B.Tech.

First Semester Examination, December-2009

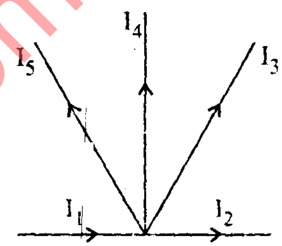
Electrical Technology (EE-101-F)

Note : Attempt any five questions.

Q. 1. (a) State Kirchoff's laws of current and voltage.

Ans. For complex circuit computations, the following two laws first stated by Gustav R. Kirchoff are indispensable :

(i) First Law (Point or Current Law) : "The sum of currents entering a junction is equal to the sum of the currents leaving the junction."



Junction

If the currents towards a junction are considered positive and those away from the same junction negative then this law states that the algebraic sum of all currents meeting at a common junction is zero.

$$\Sigma \text{ current entering} = \Sigma \text{ current leaving}$$

$$I_1 + I_3 = I_2 + I_4 + I_5$$

Or $$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

(ii) Second Law (Mesh or Voltage Law) : The sum of the emf around any closed loop of a circuit equal the sum of potential drops in that loop. Considering a rise of potential as positive (+) and a drop of potential as negative (–) the algebraic sum of potential difference around a closed loop of a circuit is zero—

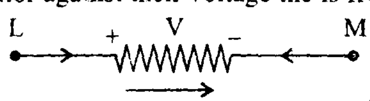
$$\Sigma E - \Sigma IR \text{ drops} = 0 \text{ (around closed loop)}$$

$$\Sigma E = \Sigma IR$$

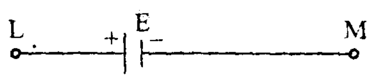
$$\Sigma \text{ Potential rises} = \Sigma \text{ Potential drops}$$

In tracing through any single circuit whether it is by itself or a part of a network, the following rules must be applied :

(i) A voltage drop exists when tracing through a resistance with or in the same direction as the current or through a battery or generator against their voltage the is from (+ve) to (–ve), refer figure



Fall in voltage (–V)



Fall in voltage (–E)

(ii) A voltage rise exist when tracing through a resistance against or in opposite direction to the current or through a battery or a generator with their voltage that is from (–ve) to (+ve).

Q. 1. (b) State maximum power transfer theorem.

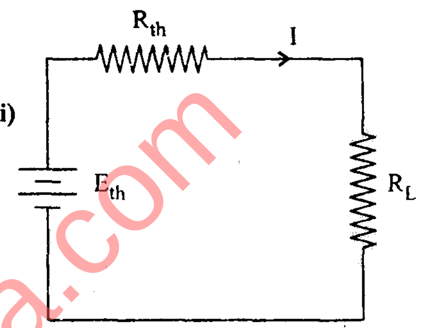
Ans. Maximum Power Transfer Theorem : This theorem is particularly useful for analyzing communication networks.

"Maximum power output is obtained from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load."

Any network can be converted into a single voltage source by the use Thevenin's theorem. The maximum power transfer theorem axis at finding R_L such that the power dissipated in R_L is maximum.

$$P = I^2 R_L$$

$$= \left(\frac{E_{th}}{R_{th} + R_L}\right)^2 R_L \quad \text{..... (i)}$$



For P to be maximum

$$\frac{dP}{dR_L} = 0$$

Differentiating equation, we have

$$\frac{dP}{dR_L} = \frac{E_{th}^2[(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)]}{(R_{th} + R_L)^4}$$

$$\frac{E_{th}^2[(R_{th} + R_L)^2 - 2R_L(R_{th} + R_L)]}{(R_{th} + R_L)^4} = 0$$

From which $\quad R_L = R_{th}$

It is worth noting that under these conditions the voltage across the load is half the open circuit voltage at the terminals L & M

Maximum power $\quad P_{max} = \left[\frac{E_{th}}{R_L + R_L}\right]^2 R_L = \frac{E_{th}^2}{4R_L}$

Q. 1. (c) What are crest and peak factor?

Ans. Crest & Peak Factor : The a.c. voltage and current wavelength completes a certain no. of cycles in one second, each cycle comprising an identical positive and negative half cycle. The value of the quantities varies from instant to instant peaking at a certain instant only. The rms voltage of the a.c voltage or current is often used in practice to specify the quantity. It is also called effective or virtual value of alternative quantity. General expression for calculating of rms val :

$$I_{eff} = I_{rms} = \sqrt{\text{avg } i^2(t)} = \sqrt{\left(\frac{i_1^2 + i_2^2 + \ldots\ldots. i_n^2}{n}\right)}$$

General expression for the average value of current wave T_x

$$I_{avg} = \frac{i_1 + i_2 + \ldots\ldots i_n}{n}$$

Consider an alternating current of sinusoidal waveform flowing through resistance R

$$i = I_m \cos \omega t = I_m \frac{\cos 2\pi}{T} t$$

Instantaneous power dissipation $p = i^2 R$

Average power dissipation over one cycle

$$P = \left(\frac{1}{T} \int_0^T i^2 dt \right) R = I^2 R$$

Where,
$$I = \left[\frac{1}{T} \int_0^T i^2 dt \right]^{1/2},$$

$$= \left[\left(\frac{1}{T} \int_0^T I_m^2 \cos^2 \frac{2\pi}{T} t \right) dt \right]^{1/2}$$

$$= \frac{I_m}{\sqrt{z}}$$

$$I_{avg} = \frac{2I_m}{\pi}$$

The form factor
$$= \frac{I_{(rms)}}{I_{avg}}$$

$$= \frac{\pi}{2\sqrt{2}} = 1.11 \text{ for sinusoidal current and voltage}$$

$$\text{Peak factor} = \frac{I_m}{I_{(rms)}} = \sqrt{2} \text{ for sinusoidal current/voltage}$$

This is also called crest factor or amplitude factor.

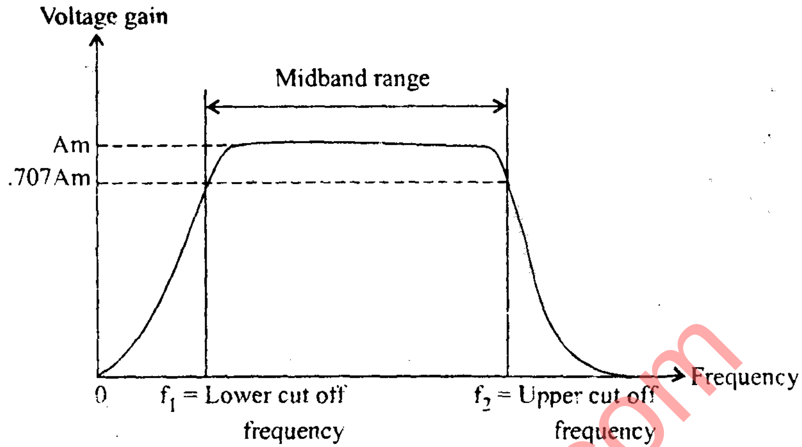
Q. 1. (d) What are cut-off frequencies?

Ans. Cut-off Frequency : The frequency response of an amplifier is mainly presented in the form of a graph that shows output amplitude plotted Vs frequency. Phase angle variation is sometime plotted on the same graph. Figure shows a typical plot of voltage gain of an a.c. amplifier Vs frequency. Notice that the gain is 0 at dc then rises as frequency increases levels off for further increases in frequency, and then begins to drop again at high frequency.

The frequency range over which the gain is more or less constant is called midband range and the gain in that range is designated Am. As shown in figure the low frequency at which the gain equals $(\sqrt{2}/2)Am$ =.707 Am is called lower cut off frequency and designated f_1. The high frequency at which the gain once again drop to 0.707 Am is called upper cut off frequency and designated f_2. The bandwidth of the amplifier is defined to be the difference between the upper and lower cut-off frequencies

$$\text{Bandwidth} = BW = f_2 - f_1$$

The points on the graph in figure where the gain is 0.707Am are called half power points and cut off frequencies is called half power frequencies.



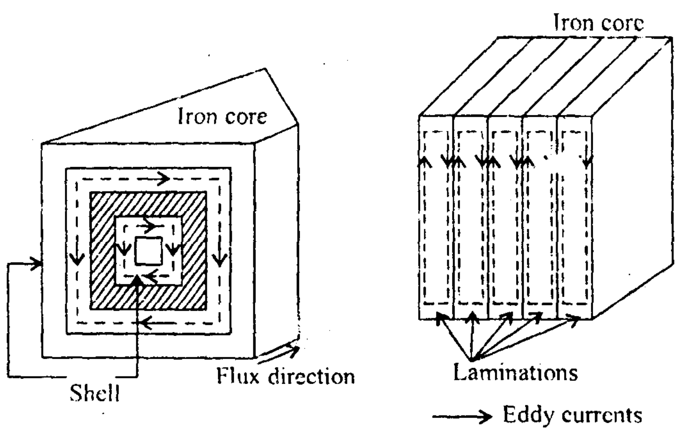
Q. 1. (e) What are unbalanced circuits?

Ans. Unbalanced Circuit : Much of the simplifying symmetry goes out the window when a three phase circuit is unbalanced. A circuit sometimes become unbalanced because the generator voltages do not form a symmetrical three phase set. But the unbalanced state often results from loads with unequal impedances.

An unbalance exist in a circuit when the impedances in one or more phases differ from the impedances of the other phases. In such a case, line or phase current are different and are displaced from one another by unequal cycles. We have considered balance loads connected to balanced systems. It is enough to solve problems, considering one phase only on balance loads, the conditions on other two plane being similiar. Problems on unbalanced three phase loads are difficult to handle because conditions in the three phases are different

Q. 1. (f) What are eddy currents?

Ans. Eddy Current : When a magnetic core carries a time varying flux, voltage is induced in all possible paths enclosing the flux. The result is the production of circulating currents in the core (all magnetic material are conductors). These currents are known as eddy current and have power loss (I^2R) associated with them called eddy current loss



The eddy current loss depend upon the resistivity of the material and lengths of the paths of circulating currents for a given cross section.

Applications of Utilization of Eddy Current :

(i) In induction energy meters eddy current bracking is used.

(ii) Eddy current heating is used for heating materials.

(iii) Eddy current damping is used in PMMC instruments.

