

FIRST SEMESTER EXAMINATION, 2008-09

ENGG. MECHANICS

Time : 3 Hours]

[Total Marks : 100

Note : (i) This paper is in three sections. Section A carries 20 marks. Section B carries 30 marks and Section C carries 50 marks.

(ii) Attempt all questions. Marks are indicated against each questions part.

(iii) Assume missing data suitably, if any.

SECTION-A

Q.1. You are required to answer all the parts : $2 \times 10 = 20$

Choose correct answer for the following two parts :

(a) Mass moment of inertia of a thin circular disc about its polar axis is

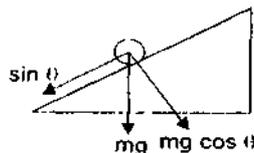
- (i) $\frac{MR^2}{2}$ (ii) $\frac{MR^2}{4}$
 (iii) $\frac{MR^4}{2}$ (iv) $\frac{MR^4}{4}$

Ans. (i)

Q. 1 (b). When a body slides down an inclined surface (making angle θ with horizontal), the acceleration of the body is

- (i) g
 (ii) $g \cos \theta$
 (iii) $g \sin \theta$
 (iv) none of the above

Ans. (iii).



because from this figure $g \sin \theta$ is responsible to accelerate the body

Fill in the blanks for the following three parts :

You will be awarded full marks, if all the criteria in a part are correct (otherwise will be awarded zero).

(c) In method of sections (truss), the section must pass through not more than members; and the equations of equilibrium are given as

Ans. Three, $\sum F_x = 0, \sum F_y = 0, \sum M = 0$

(d) A body of mass 2.5 kg is moving with a constant velocity of 5 m/s. In order to bring it to rest at a distance of 4m, the work done is..... and force required is

Ans. $m = 2.5 \text{ kg}, u = 5 \text{ m/sec}, v = 0, s = 5 \text{ m}$

$$v^2 = u^2 - 2fs$$

$$\therefore f = \frac{5^2}{2 \times 4} = 3.125 \text{ m}^2/\text{sec}$$

$$R = m \times f = 2.5 \times 3.125 = 7.813 \text{ N}$$

(Force required)

$$\text{Work done} = P \times S = 7.813 \times 4$$

$$= 31.25 \text{ N-m } (-ve)$$

(e) Maximum possible value (theoretical) for the poisson's ratio is and the actual value for the cork is approximately

Ans. 0.5, 0.3

Match the column for the following three parts :

You will be awarded full marks, if all the matches in a part are correct (otherwise will be awarded zero).

(f) Match the following columns :

Column I	Column II
(i) Non uniform straight line motion	(P) $a_n = 0$ and $a_t = 0$
(ii) Non uniform curvilinear motion	(Q) $a_n \neq 0$ and $a_t \neq 0$
(iii) Uniform straight line motion	(R) $a_n \neq 0$ and $a_t = 0$
(iv) Uniform curvilinear motion	(S) $a_n = 0$ and $a_t \neq 0$

Where a_n is radial acceleration and a_t is linear acceleration.

Ans. (i) (S)

(ii) (Q)

(iii) (P)

(iv) (R)

(g) Match the following columns :

Column I	Column II
(i) Uniformly distributed load	(P) Shear force remains constant
(ii) Uniformly varying load	(Q) Shear force varies suddenly
(iii) Concentrated load	(R) Shear force varies linearly
(iv) No load	(S) Bending moment varies linearly

Ans. (i) (S)

(ii) (R)

(iii) (P)

(iv) (Q)

(h) Match the following columns

Column I	Column II
(i) Rectangle	(P) $\frac{\pi D^4}{64}$
(ii) Triangle	(Q) $\frac{bh^3}{12}$
(iii) Circle	(R) $\frac{\pi R^4}{8}$
(iv) Semicircle	(S) $\frac{bh^3}{36}$

Column II gives the value of moment of inertia about a centroidal axis.

Ans. (i) (Q)

(ii) (S)

(iii) (P)

(iv) (R)

(i) Choose correct answer for the following two parts :

(i) STATEMENT-1

As compared to a solid shaft of outer diameter D , a hollow shaft of same outer diameter will have smaller angle of twist per unit length when subjected to same torque.

and

STATEMENT-2

For the same cross sectional area a hollow shaft has larger polar moment of inertia as compared to solid shaft.

(i) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1.

(ii) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(iii) STATEMENT-1 is True, STATEMENT-2 is False.

(iv) STATEMENT-1 is False, STATEMENT-2 is True.

$$\text{Ans. (iii)} \quad \frac{T}{\pi D^4} = \frac{C\theta}{l}$$

$$\frac{Tl}{C \times \frac{\pi D^4}{32}} = \frac{32Tl}{C\pi D^4}$$

$$\text{for solid shaft } I_p = \frac{\pi D^4}{32}$$

$$\theta_2 = \frac{32Tl}{C\pi(D^4 - d^4)} \quad \text{for hollow shaft}$$

$$I_{p2} = \frac{\pi(D^4 - d^4)}{32}$$

$I_{p1} > I_{p2}$ ∴ Statement 2 is false

(i) is true.

