring, a = acceleration of block Q.

Acceleration of block P is half of the the acceleration of block Q so that $a/2$ considering the motion of block Q

Net force of block $Q = (T - 133.5)$ in upwards direction

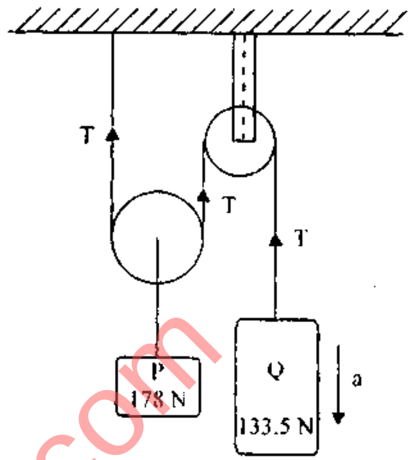
Acceleration of motion of block $Q = a$

$$(T - 133.5) = \frac{133.5}{9.81}a$$

$$\therefore \quad T = 133.5 + \frac{133.5}{9.81}a \quad \ldots(1)$$

Considering the motion of block P

Net force on block $P = (178 - 2T)$ in down direction.



Acceleration on of block A $= \dfrac{a}{2}$

$$(178 - 2T) = \frac{178}{9.81} \times \frac{a}{2}$$

$$\therefore \qquad 2T = 178 - \frac{178}{9.81} \times \left(\frac{a}{2}\right) \qquad \text{...(2)}$$

Multiplying equation (1) by 2 both sides

$$2T = 133.5 \times 2 + \frac{133.5 \times 2}{9.81} a \qquad \text{...(3)}$$

Substracted equ. (2) – (3) we get

$$178 - 133.5 \times 2 - \frac{178}{9.81}\left(\frac{a}{2}\right) - \frac{133.5 \times 2}{9.81} a = 0$$

$$\therefore \qquad 89 = 9.07\, a + 27.22\, a$$

$$\therefore \qquad a = 2.45 \text{ m/sec}^2$$

Q. 7. Attempt any two parts of the following: **$(5 \times 2 = 10)$**

Q. 7. (a) Define Poisson's ratio. Prove that its value lies between zero and half.

Ans. When a homogeneous material is loaded within its elastic limit the ratio of the lateral strain to the linear strain is constant and is known as Poission ratio.

It is denoted by μ or $\dfrac{1}{m}$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}}$$

$$\therefore \qquad \mu \text{ or } \frac{1}{m} = -\frac{e_L}{e}$$

$$\therefore \text{Lateral strain} = -\mu \times e = \frac{1}{m} \times e$$

Q. 7. (b) Determine the dimensions of a simply supported rectangular steel beam 6 m long to carry a brick wall 250 mm thick and 3 m high, if the brick weighs 20 kN/m^3 and maxmium permissible bending stress is 800 N/cm^2. The depth of a beam is 1.50 times its width.

Ans. Volume of brick wall $= 6 \times \dfrac{250}{1000} \times 3 = 4.5 \text{ m}^3$

Weight of brick 20 kN/m^3

Total weight of brick $20 \times 4.5 = 90$ kN $\therefore \omega = \dfrac{90}{6} = 15$ kN/m

Bending stress $\sigma_b = 800 \text{ N/cm}^2 = \dfrac{800}{(.01)^2} \text{ N/m}^2 = 8 \text{ Mpa} = 8 \times 10^6 \text{ N/m}^2$

For simply supported beam carry U.D.L

$$M_{max} = \frac{\omega l^2}{8} = \frac{\omega \times 6^2}{8} = 4.5\,\omega = 4.5 \times 15$$

For rectangular steel beam $I = \dfrac{bd^3}{12}$

$$y = \frac{d}{2}$$

$d = 1.5\,b$ is given in the problem.