Q. 1. (g) Explain, what is slip?

Ans. Slip : The rotor speed must always remain less than the synchronous speed. The difference between the synchronous speed and the rotor speed is known as slip. It usually expressed as a fraction of the synchronous speed.

Thus, slip is
$$S = \frac{N_s - N}{N_s}$$
$$N = N_s(1 - S)$$

N_s = Synchronous speed (r.p.m)

N = Motor speed (r.p.m)

In practice the value of slip is very small. At no load slip is around 1% or so and at full load it is around 3%. For large efficient machines the slip at full load may be around 1% only. The induction motor is therefore a motor with substantly constant speed and fills the same role as d.c. shunt motor :

(i) When the rotor is stationary its speed is zero and $S = 1$. The rotor cannot run at synchronous speed because then there will be no rotor emf and no rotor current and torque. If the rotor is to run at synchronous speed an external torque is necessary. If the rotor is driven such that $N > N_s$, the slip become negative, the rotor torque opposes external driving torque and the machine acts as induction generator.

(ii) The induction motor derives, its name from the fact that the current in the rotor circuit is induced from the stator. There is no external connection to the rotor except for some special purposes.

Q. 2. (a) Derive expressions for converting Delta circuit to star circuit.

Ans. Derive Expression for Converting Delta to Star (Δ to Y) : Consider the two networks, if these are to be identical. then the resistance between any pair of lines should be the same when the third line is kept open.

In Case of Δ Connection : **In Case of Y Connection :**

Therefore

$$R_{AB} = R_{ab} || (R_{bc} + R_{ca}) \qquad = R_a + R_b$$
$$R_{BC} = R_{bc} || (R_{ab} + R_{ca}) \qquad = R_b + R_c$$
$$R_{CA} = R_{ca} || (R_{bc} + R_{ab}) \qquad = R_c + R_a$$

Or
$$R_a + R_b = \frac{R_{ab}(R_{bc} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}}$$

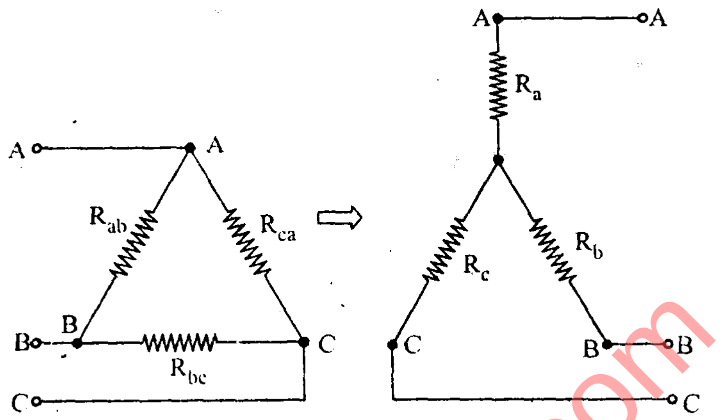


Fig. Delta to Star conversion

On solving parallel combination of $(R_{ab} || (R_{bc} + R_{ca})$ ..... **(i)**

$$R_b + R_c = \frac{R_{bc}(R_{ab} + R_{ca})}{R_{ab} + R_{bc} + R_{ca}} \quad \text{..... (ii)}$$

$$R_c + R_a = \frac{R_{ca}(R_{bc} + R_{ab})}{R_{ab} + R_{bc} + R_{ca}} \quad \text{..... (iii)}$$

Solving these three equations for R_a, R_b, R_c in terms of R_{ab}, R_{bc}, R_{ca}, we get

$$R_a = \frac{R_{ab} \cdot R_{ca}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab} \cdot R_{ca}}{\Sigma R_{ab}} \quad \text{.... (a)}$$

$$R_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab} + R_{bc} + R_{ca}} = \frac{R_{ab} \cdot R_{bc}}{\Sigma R_{ab}} \quad \text{.... (b)}$$

$$R_c = \frac{R_{bc} \cdot R_{ca}}{R_{ab} + R_{bc} + R_{ca}} + \frac{R_{bc} \cdot R_{ca}}{\Sigma R_{ab}} \quad \text{.... (c)}$$

Equations (a), (b) & (c) are the required solution.

Q. 2. (b) Differentiate between :

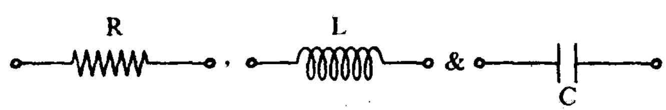
(i) Lumped parameter & distributed parameters

(ii) Mesh & Loop

(iii) Active and passive components & sources

Ans. (i) Lumped Parameter : "The parameters in the network which are physically separable one known as lumped parameters."

Example :



Distributed Parameters : "The parameters in the network like R, L etc. equation not be physically separable for analysing purpose."

Example : Transmission line

(ii) Mesh & Loop :

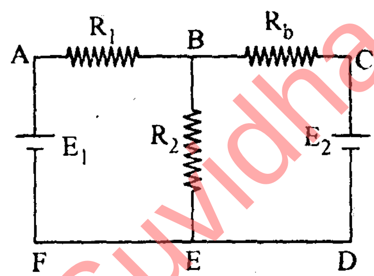
Mesh : It is the most elementary form of loop i.e., which cannot be further divided in other loops. (Mesh is the smallest close path).

Example : In figure, only ABEFA & BCDEB qualify the criteria of mesh. But ABCDEFA cannot because it encloses the ABEFA & BCDEB loops.

Note : All the meshes are loops but all the loops are not meshes.

Loop : It is any closed path of the network.

Example : In figure, there are three (3) possible closed path in loop—ABEFA, BCDEA & ABCDEFA.



(iii) Active & Passive Component : "The components which supplies the electrical energy to the circuit, are known as active components or elements."

Example : Current force, voltage forces, battery, cell etc.

"The components which receives electrical energy and then either converts it into heat (resistance) or stores in magnetic field (inductance) or electric field (capacitance)."

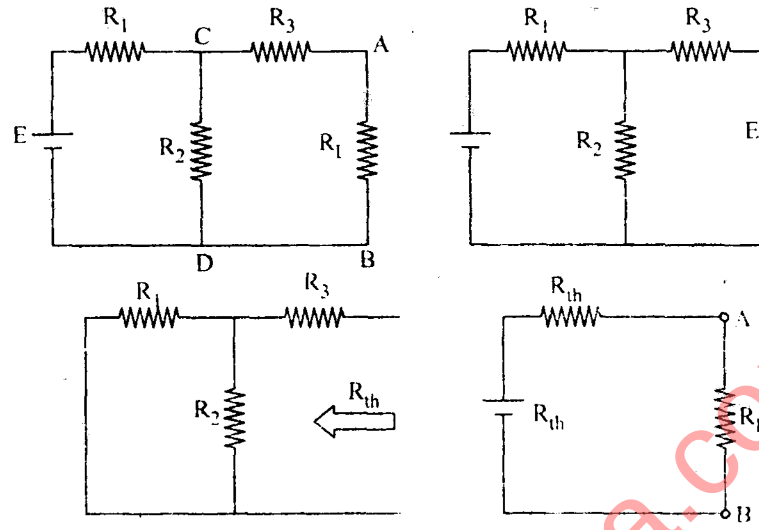
Example : R(resistance), L(inductance), C(capacitance).

Q. 3. (a) State and explain Thevenin's theorem.

Ans. Thevenin's Theorem : It states that for the purpose of determining the current in a resistor, R_L connected across two terminals of a network which contains sources of e.m.f. and resistors, the network can be replaced by a single source of e.m.f. and a series resistor R_{th}. This e.m.f., E_{th} is equal to potential difference between the terminals of the network when the resistor, R is removed : the resistance of series resistor, R_{th} is equal to the equivalent resistance of the network with the resistor, R removed C or it is sometimes called, "the resistance of the network when viewed from the terminals under consideration."

Hence $$I = \frac{E}{R_L + R_{th}} \quad \text{.... (i)}$$

Explanation : Let us consider the circuit



The following steps are required to find current therefore the load resistance R_L :

(i) Remove R_L from the circuit terminals A & B and redraw the circuit obviously the terminals have been open circuited.

(ii) Calculate the open circuit voltage ($V_{oc} = E_{th}$) which appears across terminals A & B, when they are open i.e., when R_L is removed. This voltage is E_{th}. A little thought will reveal that

$$E_{th} = \frac{ER_2}{R_1 + R} \qquad \text{.... (ii)}$$

$$\left[\because I = \frac{E}{R_1 + R} \text{ \& } E_{th} = IR_2 = \frac{ER_2}{R_1 + R_2}\right]$$

(iii) Short circuit the battery and find the thevenin resistance R_{th} of the network as seen from the terminals A & B.

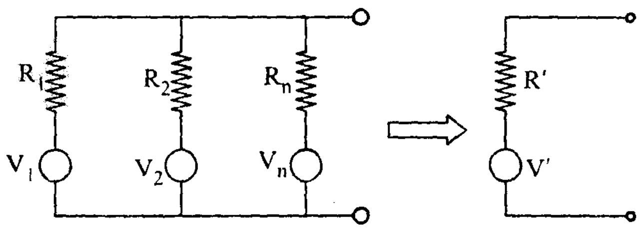
$$R_{th} = \frac{R_1 R_2}{R_1 + R_2} + R_3 \qquad \text{.... (iii)}$$

(iv) Connect R_L back across the terminals A & B from where it was temporarily removed earlier.

Current through R_L is given by $\quad I = \frac{E_{th}}{R_{th} + R_L}$ ... (iv)

Q. 3. (b) State and explain Millman's theorem.

Ans. Millman's theorem states that in any network, if the voltage sources $V_1, V_2 ...... V_n$ in series with internal resistances $R_1, R_2 .... R_n$ respectively, are in parallel then these sources may be replaced by a single voltage source V in series with R as shown in figure.

