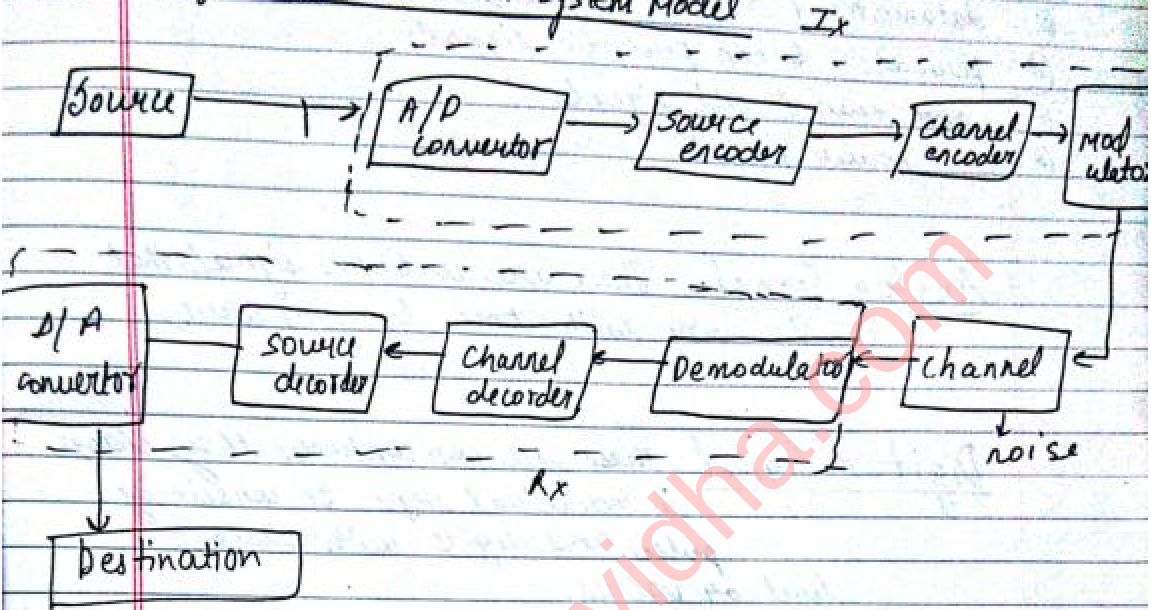
Digital communication System Model



Signals

Signal may be defined function of one or more independent variable typically it contains information about the behaviour & nature of a physical quantity for eg : voice, video, light voltage, current, stop crisis.

Classification of Signals :

1. Analog & digital signal
2. Real & complex signal
3. Continuous & discrete signals

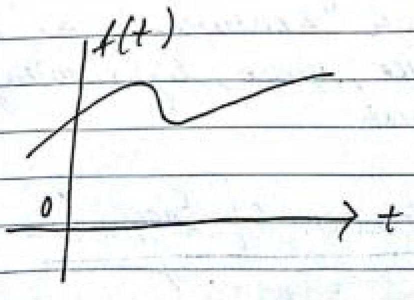
datamistic & random signals
periodic & non periodic signals
~~seed~~ even & odd signals
power signal & energy signal

Analog Signal These are continous signals that vary with time. eg. sine wave, cosine wave.

Digital Signal These are non continous they change in individual steps. It consist of pulses and digits with discrete level or values.

Continous time & Discrete Signal

A signal that varies ~~with the tim~~ continously with time is called continous time signal



Those signals that are defined at discrete time or the discrete time signal has value defined only at discrete values in time is known

Real & Complex Signal :

If a signal value is a real no. then, that signal is a real signal $x(t) = 30 \cos 20t$

If a signal value of a signal is complex no, then it is complex signal
$$x(t) = \sqrt{30} \times c \sin 20t$$

Random & Dataministic Signal :-

A dataministic signals are those signals characters ties w.r.t time are known at any instant of time. For eg sine wave, cosine wave..
$$x(t) = 30 \cos 30t \;..\; etc.$$

A Random signal is a signal which has some degree of uncertainity w.r.t its value at any instant of independent variable. for eg temp, noise, pressure .. etc.