(j) STATEMENT-1

In the case of pure bending the radius of curvature of neutral axis varies linearly.

and

STATEMENT-2

In pure bending the magnitude of bending moment remains constant.

(i) STATEMENT-1 is True, STATEMENT-2 is True. STATEMENT-2 is a correct explanation for STATEMENT-1.

(ii) STATEMENT-1 is True, STATEMENT-2 is True, STATEMENT-2 is NOT a correct explanation for STATEMENT-1.

(iii) STATEMENT-1 is True. STATEMENT-2 is False.

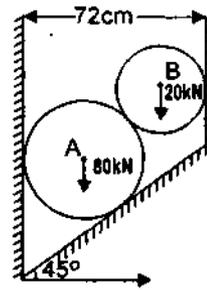
(iv) STATEMENT-1 is False, STATEMENT-2 is True.

Ans. (iv)

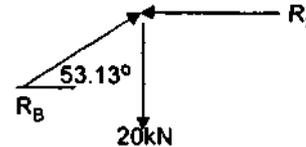
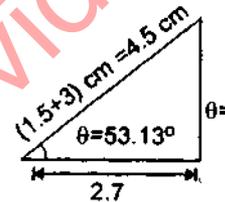
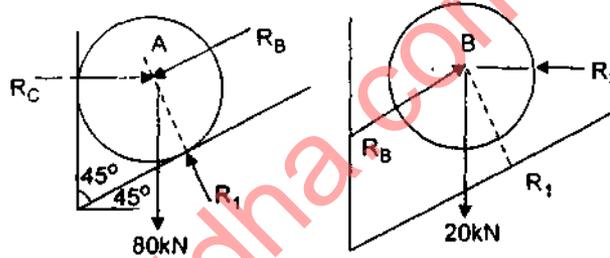
Section B

Q.2 Answer any three parts of the following: $10 \times 3 = 30$

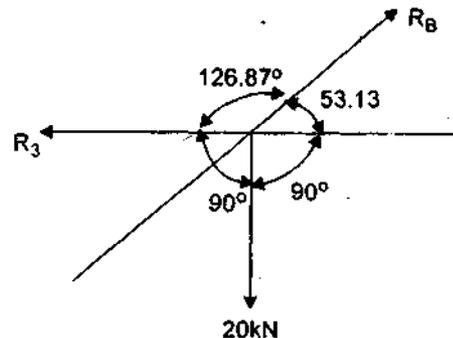
(a) Two cylinders B and A of diameters 3 cm and 6 cm weighing 20 kN and 80 kN respectively are placed as shown in figure. Assuming all the contact surfaces to be smooth, find the reactions at the walls.

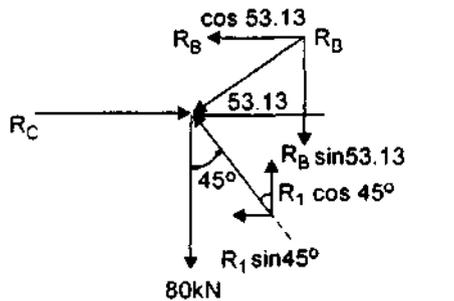


Ans. Fig.



From Larn's theorem





$$\frac{R_B}{\sin 90^\circ} = \frac{20}{\sin 126.87} = \frac{R_3}{\sin (53.13 + 90^\circ)}$$

$$\therefore R_B = 25 \text{ kN}$$

$$R_3 = 15 \text{ kN}$$

By equilibrium condition,

$$\sum F_x = R_C - R_B \cos 53.13 - R_1 \sin 45^\circ = 0 \quad \dots(1)$$

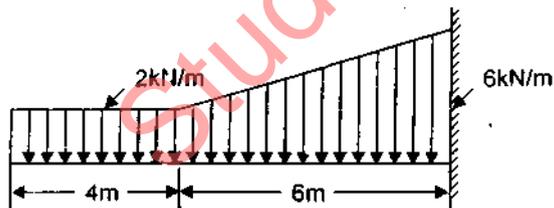
$$\sum I_y = -80 + R_1 (0.5 \times 4.5) - R_B \sin 53.13 = 0 \quad \dots(2)$$

$$\therefore R_1 = 141.42 \text{ kN}$$

$$R_C = 115 \text{ kN}$$

Reactions are 141.42 kN, 115 kN, 25 kN and 15 kN

Q. 2 (b) Find the shear force and moment equation for the cantilever beam shown in figure. Also sketch the shear force and

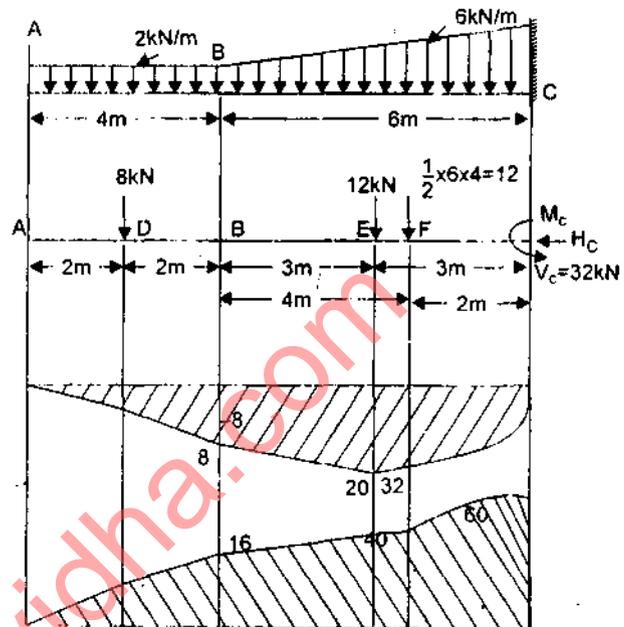


bending moment diagram.