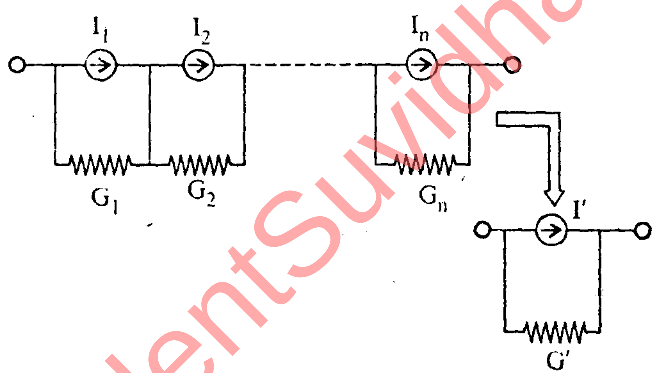


Where,
$$V = \frac{V_1 G_1 + V_2 G_2 + \ldots\ldots V_n G_n}{G_1 + G_2 + \ldots\ldots G_n}$$

Here, G_n is the conductance of the n^{th} branch

&
$$R' = \frac{1}{G_1 + G_2 + \ldots\ldots G_n}$$

As similar theorem can be stated for n-current sources having internal conductances which can be replaced by a single current source I' in parallel with an equivalent conductance.

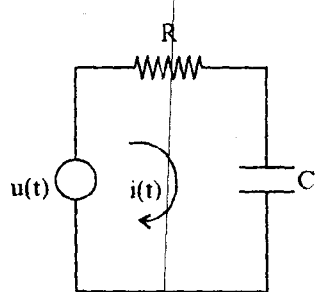


Where,
$$I' = \frac{I_1 R_1 + I_2 R_2 + \ldots\ldots\ldots I_n R_n}{R_1 + R_2 + \ldots\ldots R_n}$$

&
$$G' = \frac{1}{R_1 + R_2 + \ldots\ldots R_n}$$

Q. 4. (a) Explain the behaviour of R-C circuits to step response.

Ans. Response (Step response) of R.C. Circuit : The figure shows a series RC circuit. It is assumed that the initial energy in the network is zero, and the step input is shown by $u(t)$.



If the excitation is a step voltage, the physical analogy is that of a switch-closing at time $t = 0$, which connects a low voltage battery to the circuit.

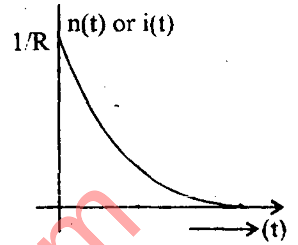
Consider the circuit of figure (i). The voltage equation can be written as

$$u(t)=Ri(t)+\frac{1}{C}\int_0^t i(t)dt \qquad \text{... (i)}$$

By laplace transform method equation (i) can be written in S-domain as

$$1=RI(s)+\frac{1}{C}\left[\frac{I(s)}{s}\right]$$

$$I(s)=\frac{1/R}{s-1/RC}$$



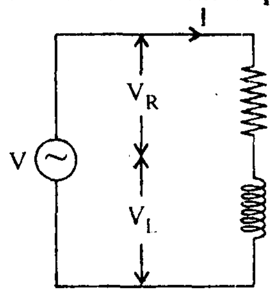
Changing above equation to time domain

$$\boxed{i(t)=\frac{1}{R}e^{\frac{1}{Rc}t}} \qquad \text{.... (ii)}$$

The time response of equation (ii) is drawn here under

Q. 4. (b) Explain the concept of voltage and current in RL, RC & RLC circuits with the help of phasor diagrams.

Ans. Series R-L Circuit :



Let V = Supply voltage

I = Circuit current

V_R = Voltage drop across R

$V_R = I.R$

V_L = Voltage drop across L

$V_L = I.X_L$

$$X_L = 2\pi fL$$

$$V_L = I.2\pi fL$$

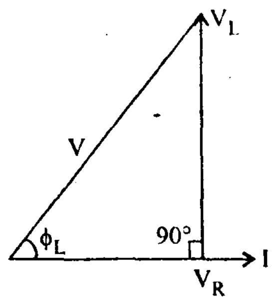
$$\bar{V} = \bar{V}_R + \bar{V}_L$$

$$V=\sqrt{V_R^2+V_L^2}$$

$$=\sqrt{I^2R^2+I^2X_L^2}$$

$$V=I\sqrt{R^2+X_L^2}$$

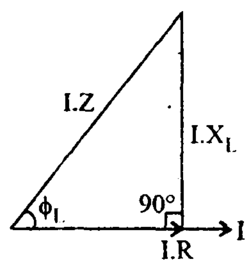
$$\frac{V}{I}=\sqrt{R^2+X_L^2}$$



Since I is common to both element R & L, this is used as reference phasor. The voltage is in phase with I & V_L leads by 90°

Let $\qquad Z_L \overset{\Delta}{=} \sqrt{R^2+X_L^2}$

The quantity Z_L is called the impedance of a series R-L circuit



$$\boxed{\frac{V}{I} = Z_L}$$

From impedance Δ,

$$\tan \phi_L = \frac{X_L}{R}$$

$$\cos \phi_L = \frac{R}{Z}$$

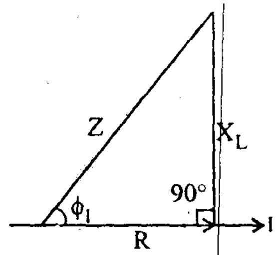


Fig. Impedance triangle for a series R-L circuit

Series R-C Circuit :

Let

V = Supply voltage

I = Supply current

V_R = Voltage across R

$V_R = I.R$

V_C = Voltage across C

$V_C = I.X_C$

$$X_C = \frac{1}{2\pi fc}$$

$$V_C = \frac{I}{2\pi fc}$$

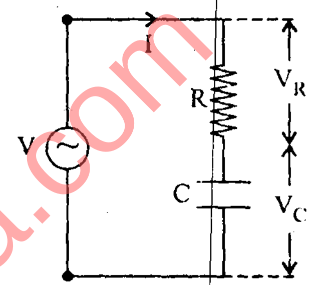
$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$V = \sqrt{V_R^2 + V_C^2}$$

$$= \sqrt{I^2 R^2 + I^2 X_C^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_C^2}$$



I is common to takes as a reference

I leads V by an angle 90°

Let $\quad Z_C \stackrel{\Delta}{=} \sqrt{R^2 + X_C^2}$

The quantity Z_C is called the impedance of series R-C circuit

$$\boxed{\frac{V}{I} = Z_C}$$

From impedance Δ,

$$\tan \phi_c = \frac{X_C}{R}$$

$$\cos \phi_c = \frac{R}{Z}$$

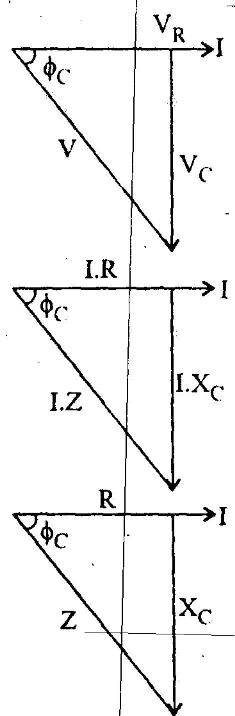


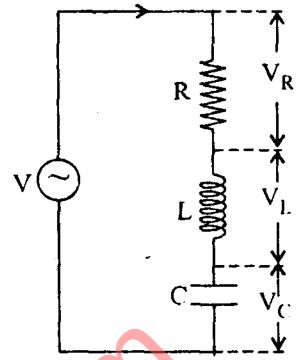
Fig. Impedance triangle of series R-C circuit

Series R-L-C Circuit : There are four voltages in the circuit,

V_R in phase with I

V_L leads I by 90°

V_C lags I by 90°



V in the phasor sum of V_R, V_L & V_C

$$V = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Thus, V_L & V_C are in opposite directions. The resultant of V_L & V_C is therefore the arithmetical difference between them.

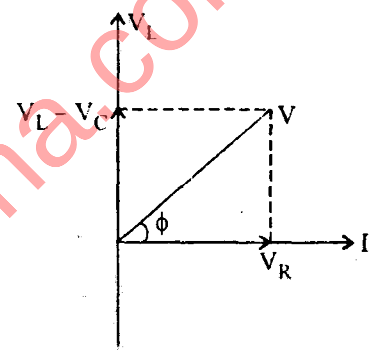
There are three possible cases of the series RLC circuit

(a) $V_L > V_C$ i.e., $X_L > X_C$

(b) $V_L < V_C$ i.e., $X_L < X_C$

(c) $V_L = V_C$ i.e., $X_L = X_C$

Case I : $V_L > V_C$



When $X_L > X_C$, the circuit is predominantly inductive

$$V^2 = V_R^2 + (V_L - V_C)^2$$

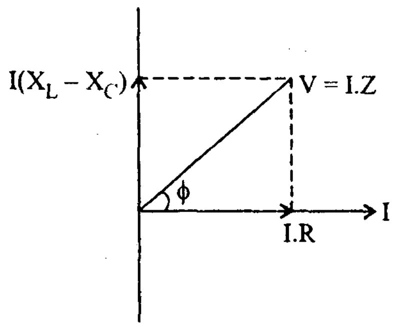
$$= (I.R)^2 + \{(IX_L) - (IX_C)\}^2$$

$$= I^2R^2 + I^2[(X_L - X_C)^2]$$

$$V^2 = I^2[R^2 + (X_L - X_C)^2]$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$



Let $$Z \overset{\Delta}{=} \sqrt{R^2 + (X_L - X_C)^2}$$

Z is the impedance of series RLC circuit

$$\boxed{\frac{V}{I} = Z}$$

From impedance Δ

$$\tan\phi = \frac{(X_L - X_C)}{R}$$

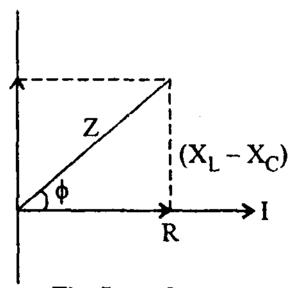
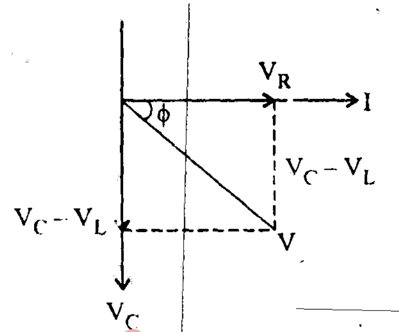


Fig. Impedance Δ of series RLC circuit

$$\boxed{\cos\phi = \frac{R}{2}}$$



Case II : $V_L < V_C$

When $X_C > X_L$, the circuit is predominantly capacitive

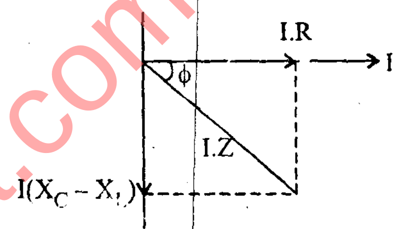
$$V^2 = V_R^2 + (V_C - V_L)^2$$

$$= I^2R^2 + [IX_C - IX_L]^2$$

$$= I^2R^2 + [I(X_C - X_L)]^2$$

$$V^2 = I^2[R^2 + (X_C - X_L)^2]$$

$$V = I\sqrt{R^2 + (X_C - X_L)^2}$$

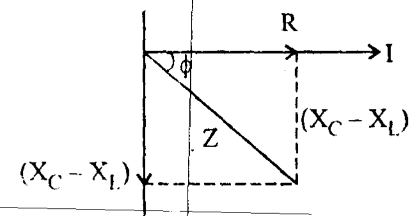


Let $$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

The quantity Z is called the impedance of series RLC circuit.