By using bending equation $\dfrac{M}{I} = \dfrac{\sigma_b}{y}$

$$\frac{4.5 \times 15}{\frac{bd^3}{12}} = \frac{8 \times 10^6}{\frac{d}{2}}$$

$$\therefore \quad \frac{bd^3}{12} \times \frac{2}{d} = \frac{4.5 \times 15}{8 \times 10^6}$$

$$bd^2 = (50.63 \times 10^{-6})\ m^3$$

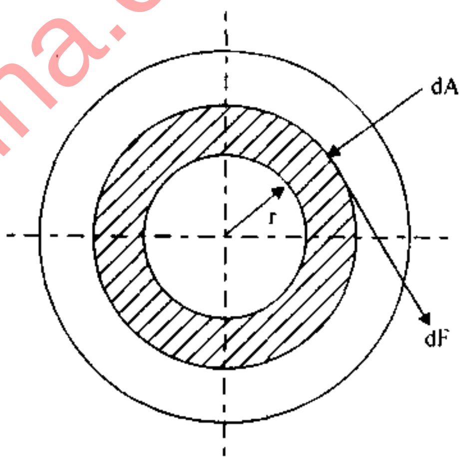
$$b \times (1.5\,b)^2 = (50.63 \times 10^{-6})$$

$$\therefore \quad b^3 = 22.5 \times 10^{-6}$$

$$b = 28.23\ mm$$

$$d = 1.5\,b = 42.35\ mm$$

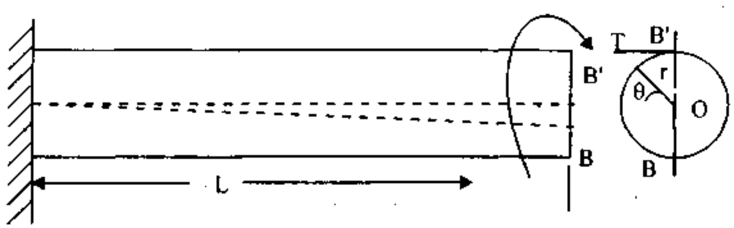
are the dimension of beam.



Q. 7. (c) For torsion of a circular shaft, derive the torsion equation. State all the assumptions at the beginning.

Ans. Assumption for torsion equation.

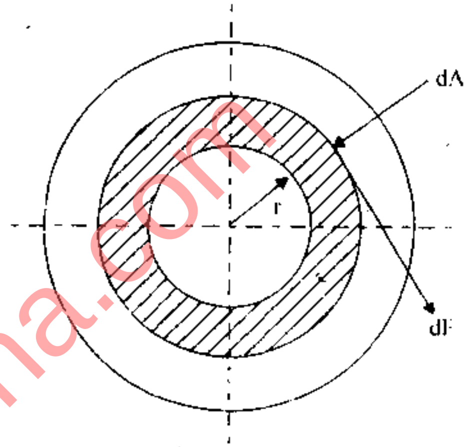
1. A plane section material perpendicular to the axis of the circular member remains plane after the torques are applied and wrapage or distortion occurs.
2. Circular sections remain circular.
3. In a circular member subjected to torque, shearing strains vary linearly from the central axis.
4. Shaft is loaded by twisting couples in planes that are perpendicular to the axis of shaft.



5. Stresses don't exceed the proportional limit. Thus it follows the shearing stress is proportional to shearing strain.

Consider of a shaft of radius R and length L is subjected to a torque T on the free end and other end is fixed. Initially the line of shaft is horizontal AB before twisting and after twisting it takes position AB'.

Angle of twist $\angle BAB' = \phi$ and shear strain $\angle BOB' = \theta$



From longitudinal section $\tan\phi = \dfrac{BB'}{L}$

$$BB' = L\phi \text{ as } \tan\phi = \phi$$

For cross section length of arc= Radius × Angle

$\tan\theta = \theta$ consider

$BB' = r\theta$

$$L\phi = r\theta$$

$$\phi = \frac{r\theta}{L}$$

Shear modulus $G = \dfrac{\text{shear stress}}{\text{shear strain}} = \tau/\phi$

$$\phi = \tau/G$$

$$\frac{\tau}{G} = \frac{r\theta}{L}$$

$$\frac{\tau}{r} = \frac{G\theta}{L}$$

Maximum shear stress occur at radius R

$\therefore \qquad \tau/R = \dfrac{G\theta}{L}$

Now consider an elemental ring of area dA at radius r from centre O.

Shear force on elemental ring = Stress × area

$$dF = \frac{G\theta}{L} r \times dA$$

Torsional moment n ring $dT = dF \times r = \left(\dfrac{G\theta}{L} r\,dA\right) \times r$

$$dT = \frac{G\theta}{L} r^2 dA$$

Now torsional moment on whole section

$$\int dt = \frac{G\theta}{L} \int r^2 dA$$

but we know $\int r^2 dA$ = polar moment of interia = J

$$T = G\theta/L \times J$$

$$\frac{T}{J} = \frac{G\theta}{L}$$