Ans. The loading on the beam can be considered to be composed of U.D.L. of 2 N/m and gradually varying load of zero intensity at B and 4 kN/m at C.

$$\sum M_C = H_C \times 0 + V_C \times 0 + M_C + 12 \times 2 + 12 \times 3 + 8 \times 8 = 0$$

$$M_C = -124 \text{ kN-m}$$



$$V_C = -12 + 12 + 8 = 32 \text{ kN}$$

S.F. Calculation :

S.F. between DA = 0

S.F. between BD = -8 kN

S.F. between BE = -8 kN

S.F. between FE = -20 kN

S.F. between FC = -20 + 12 = -8 kN

S.F. at C = -32 + 32 = 0

B.M. Calculation :

B.M. of point A = 0

B.M. at point D = 0

B.M. of point B = $8 \times 2 = 16 \text{ kN-m}$.

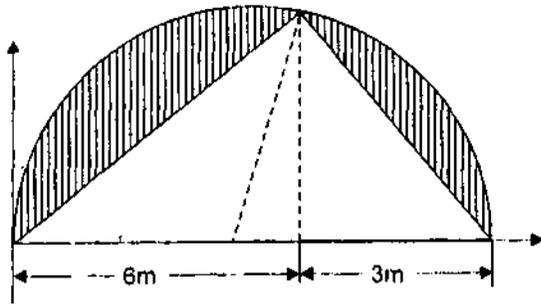
B.M. of point E = $8 \times 5 = 40 \text{ kN-m}$

B.M. of F = $8 \times 6 + 12 \times 1 = 60 \text{ kN-m}$

B.M. of point

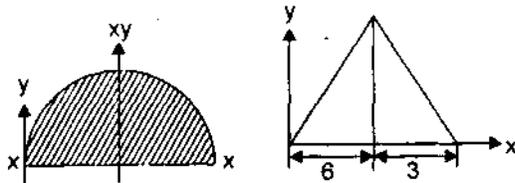
C = $-8 \times 8 + 12 \times 3 + 12 \times 2 = 0 \text{ kN-m}$

Q.2 (c) A triangle is removed from a semicircle as shown in figure. Locate the centroid of the remaining object.



Ans. Area (A) $x_1 y_1 A_1 x_2 A_2 y_2$

$$A_1 = \frac{\pi}{8} D^2 = 31.81 \text{ cm}^2$$



$$x_1 = \frac{9}{2} = 4.5 \text{ cm}$$

$$y_1 = \frac{4\pi}{3\pi} = 1.9$$

$$A_1 x_1 = 142.145 \text{ cm}^3$$

$$A_1 y_1 = 60.759 \text{ cm}^3$$

$$A_2 = \frac{1}{2} \times 9 \times 4.5 = 20.25 \text{ cm}^2$$

$$x_2 = 4.5 \text{ cm}^2$$

$$y_2 = \frac{4.5}{13} = 1.5 \text{ cm}$$

$$A_2 x_2 = 91.125 \text{ cm}^3$$

$$A_2 y_2 = 30.375 \text{ cm}^3$$

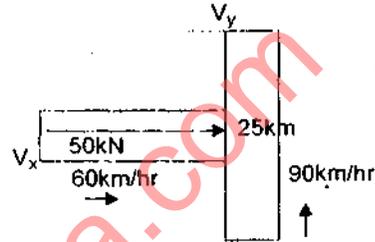
$$\therefore \bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2} = \frac{142.145 - 91.125}{31.81 - 20.25} = 4.5 \text{ cm}$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{60.759 - 30.375}{31.81 - 20.25} = 2.628 \text{ cm}$$

Q.2 (d) A vehicle weighing 50 kN moves with a velocity of 60 km/hr along x-direction. Another vehicle weighing 25 kN moving

along y-direction with a velocity of 90 km/hr, collides with it. If two vehicles get entangled after collision, determine their common velocity.