From impedance Δ,

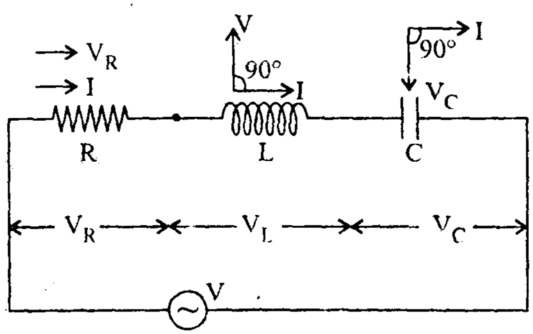
$$\tan\phi = \frac{(X_C - X_L)}{R}$$



$$\boxed{\cos\phi = \frac{R}{Z}}$$

Q. 5. (a) Derive results of resonant frequency, bandwidth and impedance in case of series RLC circuits.

Ans. Consider a circuit containing a resistance R, and inductance L and a capacitance C connected in series as shown in the figure.



V_R = Voltage drop across R & I & V are in phase

V_L = Voltage drop across L & V leads I by an angle 90°

V_C = Voltage drop across C & I leads V by an angle 90°

(Under the condition of resonance, the inductive reaction (X_L) and the capacitive reactance (X_C) should be made equal and voltage and the current are in phase i.e., the power factor becomes unity and network is purely resistive) and the frequency at which it occurs is known as resonant frequency for

$$X_L = X_C$$

(i) Net reactance $X = X_L - X_C = 0$

(ii) Impedance of the circuit

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + 0} \Rightarrow \boxed{Z = R}$$

If resonant frequency is denoted by f_r, then

$$X_L = \omega L = 2\pi f_r L$$

& $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f_r C}$$

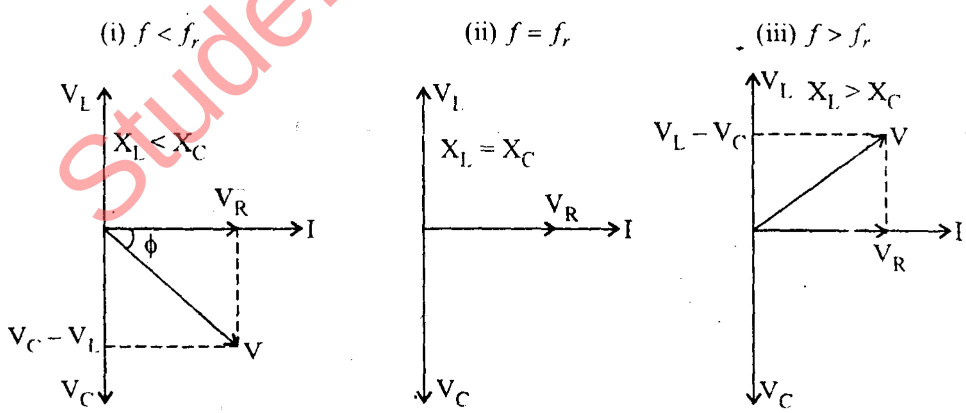
For resonance $$X_L = X_C$$

$\therefore$ $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

Resonant Frequencies in Series :

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}} \text{ or } \omega_r = \sqrt{1/LC}$$

Phasor diagrams for series RLC circuit shown in figure at three different frequencies with L and C kept constant.

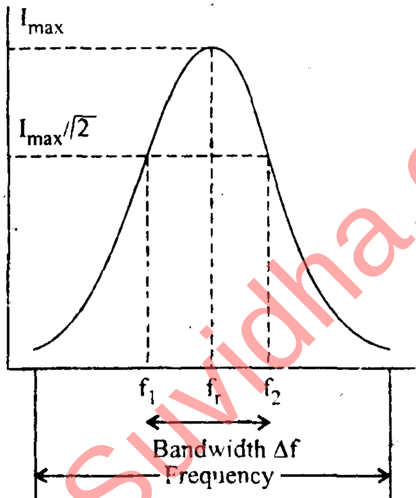


Note : Since in this resonance the voltages is maximum. It is called voltage resonance. The series resonance is also called acceptor circuit. Such a circuit accepts current at one particular frequency but rejects current of other frequencies.

Bandwidth Determination : The bands of frequencies while lie between points on either side of the resonant frequency where current is $1/\sqrt{2}$ times of max current I_{max}.

Bandwidth $\Delta f = f_2 - f_1$ and f_1 & f_2 at the limits of the bandwidth are called half power points on the frequency scale.

The impedance of the tuned circuit must be $\sqrt{2}$ times its impedance at resonance so that the current is $I_{max}/\sqrt{2}$. But the impedance at resonance $Z = R$ so at half power points the impedance is $\sqrt{2}R$, since $Z = \sqrt{R^2 + X^2}$, therefore $\sqrt{2}R = \sqrt{R^2 + X^2}$ or $X = R$. Now this information cannot be used to obtain an expression for the bandwidth.



The reactance at linear half power frequency X_1

$$X_1 = \omega_1 L - \frac{1}{\omega_1 C} = -R$$

(Minus sign appears on the right hand side of the equation because below resonance the capacitance reactance exceeds the X_L)

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$

Example :

$$\omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$= -\alpha \pm \sqrt{\alpha^2 + \omega_r^2}$$

Where, $\alpha = \dfrac{R}{2L}$ & $\omega_r = \sqrt{1/LC}$

On solving, we get

Lower half power frequency $\omega_1 = -\alpha + \sqrt{\alpha^2 + \omega_r^2}$ .... (i)

Similarly the upper half power frequency

$$X_2 = \omega_2 L - \frac{1}{\omega_r C} = +R$$

Or $$\omega_2^2 - \frac{R}{L}\omega_r - \frac{1}{LC} = 0$$

Or $$\omega_2 = \alpha + \sqrt{\alpha^2 + \omega_r^2}$$

So the equation for the bandwidth becomes

$$\omega_{bw} = \omega_2 - \omega_1 = \alpha + \sqrt{\alpha^2 + \omega_r^2} - \left[-\alpha + \sqrt{\alpha^2 + \omega_r^2}\right.$$

$$= 2\alpha = \frac{R}{L}$$

& bandwidth $$\Delta f = f_2 - f_1 = \frac{\omega_2 - \omega_1}{2\pi}$$

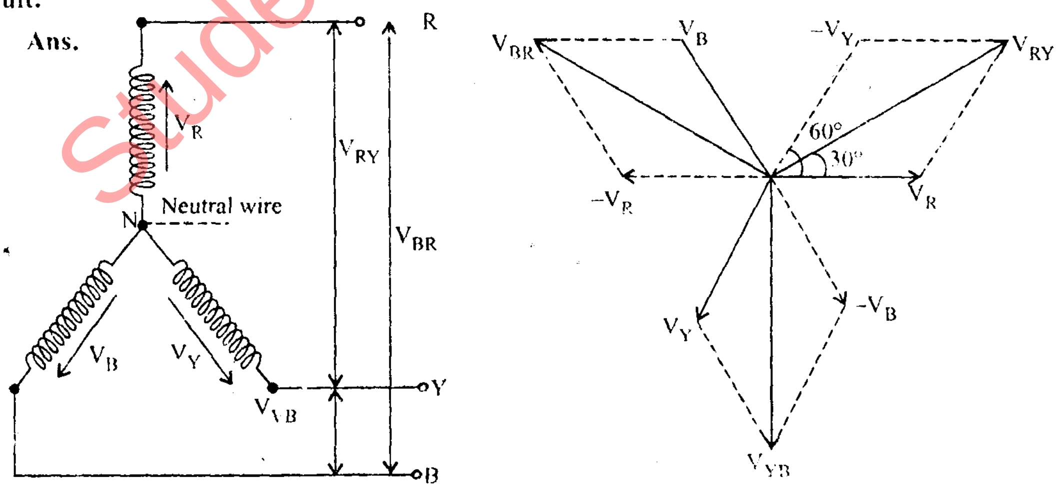
$$\boxed{\Delta f = \frac{R}{2\pi L}\text{ Hz}}$$

Lower half power frequency $$\boxed{f_1 = f_r - \frac{\Delta f}{2} = f_r - \frac{R}{4\pi^2}\text{ Hz}}$$

Upper half power frequency $$\boxed{f_2 = f_r + \frac{\Delta f}{2} = f_r + \frac{R}{4\pi L}\text{ Hz}}$$

Q. 5. (b) Derive relation between I_c & I_{Ph}, V_c & V_{Ph} in case of star connected three phase circuit.

Ans.



(a) Connection diagram **(b) Phase diagram of line & phase voltage**

Fig. Three phase star connected system

The three phase 4 wire star connected system is shown in figure (a). The voltage between any line and the neutral point i.e., voltage across the phase winding is called phase voltage (V_{Ph}) while the voltage between two line is called the line voltage (V_L).

The neutral point is usually earth connected.

In figure (a) the three phase are numbered as usually done R, Y & B indicate the three neutral columns red, yellow and blue respectively.

In figure (b), the emf induced in 3 phase are shown by the phases.