Ans. Let the velocity of the vehicles after collision be V_x in x-direction (V_y in y direction). Applying impulse momentum equation along x-direction we get



$$50 \times 60 + 0 = \frac{(50 + 25)}{9.81} \times V_x$$

$$\therefore V_x = 40 \text{ km/hr}$$

Applying impulse momentum equation in y-direction,

$$0 + \frac{25 \times 90}{9.8} = \frac{(25 + 50)}{9.8} \times V_y \therefore V_y = 19.57$$

km/hr

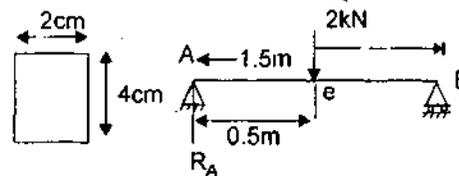
\therefore Resultant velocities

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{40^2 + 19.57^2}$$

$$= 44.53 \text{ km/hr}$$

Q.2 (e) Why I-section beam is preferred over a rectangular-section beam? A simply supported beam, 2 cm wide by 4 cm high and 1.5 m long is subjected to a concentrated load of 2 kN (perpendicular to beam) at a point 0.5 m from one of the supports. Determine : (i) the maximum fiber stress and (ii) the stress is a fiber located 1 cm from the top of the beam at mid-span.

Ans. Bending stress (σ_b) = $\frac{M_y y}{I}$



For rectangular section $I = \frac{bd^3}{12}$ where b and d are width and depth of the section. For I section value of I is less than previous one for same b and d . So bending stress is increased in I section. for this reason I reaction beam is preferred over a rectangular.

$$I = \frac{bd^3}{12} = \frac{2 \times 4^3}{12} = 10.667 \text{ cm}^3$$

Bending moment: $R_A + R_B = 2$,

$$\sum M_A = 2 \times 0.5 - R_B \times 1.5 = 0$$

$$\therefore R_B = 0.667 \text{ kN}, R_A = 1.333 \text{ kN}$$

B.M. at point A = 0,

$$\text{B.M. at point C} = 1.333 \times 0.5$$

$$= 0.6665 \text{ kN-m}$$

$$\text{B.M. at point B} = 1.333 \times 1.5 - 2 \times 1 = 0$$

$$\therefore \text{Max. Bending moment (M)} = 0.6665 \text{ kN-m}$$

$$\text{kN-m} = 0.6665 \times 10^2 \text{ kN-m.}$$

(i) Maximum bending stress

$$\frac{M_y}{I} = \frac{0.6665 \times 10^2 \times 2}{10.667}$$

$$= 12.496 \text{ kN/cm}^2$$

(ii) Stress in a fiber located 1 cm from the top of the beam at mid span,

$$\frac{0.6665 \times 10^2 \times 0.5}{0.1667} = 199.91 \text{ k-N/cm}^2$$

Section-C

Q.3. Answer any two parts of the following: $5 \times 2 = 10$

(a) Explain the following:

(i) Lami's theorem

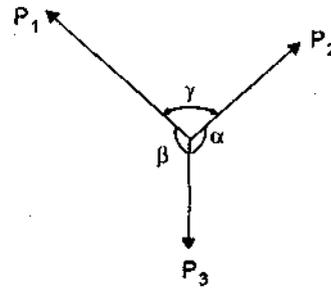
(ii) Principle of transmissibility of forces

(iii) Conditions of equilibrium for coplanar forces and concurrent forces

Ans. (i) **Lami's Theorem** – If a body is in equilibrium under the action of three forces, each force is proportional to the sine of the angle between the other two forces.

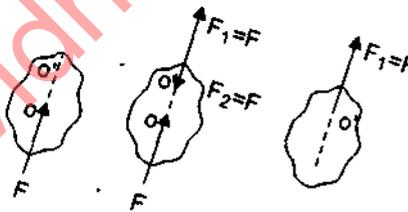
For the system of forces shown in Figure

$$\frac{P_1}{\sin \alpha} = \frac{P_2}{\sin \beta} = \frac{P_3}{\sin \gamma}$$



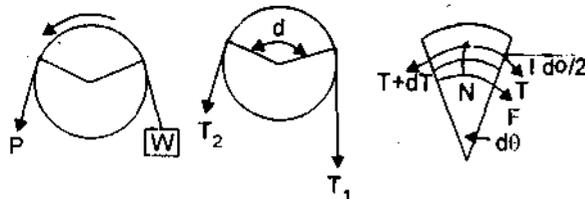
(ii) **Principle of Transmissibility of forces** – It states that if a force acting at a point on a rigid body is shifted to any other point which is on the line of action of the force the external effect on the force on the body remains unchanged.

$$(iii) \sum F_x = 0, \sum F_y = 0, \sum M = 0$$



Q.3 (b) Derive the following expression for the belt, where all symbols have their usual meaning, $T_1 > T_2$, in belt-pully arrangement for power transmission $\frac{T_1}{T_2} = e^{\mu \theta}$

Ans.