From the phasors, potential difference between line R & Y.

$$V_{RY} = V_R - V_Y \text{ (phasor difference)}$$

Or
$$V_{RY} = V_R + (-V_Y) \text{ (phasor sum)}$$

Since angle between V_R and $(-V_Y)$ is 60°

∴ From phasor diagram
$$V_{RY} = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60°} \quad \text{.... (a)}$$

Let
$$V_R = V_Y = V_B = V_P \text{ (all are phase voltage)}$$

Then from equation (a)
$$V_{RY} = \sqrt{V_P^2 + V_P^2 + 2V_P V_P \times \frac{1}{2}} = \sqrt{3}V_{Ph}$$

In balanced star connected system V_{RY}, V_{YB}, V_{BR} are equal in magnitude and are known line voltage

$$V_{RY} = V_{YB} = Y_{BR} = V_L$$

Similarly potential difference between line Y and B & B, R

Or
$$V_{YB} = V_Y - V_B = \sqrt{3}V_{Ph}$$

$$V_{BR} = V_B - V_R = \sqrt{3}V_{Ph}$$

Is
$$\boxed{V_L = \sqrt{3}V_{Ph}} \quad \text{.... (i)}$$

Since in star connected system each line conductor is connected to separate phase, so current through the lines and phase are same i.e., line current = phase current

$$\boxed{I_C = I_{Ph}}$$

Q. 6. (a) Explain how three phase power can be measured by two wattmeter method?

Ans. Two Wattmeter Method : Balanced or unbalanced load figure show connection diagram for star connected and delta connected loads respectively. In this method the current coils of the two wattmeters are inserted in any two lines and the pressure coil of earth joined to the third line.

It can be proved that the sum of the instantaneous powers indicated by W_1 & W_2 gives the instantaneous power absorbed by the three loads L_1, L_2 & L_3. Let us consider a star connected load.

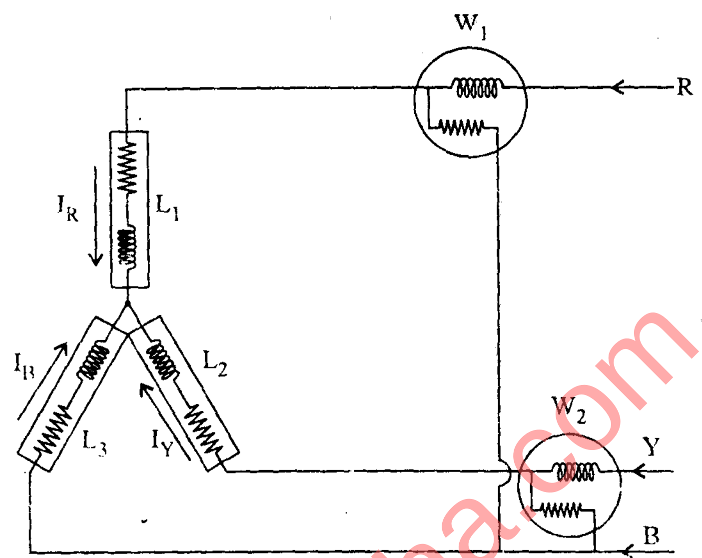


Fig. Star connected load

Keeping in mind that it is important to take the direction of the voltage though the circuit the same as that taken for the current which establishing the reading of the two wattmeters.

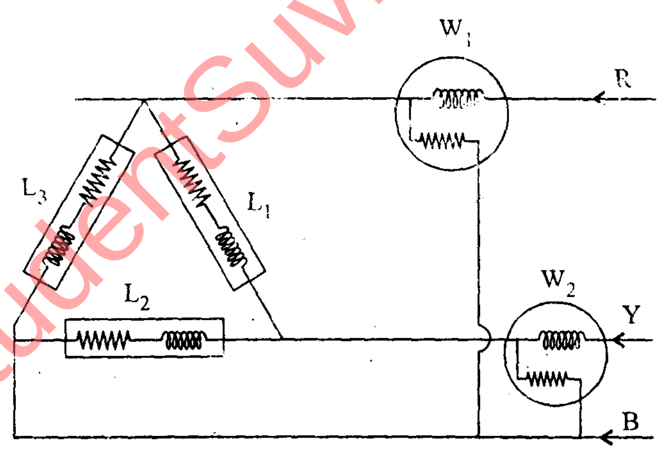


Fig. Delta connected load

Instantaneous current through $W_1 = i_R$

Instantaneous potential difference across

$$W_1 = e_{RB}$$

$$= e_R - e_B$$

Instantaneous power read by $W_1 = i_R(e_R - e_B)$

Instantaneous current through $W_2 = i_Y$

Instantaneous potential difference across

$$W_2 = e_{YB} = e_Y - e_B$$

Instantaneous power read by $\quad W_2 = i_Y (e_Y - e_B)$

$$W_1 + W_2 = i_R (e_R - e_B) + i_Y (e_Y - e_B)$$

$$= i_R e_R + i_Y e_Y - e_B (i_R + i_Y)$$

Now according to Kirchoff law

$$i_R + i_Y + i_B = 0$$

$$i_R + i_Y = -i_B$$

$$W_1 + W_2 = i_R e_Y + i_Y e_Y + i_B e_B$$

$$= P_1 + P_2 + P_3$$

Where, P_1 = Power absorbed by load L_1

P_2 = Power absorbed by load L_2

P_3 = Power absorbed by load L_3

$W_1 + W_2$ = Total power absorbed

This proof is true whether the load is balanced or unbalanced.

In case, the load is star-connected then it should have no neutral connection and if it has a neutral connection that it should be exactly balance so that in each case there is no neutral current in otherwise Kirchoff law will give.

$$i_R + i_Y + i_B + i_N = 0$$

In the above derivation we have considered the instantaneous reading. In fact the moving system wattmeter due to its inertia, cannot quickly follow the variations taking place in cycle, hence it indicates the average power.

Total power $\quad = W_1 + W_2$

$$= \frac{1}{T} \int_0^T i_R e_{RB} \, dt + \frac{1}{T} \int_0^T i_Y e_B \, dt$$

Q. 6. (b) Draw and explain the construction of transformer and derive emf. equation.

Ans. Transformer Construction : All transformer have the following essential elements :

(i) Two or more electrical windings insulated from each other and from the core.

(ii) A core which in case of a single phase distribution transformers usually comprises cold rolled silicon steel strip.

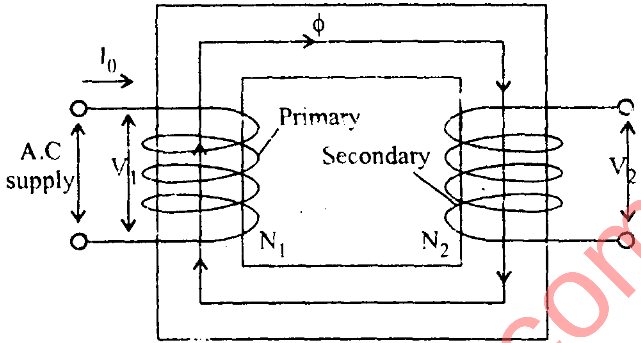
Other Necessary Parts : (i) A suitable container for the assembled core and winding.

(ii) A suitable medium for insulating the core and its winding from each other and from container.

(iii) Suitable bushing for insulating and bringing the terminals of the windings out of case.

The two types of transformers are :

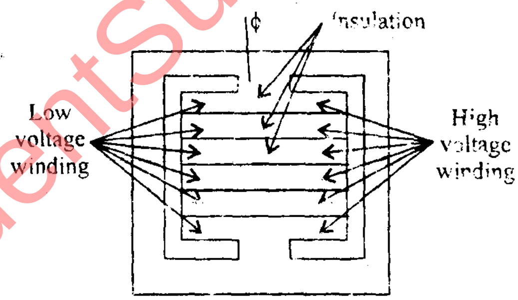
(i) Core Type Transformer : The complete magnetic circuit of the core type transformer is in the slope of the hollow rectangle as shown in which I_0 is the no load current and ϕ is the flux produced by it. N_1 & N_2 be the no. of turns on primary and secondary sides respectively.



The core is made up of silicon steel laminations which are either rectangular or L-shaped. In order to provide maximum linkage between windings, the group on each leg is made up of both high tension and low tension coils.

By placing the high voltage winding around the low voltage winding, only one layer of high voltage insulation is required, that between the two coils. If high voltage coils were adjacent to the core, an additional high voltage insulation layer would be necessary between the coil and iron core.

(ii) Shell Type Transformer :



Shell type

In the shell type construction the iron almost entirely surrounds the copper. The core is made up of E-shaped or F-shaped laminations which are stacked to give a rectangular figure eight. All the windings are placed on the central leg and in order to reduce leakage, each high side coil is adjacent to a low coil. The coil actually occupy the entire space of both windows are flat or pencake in shape and are auxiliary constructed of strip copper again to reduce the account the high voltage insulation required, the low voltage coils are placed adjacent to the iron core.

Emf Equation of a Transformer :

N_1 = No. of turns in primary

N_2 = No. of turns in secondary

ϕ_m = Maximum flux in the core

$$= B_m \times A$$

B_m = Maximum flux testing

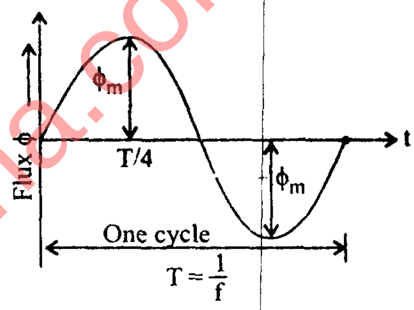
A = Area

f = Frequency of ac input

Refer figure since the flux increase from its zero value to maximum value ϕ_m in one quarter of the cycle in $T/4$ sec.

Average rate of change of flux $= \dfrac{\phi_m}{1/4f} = 4f\phi_m$ Wb/s or volt

If flux of varies sinusoidally then r.m.s. value of emf is obtained by multiplying the average value with form factor.