A load W being pulled by a force P over fixed drum. Let the force on slack side T_2 and tight side T_1 . T_1 is more than T_2 the angle of contact between rope and drum. Now consider an elemental length of rope. Let T be the force on slack side and $T + dT$ on tight side. There will be normal reaction N on the rope in the

radial direction and functional force $F = \mu N$ in the tangential direction. Then

$$\Sigma \text{ forces in radial direction} = 0$$

$$N - T \sin \frac{d\theta}{2} - (T + dT) \sin \frac{d\theta}{2} = 0$$

Since $d\theta$ is small $\sin \frac{d\theta}{2} \approx \frac{d\theta}{2}$

$$N \cdot T \frac{d\theta}{2} - (T + dT) \frac{d\theta}{2} = 0$$

$$\text{i.e., } N = \left(T + \frac{dT}{2} \right) d\theta$$

From law of friction

$$F = \mu N = \mu \left(T + \frac{dT}{2} \right) d\theta$$

where μ is coefficient of friction

Now Σ Forces in tangential direction = 0

$$(T + dT) \cos \frac{d\theta}{2} - F + T \cos \frac{d\theta}{2}$$

Since $\frac{d\theta}{2}$ is small, $\cos \frac{d\theta}{2} = 1$

$$T + dT - F + T$$

$$\therefore dT = F = \mu \left(T + \frac{dT}{2} \right) d\theta$$

$$\therefore dT = \mu \left(T + \frac{dT}{2} \right) d\theta$$

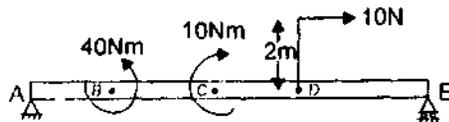
Integrating both side

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu d\theta \ln \frac{T_1}{T_2} = \mu \theta \therefore \frac{T_1}{T_2} = e^{\mu \theta}$$

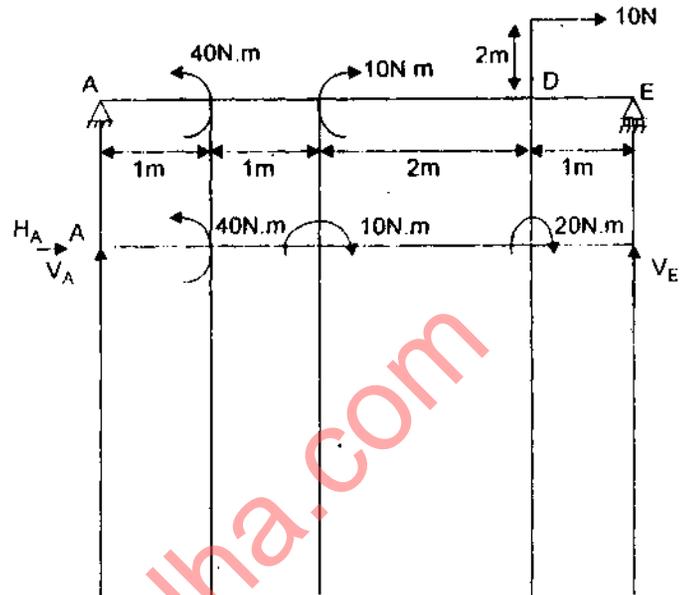
$$\therefore T_1 = T_2 e^{\mu \theta} \text{ as } T_1 > T_2$$

Q.3 (c) Determine the reactions at A and

E.

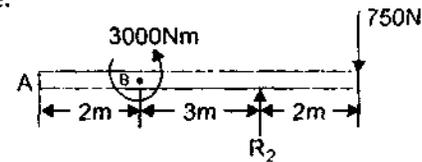


Ans. Data insufficient. So problem will not solve.

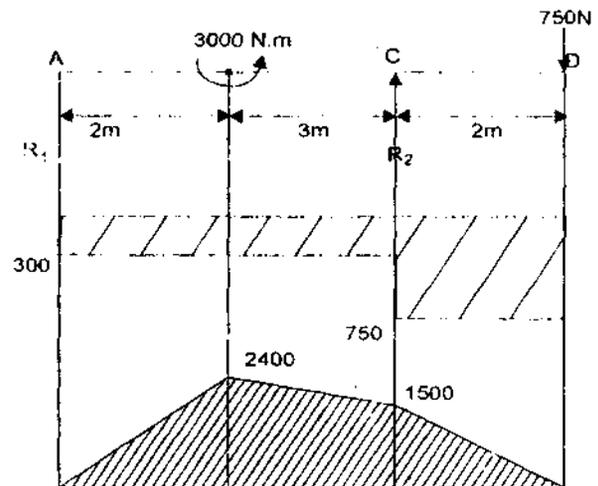


Q.4. Answer any one part of the following:

(a) Draw the shear force and bending moment diagram for the beam shown in figure.



$$\text{Ans. } \Sigma M_A = 3000 + R_2 \times 5 - 750 \times 7 = 0$$



$\therefore R_2 = 450 \text{ N}$
 $\sum F_y = 0 \quad R_1 + R_2 - 750 = 0$
 $\therefore R_1 = 300 \text{ N}$

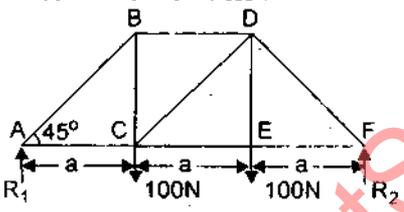
Calculation for S.F. diagram

S.F. between BA = + $R_1 = 300 \text{ N}$
 S.F. between BC = 300 N
 S.F. of CD = 300 + 450 = 750 N
 S.F. at D = 0