But form factor $= \dfrac{\text{r.m.s. value}}{\text{Average value}} = 1.11$

r.m.s. value of emf $= 1.11 \times 4f\phi_m = 4.44 f\phi$

Now r.m.s. value of induced emf in the whole of primary winding

$$E_1 = 4.44 f\phi_m N_1$$

Similarly r.m.s. value of induced emf in the secondary is

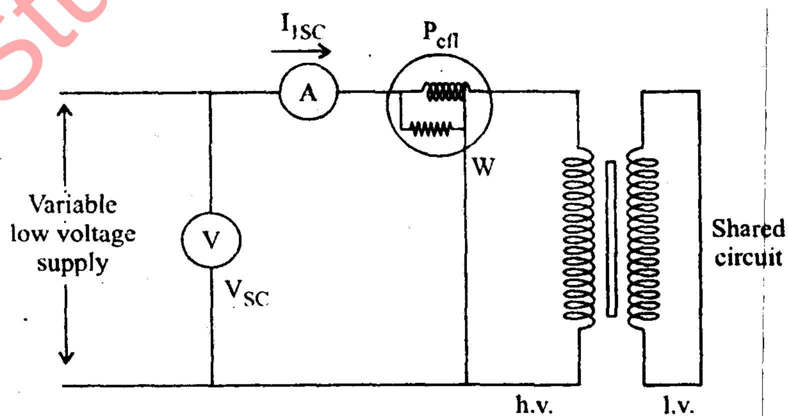
$$E_2 = 4.44 f\phi_m N_2$$

In an ideal transformer on no load

$$V_1 = E_1 \;\; \& \;\; V_2 = E_2$$

Q. 7. (a) Explain how S.C. test is performed and why?

Ans. SC Test or Impedance Test : In the SC test, usually the low voltage side is short circuited by a thick conductor.



An ammeter, a voltmeter and a wattmeter are connected on the high voltage (h.v.) side.

The reasons for short circuiting the l.v. side and taking measurement on the hv side are as follows :

(i) The rated current on hv side is lower than that on lv side, this current can be safely measured with the available laboratory ammeter.

(ii) Since the applied voltage is less than 5% of the rated voltage of the winding, greater accuracy in the reading of the voltmeter is possible when the hv side is used as a primary.

The high voltage winding is supplied at the reduced voltage from a variable supply. The supply voltage is gradually increased until full load primary current flows, when the rated full load current flows in the primary winding rated full load current will flow in the secondary winding by transformer action.

Readings of the ammeter, voltmeter and wattmeter are noted.

The readings of the instruments in a SC test are as follows :

(i) Ammeter reading = Full load primary current, I_{1SC}

(ii) Voltmeter reading = Short circuit voltage V_{1SC}

Wattmeter Reading : Full load copper loss of the transformer page (since the applied voltage is low (usually 5 to 10% of normal rated supply voltage). The flux ϕ produced is low, also, core loss is nearly proportional to flux^2, the core loss is so small that it can be neglected. However, the windings are carrying normal full load currents and therefore input is supplying the normal full load copper losses. Thus, the wattmeter gives the full load copper loss. P_{cfl}, the output voltage V_2 is zero because of is short circuit.

Consequently whole of the primary voltage is used in supplying the voltage drop in the total impedance Z_{1e} referred to the primary.

$$V_{1SC} = I_{1SC} Z_{1e}$$

From this test, the following calculation can be made :

(i) Equivalent resistance, $R_{eq} = \dfrac{P_{cfl}}{I_{1SC}^2}$

(ii) Equivalent impedance, $Z_{eq} = \dfrac{V_{1SC}}{I_{1SC}}$

(iii) Equivalent reactance, $X_{eq} = \sqrt{Z_{eq}^e - R_{eq}^e}$

& $\cos\phi_{SC} = \dfrac{R_{eq}}{Z_{eq}}$

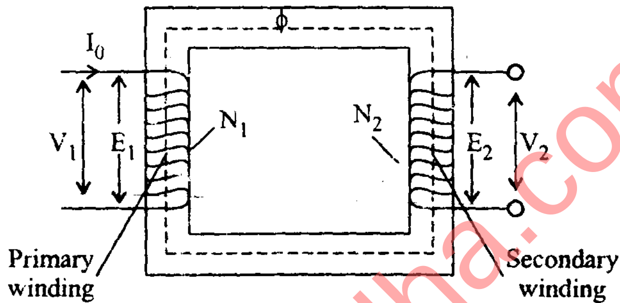
The above values are referred to the hv side in above case, if desired, the values could be easily calculated referred to other side also.

The purpose of this test is to determine full load copper loss and equivalent resistance and equivalent reactance referred to metering side (hv side).

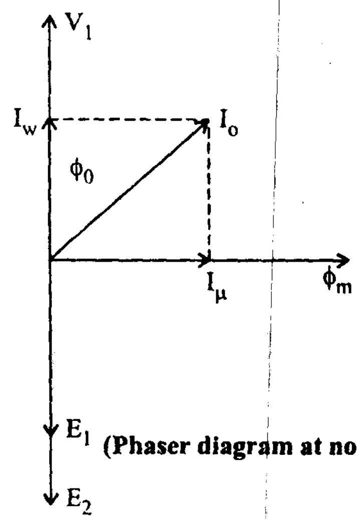
Q. 7. (b) Draw and explain the phasor diagram of transformer with no load and full load.

Ans. Transformer on No Load : A transformer is said to be on no load when the secondary winding is open circuited. The secondary current is thus zero. (Thee is no load on secondary).

When an alternating voltage is applied to the primary, a small current I_0 i.e., no load current of the transformer flows in the primary. It is made up of two components I_μ & I_w. The component I_μ is called the magnetizing component which magnetizes the core. In other words, it set up a flux in the core and therefore I_μ is in the phase with ϕ_M . The current I_μ is also called reactive or wattless components of no load current.



The component I_w supplies the hysteresis and eddy current losses in the core and the negligible I^2R loss in the primary winding. The current I_w is called active component or wattful component of no load current and it is in phase with the applied voltage V_1.



(Phaser diagram at no load)

For a transformer on no load, we have

$$\phi = \phi_m \sin \omega t$$

$$e_1 = E_{1m} \sin(\omega t - \pi/2)$$

$$e_2 = E_{2m} \sin(\omega t - \pi/2)$$

Since E_1 & E_2 are induced by the same flux ϕ_1 they will be in phase with each other. E_2 differs in magnitude from E_1 because

$$E_2 = E_1 \frac{T_2}{T_1} = \frac{E_1}{a}$$

If the voltage drops in the primary winding are neglected, E_1 will be equal and opposite to the applied voltage V_1.

I_μ is in phase with ϕ_m and

I_w is in phase with V_1

The phasor sum of I_μ & I_w is I_0.

Angle ϕ_0 is called the no load power factor angle. So that the power factor on no load is $\cos\phi_0$.

From the Phasor Diagram :

$$I_w = I_0 \cos\phi_0 \qquad \text{... (i)}$$

$$I_\mu = I_0 \sin\phi_0 \qquad \text{.... (ii)}$$

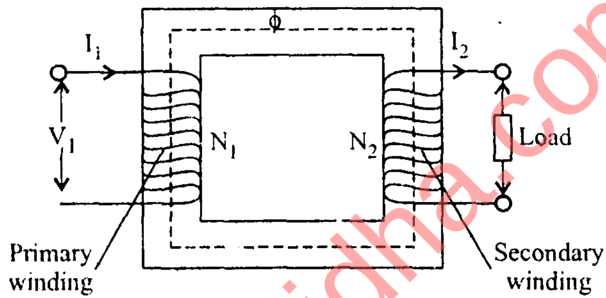
$$I_0 = \sqrt{I_w^2 + i_\mu^2} \quad \text{... (iii)}$$

$$\cos\phi_0 = \frac{I_w}{I_0} \quad \text{.... (iv)}$$

Also core loss $= V_1 I_0 \cos\phi_0 = V_1 I_w$ ... (v)

Magnetizing (reactive) voltmeters $= V_1 I_0 \sin\phi_0 = V_1 I_\mu \; V_{Ar}$ ... (vi)

Transformer On Load : When there is a load connected to secondary side of transformer, then transformer is said to be on load.



When the transformer is on load (i.e., an impedance or load is connected across the secondary terminals).

Current I_2 flows through the secondary windings, the secondary current I_2 set up its own mmf and hence creates a secondary flux ϕ_2, the secondary flux ϕ_2 opposes the main flux ϕ set up by the exciting current. I_0 the opposing secondary flux ϕ_2 weakers the main flux ϕ momentarily.

So primary back emf E_1 tends to reduced, so difference of applied voltage V_1 & E_1 increases.

Therefore, more current is drawn from the source of supply flowing through the primary winding until original value of flux ϕ is obtained and it again causes increase in back emf E_1.

Let the additional primary current be I_1'.

The I_1' is in phase opposition with secondary current I_2 and is called the counter balancing current.

The additional current I_1' set up an mmf $N_1 I_1'$ producing flux ϕ_1' in the same direction as that of the main flux ϕ and cancels the flux ϕ_2 produced by secondary mmf $N_2 I_2$.

So $$N_1 I_1' = N_2 I_2 \quad \text{or} \quad I_1' = \frac{N_2}{N_1} I_2 \quad \text{.... (a)}$$

The total primary current I_1 is therefore, phasor sum of primary counter balancing current I_1' and no load current I_0.

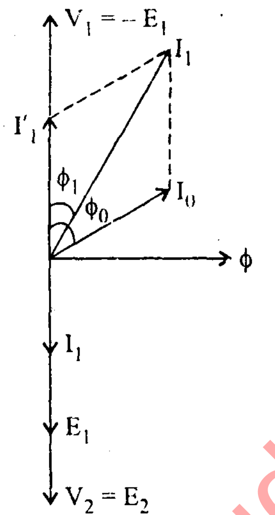
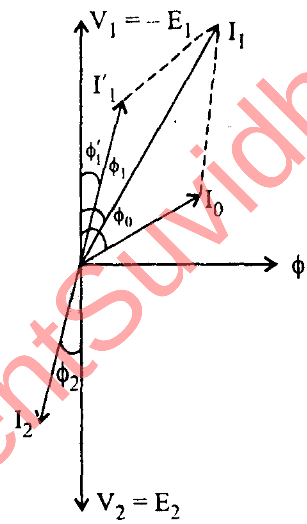
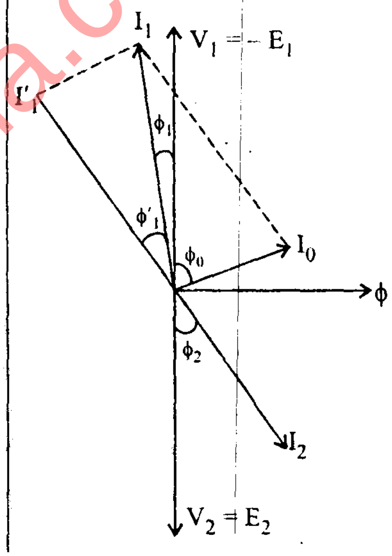
I_0 is very small so $\quad I_1 = I_1'$

$\therefore \quad I_1 = I_1' = N_2/N_1 . I_2 \;$ or $\; \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$

Since the secondary flux ϕ_2 produced in secondary mmf $N_2 I_2$ is neutralized by the flux ϕ_1' produced by mmf $N_1 I_1'$ set up counter balancing current I_1'. The flux in the transformer core remains constant from no load to full load.

Since the voltage drop in both of the winding of the transformer are assumed to be negligible.

Therefore, $V_2 = E_2 \;\&\; V_1 = -E_1$

Resistive Load	Inductive Load	Capacitive Load
Current I_2 will be in phase with V_2, the current I_1' will be in phase with V_1 and equal to I_2 in magnitude. The current I_1 is made up of I_0 and I_1' so lags behind V_1 by an angle ϕ_1.	The secondary current I_2 lags behind the secondary terminal voltage V_2 by an angle ϕ_2. The induced primary current I_1' is always in opposition of I_2.	The secondary current I_2 loads the secondary terminal voltage V_2 by an angle ϕ_2. The current I_1 is the vector sum of the current I_0 and I_1' which will be lagging behind V_1 by an angle ϕ_1.
		

Phasor diagram for the ideal transformer on load

Q. 8. (a) Explain the construction and working of synchronous motor.

Ans. Construction of Synchronous Motor : It consist of following essential parts :

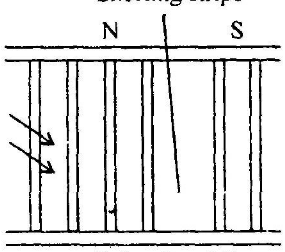
(i) A laminated stator core with three phase armature winding.

(ii) Revolving field complete with amortiseur inding and slip rings.

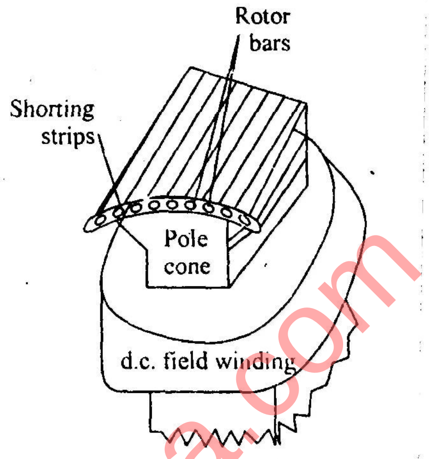
(iii) Brushes and brush holders.

(iv) Two end shields to house the bearings that support the shaft.

The stator core and windings of a synchronous motor are similiar to those of 3-phase squirrel cage induction motor. The rotor is generally a salient pole rotor. In order to eliminate hunting and to develop the necessary starting torque when a.c. voltage is applied to stator, the rotor poles contain pole phase conductors which are short circuited at their ends as shown. The field circuit leads are brought out of two slip rings mounted on the rotor shaft. Carbon brushes mounted in brush holders make contact with the two slip rings the terminal of the field circuit are brought out from the brush holders to a second terminal box mounted on the rotor frame.

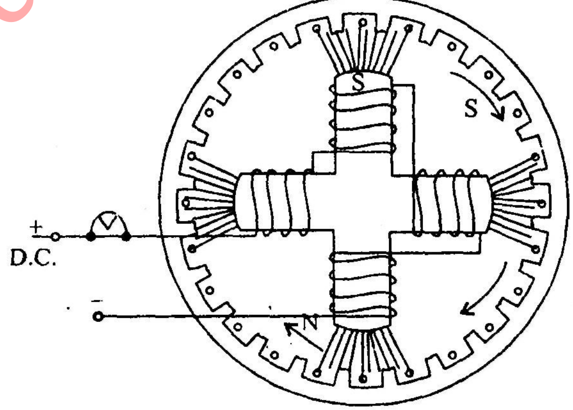


Pole of an AC synchronous motor



Damper winding

Principle of Operation : Where the stator winding of a 3-phase synchronous motor are supplied with rated 3-phase. Voltage a rotating field travelling at synchronous field is set up. The synchronous speed is found from the relation $N_S = \frac{120f}{P}$ where N_s (rpm), f and P are synchronous P speed, frequency and no. of poles respectively. This rotating magnetic field cuts across the squirrel cage winding of rotor and induces voltage and current in the bass of this winding. The resultant magnetic field of squirrel cage winding embedded in the rotor field poles reacts with the rotor field in such a manner as to cause the rotation of motor. The rotor will increase its speed to a point slightly below the synchronous speed of the stator field. The field circuit is now existed from an outside source of d.c. and magnetic poles of fixed polarity are set up in the rotor field cores. The fixed magnetic poles of the rotor are attracted to unlike poles of the rotating magnetic field set up by the stator winding.



Application : Power houses and substations, factories, 18/13 industries, constant speed equipment (fans, blowers, centrifugal pumps, motor generator sets) etc.

Q. 8. (b) Explain the working principle of D.C. machine and derive emf. equation for it.

Ans. Working Principle : It is a machine which convert electrical power into mechanical power. It is used to operate pumps, lathe etc.

When a current carrying conductor is placed in a magnetic field then it experiences a mechanical force tending to rotate and the direction of the force experienced is given by the flowing left hand rule. The magnitude of this mechanical force (F) is experienced by the conductor is given by

$$F = Bil \text{ newton}$$

l = Length of conductor (in meter)

i = Current in the conductor (in amp)

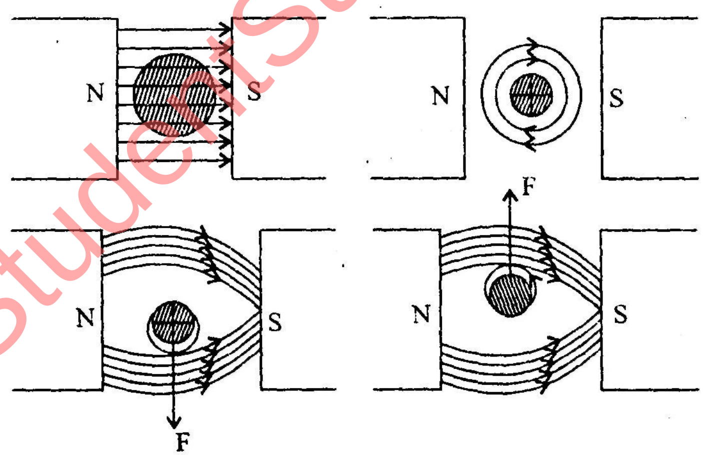
B = Magnetic flux density (in wb/m^2)

(i) Suppose a conductor is placed in a magnetic field as shown and here magnetic field passes the airgap from N-pole to S-pole.

(ii) Now the figure, conductor carries the current but no magnetic effect of the main right hand, S thumb rule or work screw rule.

(iii) Again in figure, the current carrying conductor is placed in a magnetic field. Here magnetic field of conductor supports main field upon the conductor and opposes main field under the conductor in other words the flux density increased in one side and decrease in other side. Here a force is developed on the conductor for high density flux to low density flux.

(iv) If we change the direction of current in conductor, the direction of force is also changed as shown.



E.M.F Equation : Let in a d.c. machine

P = No. of poles

ϕ = Flux generated per pole (in wb/m)

Z = Total conductor in armature

N = Speed of armature (r.p.m.)

$$A = \text{Parallel paths (in armature)}$$

According to Faraday's law,

In one conductor

Induced average e.m.f. $\quad E = \dfrac{d\phi}{dt}$ volt

Flux cut in one revolution $\quad d\phi = \text{No. of poles} \times \text{Flux/pole}$

$$= p \times \phi$$

Time taken in one revolution $\quad dt = \dfrac{60}{N}$ sec.

Here induced average emf in one conductor

$$E = \frac{d\phi}{dt} = \frac{P\phi}{60/N} = \frac{P\phi N}{60} \text{ volt}$$

When conductor are Z, total induced e.m.f.

$$E = \frac{P\phi N}{60} \times Z \text{ volt}$$

But conductor in one parallel path $= \dfrac{Z}{A}$

So induced e.m.f. $\quad E = \dfrac{\phi Z N P}{60 \times A}$ volt

$$A = \begin{cases} Z & \text{for wave winding} \\ P & \text{for loop winding} \end{cases}$$

Q. 9. Write short notes on :

(a) Induction type voltmeter

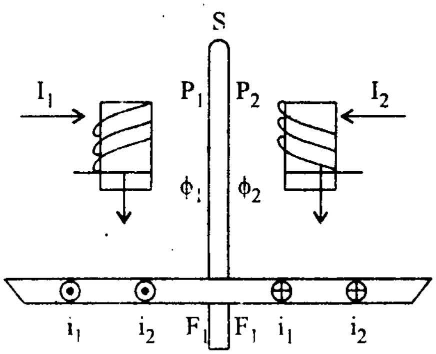
(b) Energy meter

Ans. (a) Induction Type voltmeter : Here the deflecting torque is produced due to the reaction between the flux of an a.c. magnet and the eddy currents induced by this flux.

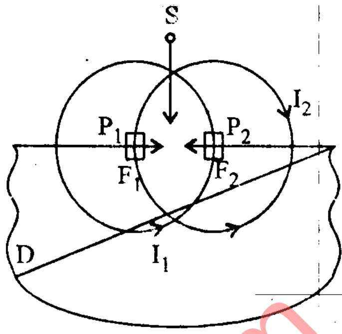
Principle of Operation : It consist of thin aluminium disc D free to rotate about an axis passing through its centre. Two a.c. magnetic poles P_1 & P_2 produce alternating fluxes ϕ_1 & ϕ_2 respectively which cut this disc. Consider any annular portion of the disc around P_1 with centre on the axis of P_1 this portion will be linked by flux ϕ_1 and so on alternating emf e_1 be induced in it. This emf will circulate an eddy current P_1 which will pass under P_2. Similarly ϕ_2 will induced an emf e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_2.

This eddy current i_2 flows under pole P_1.