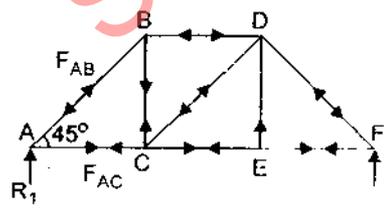
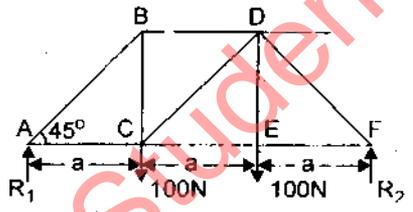
Calculation for B.M. diagram

B.M. at point A = 0
 B.M. at point B = $300 - 300 \times 2 = 2400 \text{ Nm}$
 B.C. at point
 $C = -300 \times 5 + 300 = 1500 \text{ N-m}$
 B.M. at point D = $-300 \times 7 + 3000 - 450 \times 2 = 0$

Q.4 (b) For the truss shown in figure, find the force in the members :



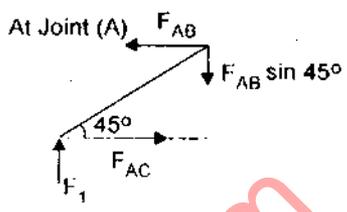
Ans.



$\sum F_{xy} = 0$
 $\therefore R_1 + R_2 = 100 + 100 = 200 \text{ N}$
 $\sum M_A = R_1 \times 100 \times a - 100 \times 2a + R_2 \times 3a = 0$

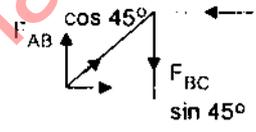
$\therefore R_2 = \frac{3000a}{3a} - 100 \text{ N}$
 $R_2 = 100 \text{ N}$

By Joint Method



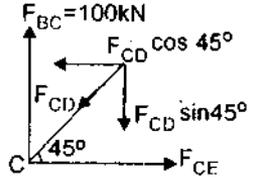
$F_{AB} \cos 45^\circ = F_{AC}$
 $F_{AB} \sin 45^\circ = R_1 = 100 \text{ N}$
 $F_{AB} = \frac{100}{\sin 45^\circ} = 141.44 \text{ N}$

At Joint (B)



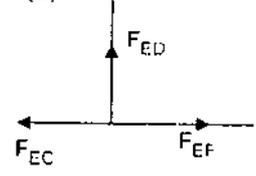
$F_{AB} \sin 45^\circ = F_{BD}$
 $\therefore F_{BD} = 100 \text{ kN}$
 $F_{AB} \cos 45^\circ = F_{BC}$
 $\therefore F_{BC} = 100 \text{ kN}$

At Joint (C)



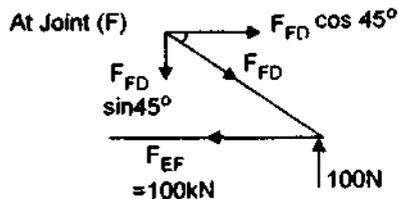
$F_{BC} = F_{CD} \sin 45^\circ$
 $F_{CD} = 141.44 \text{ N}$
 $F_{CE} = F_{CD} \cos 45^\circ = 100 \text{ N}$

At Joint (E)



$F_{ED} = 0$

$$F_{EF} = F_{CE} = 100 \text{ N}$$



$$F_{FD} \cos 45^\circ = 100 \text{ N}$$

$$F_{FD} \sin 45^\circ = 100 \text{ N}$$

$$F_{FD} = 141.44 \text{ N}$$

$$F_{AB} = 141.44 \text{ N (C)}$$

$$F_{BD} = 100 \text{ N (C)}$$

$$F_{BC} = 100 \text{ N (T)}$$

$$F_{CD} = 141.44 \text{ N (C)}$$

$$F_{CE} = 100 \text{ N (T)}$$

$$F_{ED} = 0$$

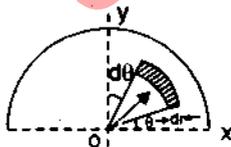
$$F_{EF} = 100 \text{ N (T)}$$

$$F_{FD} = 141.44 \text{ N (C)}$$

Q.5. (a) Answer any two parts of the following—

(a) Derive an expression for the centroid of semicircular arc.

Ans. Centroid of Semicircle—Consider the semicircular of radius R as shown in figure. Due to symmetry centroid must lie on y axis. Let its distance from diametral axis be \bar{y} . To find \bar{y} , consider on elements at a distance r from the centre O of the semicircle, radial width being dr and bound by radii at θ and $\theta + d\theta$, Area of element $= r\theta dr$



The moment about diametral axis x is given by

$$r d\theta \times dr \times r \sin \theta = r^2 \sin \theta dr d\theta$$

\therefore Total moment of area about diametral axis

$$\int_0^\pi \int_0^R r^2 \sin \theta dr d\theta = \int_0^\pi \left[\frac{r^3}{3} \right]_0^R \sin \theta d\theta$$

$$= \frac{R^3}{3} [-\cos \theta]_0^\pi = \frac{R^3}{3} [1 + 1] = \frac{2R^3}{3}$$

$$\therefore \bar{y} = \frac{\text{Moment of area}}{\text{Total area}} = \frac{\frac{2R^3}{3}}{\frac{1}{2} \pi R^2} = \frac{4R}{3\pi}$$

Q. 5. (b) Explain any three of the following—

(i) Second moment of area

(ii) Radius of Gyration

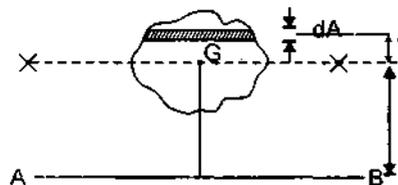
(iii) Parallel axis theorem

(iv) Perpendicular axis theorem

Ans. (i) Second moment area : It is the product of mass with the square of the perpendicular distance of its centre gravity from axis of reference.

(ii) Radius of Gyration : It is the distance from axis of reference where the whole mass (or area) is assumed to be concentrated so as not to alter the moment of inertia about the given axis.

If whole mass is concentrated of a body at a distance K from the axis of reference. The moment of inertia of the whole area A about the given axis is equal to Ak^2 .



$$\text{If } Ak^2 = I \text{ then } k = \sqrt{\frac{I}{A}}$$

k = radius of gyration about the given axis.

(iii) Parallel axis theorem—

A = Area of plane figure

I_G = Moment of inertia of the given area about C.G. $= I_{xx}$

I_{AB} = Moment of inertia of the given area about AB axis.

h = Distance between C.G. of area (i.e., xx axis) and axis AB parallel to XX axis.

$$I_{AB} = I_G + Ah^2$$

(iv) **Pendicular axis theorem**...

I_{xx} = Moment of inertia of the given area about $x-x$ axis.

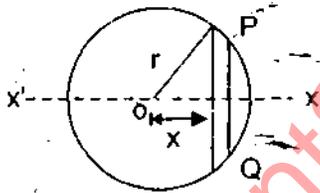
I_{yy} = Moment of inertia of the given area about $y-y$ axis.

I_{zz} = Moment of inertia of the given area about $z-z$ axis.

Then, $I_H = I_{xx} + I_{yy}$

Q.5. (c) Derive an expression for mass moment of inertia of solid sphere about axis passing through its centre.

Ans. A solid sphere of radius r is shown. Take any two reactions at a distance x and $(x + dx)$ from O . Then radius of cut part is $\sqrt{r^2 - x^2}$



Area of this elementary part = $\pi(r^2 - x^2)$

Volume of thin elementary part

$$= \pi(r^2 - x^2) dx$$

If M = Mass of sphere then

$$\text{Mass per unit volume} = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

\therefore Mass of elementary part

$$= \frac{3M}{4\pi r^3} \times \pi(r^2 - x^2) dx$$

\therefore M.I. of elementary part about $X'O'X'$

$$= \text{mass} \times \frac{(\text{Radius})^2}{2}$$

$$= \frac{3M}{4\pi r^3} \times \pi(r^2 - x^2) \cdot dx \cdot \left(\frac{r^2 - x^2}{2} \right)$$

$$= \frac{3M}{8r^3} (r^2 - x^2) dx$$

\therefore Moment of inertia of sphere

$$= \int_{-r}^r \frac{3m}{8r^3} (r^2 - x^2)^2 dx$$

$$= \frac{3m}{8r^3} \int_{-r}^r (r^4 + x^4 - 2r^2x^2) dx$$

$$= \frac{3M}{8r^3} \left[r^4(r+r) + \frac{1}{5}(r^5 + r^5) - \frac{2}{3}r^2(r^3 + r^3) \right]$$

$$= \frac{2}{5} Mr^2$$

Q. 6. Answer any one part of the following: 10

Q. 6. (a) A sphere, a cylinder and a hoop, each having the same mass and radius are released from rest on an incline. Determine the velocity of each body after it has rolled through a distance corresponding to a change in elevation h .

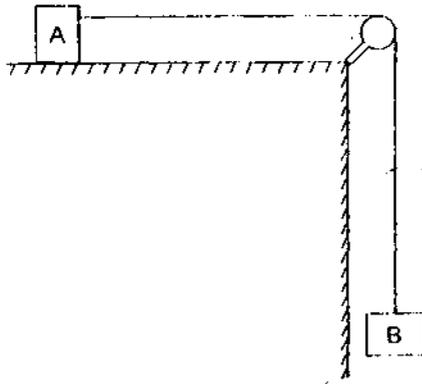
Ans. Data insufficient. So problem will not solve.

$$\text{Solid sphere velocity} = \sqrt{\frac{10gh}{7}}$$

$$\text{Solid cylinder velocity} = \sqrt{\frac{4gh}{3}}$$

$$\text{Hoop velocity} = \sqrt{gh}$$

Q.6 (b) A body of mass 25 kg resting on a horizontal table is connected by a string passing over a smooth pulley at the edge of the table to another body of mass 3.75 kg and hanging vertically as shown in fig. Initially, the friction between 25 kg mass and the table is just sufficient to prevent the motion. If an additional 1.25 kg is added to the 3.75 kg mass, find the acceleration of the masses.