The portion of the disc which is traversed by flux ϕ_1 and carries eddy current i_2 experience a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \propto \phi_1 i_2$. Similarly the portion of the disc are leging in flux ϕ_2 and carrying eddy current i_1 experiences a force $F_2 \propto \phi_2 i_1$ or $F_2 = K\phi_2 i_1$



(a)



(b)

$$F_1 \propto \phi_1 i_2 = K\phi_1 i_2$$

Let R be the effective radius at which these force act. The torque T acting on the disc being equal to the difference of the two torque. A given by

$$T = R(K\phi_1 i_2 - K\phi_2 i_1)$$

$$= K_1(\phi_1 i_2 - \phi_2 i_1) \qquad K_1 = RK$$

The value of eddy current $\quad i_1 = \dfrac{e_1}{R} = \left(\dfrac{d\phi_1}{dt/R}\right)$

$$i_1 = \frac{\frac{d}{dt}(\phi_{1m} \sin \omega t)}{R}$$

$$i_1 = \frac{\omega\phi_{1m} \cos \omega t}{R}$$

Similarly $\quad i_2 = \dfrac{\omega\phi_{2m}(\cos \omega t - \alpha)}{R}$

Where, $\phi_1 = \phi_m \sin \omega t$

$\phi_2 = \phi_{2m} \sin(\omega t - \alpha)$

After putting the value of i_1 & i_2 in equation (i), we have

$$T = \frac{K_1\omega}{R} \phi_{1m}\phi_{2m} \sin \alpha$$

From above it is clear that : (i) If $\alpha = 0$, if two fluxes are in plane the net torque is zero.

(ii) If $\sin \alpha = 1$, $\alpha = 90°$ the net torque is max. for given values of ϕ_{1m} & ϕ_{2m}.

(iii) The net torque is in such a direction so as to rotate the disc from the pole with leading flux towards the pole with lagging flux.

(iv) The value of T does not depend upon t. So we can say that it has a steady value at all times

(b) Energy Meter : An instrument which measures electrical energy is called an energy meter or watthours (wh) meter. Since the electrical energy consumed by a load adds up as the time goes on, an energy meter is an integrating meter. There are several types of energy meters. Single phase induction type energy meters are very commonly used to measure electrical energy consumed in domestic, commercial, and industrial installation.

Construction : A single phase induction type energy meter consists of the following parts :

(i) Driving system (ii) Moving system

(iii) Braking system (iv) Registering system

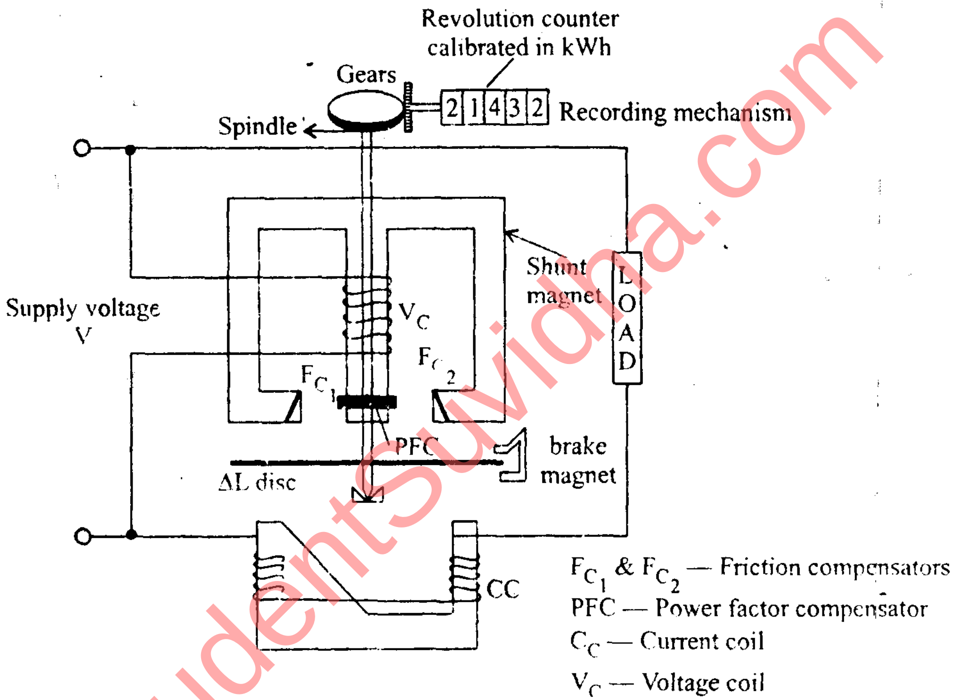


Fig. Single phase energy meter

(i) Driving System : The driving system of the energy meter consists of two silicon steel laminated electromagnets M_1 & M_2. The magnet M_1 is called the series magnet and the magnet M_2 is called the shunt magnet. The series magnet M_1 carries a coil consisting of few turns of thick wire and carries the load current. This coil is called the current coil (CC) and is connected in series with the circuit. The shunt magnet M_2 carries a coil consisting of many turns of thin wire.

This coil is called the voltage coil (VC) and is connected across the supply. The voltage coil carries a current proportional to the supply voltage.

Short-circuited (copper shading bands) are provided on the lower part of the central limb of the shunt magnet. These bands are also called power factor compensators.

(ii) Moving System : The moving system consists of a thin aluminium disc (rotor) mounted on a spindle. The disc is placed in the air gap between the series and shunt magnets so that it acts the fluxes of both the magnets.

(iii) Braking System : The braking system consists of a permanent magnet known as brake magnet. It is placed near the edge of the aluminium disc. When the disc rotates in the field of the brake magnet, eddy currents are induced in it. These eddy currents react with the flux and exert a braking or retarding torque.

(iv) Registering System : The disc spindle is connected, through a set of gears to a counting mechanism. This mechanism records a number which is proportional to the number of revolutions of the disc and indicates the energy consumed directly in kilowatthours (kWh).

Theory : When the energy meter is connected in the circuit, the current coil carries a current equal to the load current I and the voltage coil carries a current proportional to the supply voltage.

Let, V = Supply voltage

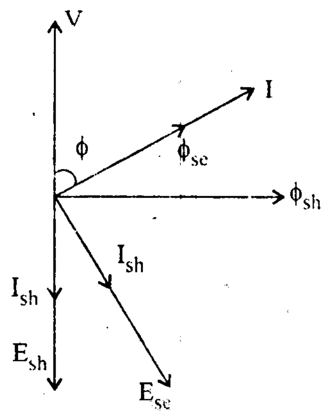
I = Load current lagging behind V by ϕ

$\therefore$ $\cos\phi$ = Load power factor

I_{sh} = Current through the shunt coil VC

I_{se} = Current through the series coils CC.

The flux ϕ_{sh} of the shunt coil (VC) is made to lag behind V by 90° with the adjustment of copper shading bands. The flux ϕ_{se} of the series coil (CC) is taken to be in phase with current I. Let e_{sh} and e_{se} be the induced emfs in the disc due to fluxes ϕ_{sh} and ϕ_{se}. The emfs ϕ_{sh} lags behind ϕ_{sh} by 90° and e_{se} lags behind ϕ_{se} by 90°. Eddy currents i_{sh} and i_{se} are set up in the disc by e_{sh} and e_{se} respectively. Since the inductance of the eddy current path in the disc is negligible, i_{sh} is in phase with e_{sh} and i_{sh} is in phase with e_{se}. The phasor diagram is shown below.



The instantaneous torque acting upon the disc will be due to the two oppositely directed torques proportional to $\phi_{sh}\ i_{se}$ and $\phi_{se}\, i_{sh}$.

$\tau_{inst} \propto \phi_{sh} i_{se} - \phi_{se} i_{sh}$: Let supply voltage be,

$$v = V_m \sin\omega t$$

Then the load current is $\quad i = I_m \sin(\omega t - \phi)$

The fluxes are $\quad \phi_{se} = K_1 I_m \sin(\omega t - \phi)$

Where, K_1 = Constant

$$\phi_{sh} = K_2 \int v\,dt$$

Where, $v = \dfrac{1}{K_2}\dfrac{d\phi_{sh}}{dt}$, being in phase opposition to the induced voltage

$$\phi_{sh} = -K_2 \frac{V_m}{\omega}\cos\omega t$$

Where, K_2 is another constant.

The eddy emf induced by the flux ϕ_{se} is

$$e_{se} = \frac{-d\phi_{se}}{dt} = -K_1 I_m \omega \cos(\omega t - \phi)$$

If the eddy current path is assumed to be purely resistance of value R.

$$i_{se} = \frac{e_{se}}{R} = \frac{-K_1 I_m \omega}{R} \cos(\omega t - \phi)$$

$$e_{sh} = \frac{-d\phi_{sh}}{dt} = -K_2 V_m \sin \omega t$$

$$i_{sh} = \frac{e_{sh}}{R} = \frac{-K_2 V_m}{R} \sin \omega t$$

The instantaneous torque in the disc is then

$$T_{inst.} \alpha \ \phi_{sh} i_{se} - \phi_{se} i_{sh}$$

$$\alpha \frac{K_2 V_m}{\omega} \cdot \frac{K_1 I_m \omega}{R} \cos \omega t . \cos(\omega t - \phi) + \frac{K_1 I_m K_2 V_m}{R} \sin(\omega t - \phi) \sin \omega t$$

$$\alpha \frac{K_1 K_2}{R} V_m I_m [\cos \omega t \cos(\omega t - \phi) + \sin \omega t \sin(\omega t - \phi)]$$

$$\alpha \frac{K_1 K_2}{R} V_m I_m [\cos \omega t - \omega t - \phi]$$

$$\alpha \frac{K_1 K_2}{R} V_m I_m \cos \phi$$

The average deflecting torque upon the disc is

$$T_\alpha \propto VI \cos \phi$$

Thus, the average deflecting torque on the disc is proportional to the active power in the circuit.

Neglecting friction in the meter, and assuming that the shunt magnet flux lags behind the supply voltage by exactly 90°, the driving torque is proportional to the power consumed in the circuit, that is,

$$T_d \propto P, \qquad T_d = K_3 P$$

The braking torque is due to eddy currents induced in the disc. Since the magnitude of eddy currents is proportional to the disc speed.

$$T_b \propto N, \ T_d = K_4 N$$

The disc will prove with steady speed when

$$\boxed{T_b = T_d}$$