Ans. Let f is acceleration of the system.

Reverse effective force (inertial force) on weight W_1 -- (mass of W_1) \times Acceleration

$$= \frac{W_1}{g} \times f$$

Reversed effective force (inertial force) on weight $W_2 = -\frac{W_2}{g} \times f$

According to D'Alembert principle, the net external force acting on the system in the direction of motion and the resultant reversed effective forces should be in equilibrium.

$$W_1 - \frac{W_1}{g} f - \frac{W_2}{g} f = 0$$

$$-W_1 - (W_1 + W_2) \frac{f}{g} = 0$$

$$f = \frac{gW_1}{W_1 + W_2} = \frac{9.8 \times (1.25 + 3.75)}{(1.25 + 3.75) + 25}$$

$$= 1.63 \text{ m}^2/\text{sec}$$

Q.7. Answer any two parts of the following— 8 \times 2 = 10

Q. 7. (a) Derive the following expression for the elastic constants :

$$K = \frac{E}{3(1 - 2\mu)}$$

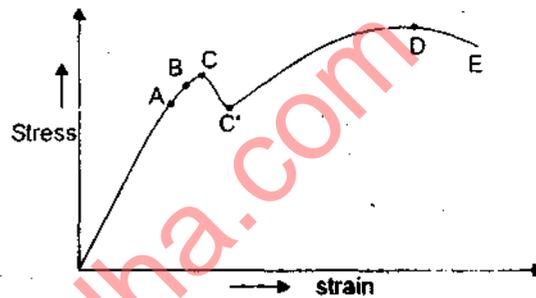
Ans. K = Bulk modulus

$$\frac{\text{Fluid pressure or hydrostatic pressure}}{\text{Volumetric strain}} = \frac{\sigma}{\epsilon_0}$$

$$\frac{\sigma}{E} = \frac{3\sigma}{3(1 - 2\mu)}$$

Q.7. (b) Plot tensile test diagram for Mild Steel and explain all salient points. Derive an expression for strain energy in terms of strain and Young's modulus.

Ans.



A \rightarrow Proportionality limit

B \rightarrow Elastic limit

C \rightarrow Upper yield point

C' \rightarrow Lower yield point

D \rightarrow Maximum or ultimate tensile stress

E \rightarrow Fracture or breaking strength

The strain energy is the energy stored in a body due to the work done on the applied load in straining it. For static load P , the change in length x is proportional to the load.

Work done for gradually applied load = Strain energy (v)

$$U = \frac{1}{2} \times P \times x$$

For a bar of uniform area of cross section A and length L

$$\sigma = \frac{P}{A}$$

$$\rightarrow P = \sigma A$$

$$\epsilon = \frac{x}{L}$$

$$L = \frac{\sigma}{\epsilon}$$

$$E = \frac{PL}{Ax}$$

$$\Rightarrow x = \frac{PL}{AE}$$

$$U = \frac{1}{2} \left(\frac{P}{A} \cdot \frac{L}{E} \right) P = \frac{1}{2} \frac{P^2 L}{AE}$$

$$\Rightarrow U = \frac{1}{2} \frac{P^2 L}{AE}$$

$$U = \frac{1}{2} \left(\frac{P}{A} \right)^2 \frac{AL}{E} = \frac{1}{2} \frac{\sigma^2 AL}{E}$$

Since, $AL = \text{Volume } (v)$

$$U = \frac{1}{2} \frac{\sigma^2 v}{E}$$

$$\therefore \frac{U}{v} = \frac{1}{2} \frac{\sigma^2}{E}$$

Q.7 (c) Calculate the minimum diameter of a solid steel shaft which is not allowed to twist more than 3° in a 6 m length when subjected to a torque of 12 kN-m. Also calculate the maximum shearing stress developed. Take $G = 83 \text{ GPa}$.

Ans. $\theta = 3^\circ$, $l = 6 \text{ m}$, $T = 12 \text{ kN-m}$,
 $G = 83 \text{ GPa}$

$$\frac{T}{I_p} = \frac{C\theta}{l}$$

$$\frac{12 \times 10^3}{\frac{\pi}{32} d^4} = \frac{83 \times 10^9 \times 0.52}{6}$$

$\therefore d = 114 \text{ mm}$

Shearstress f_s

$$\frac{T}{I_p} = \frac{C\theta}{l} = \frac{f_s}{R}$$

$$f_s = \frac{T \times R}{I_p} = \frac{12 \times 10^3 \times \frac{114}{2}}{\frac{\pi}{32} (114)^2}$$

$$= 536.1 \times 10^3 \text{ N/m}^2$$

$$= 536.1 \text{ k-N/m}^2$$

$$f_s = \frac{C\theta R}{l} = \frac{83 \times 10^9 \times 0.52 \times \frac{114}{2}}{6}$$

$$= 41002 \text{ k-N/m}^2 = 41 \text{ MPa}$$

So maximum shearing stress developed
 $41002 \text{ kN/m}^2 = 41 \text{ MPa}$