Periodic & Non Periodic Signal

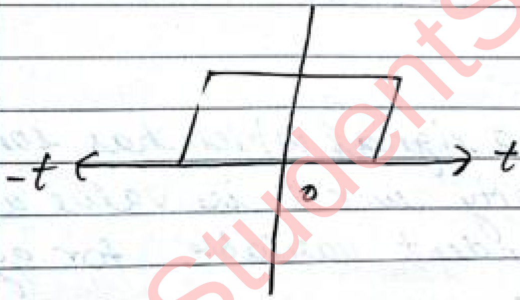
Those signals which are repeat itself by a definite timeperiod for eg sine, cosine, pulses rectangular wave, is known to be periodic signal.

The signal which does not repeat after the certain time period is known to be non periodic signal.

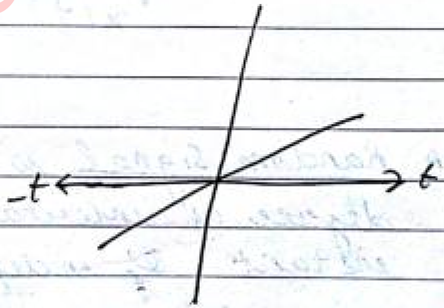
Even & Odd Signals

A signal $x(t)$ is called even signal if this identical to its folded counter part it means its reflection about the origin.

odd signals are those which exhibit a symmetry



even ~~odd~~ signal



odd signal

Energy & Power Signal

If a signal statisfies the following cond^n :-

• ... energy finite $0 < E < \infty$

then, that signal is Energy Signal

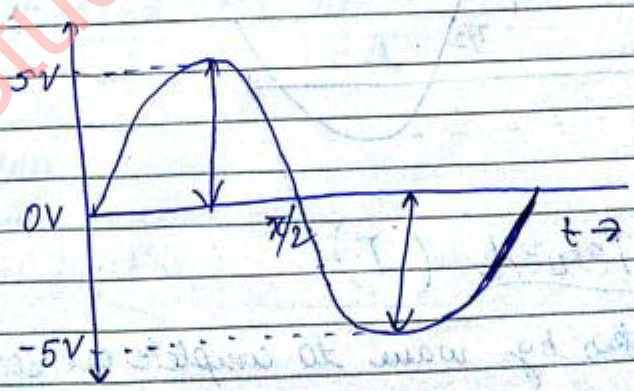
& for Power signal
- signal has energy infinite $E = \infty$
- power finite $0 < P < \infty$

Properties of S/gs

- Amplitude
- frequency
- phase
- wavelength
- Time period

Amplitude

Amplitude of a signal is signal strength at a particular time. (A)



Amplitude - 5

Frequency

no. of cycles per unit time (f)

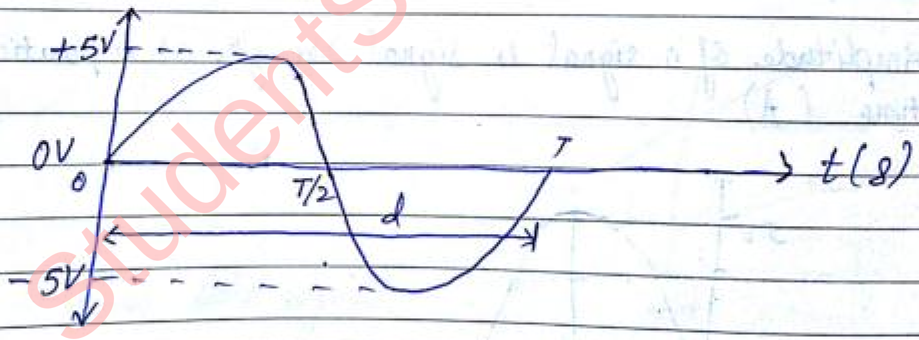
↳ linear frequency (f)

↳ Angular frequency (ω)

$$\boxed{f = 2\pi\omega}$$

Wavelength (λ)

~~distnar~~ distance travelled by the wave in one cycle or in one time period.



Time Period (T)

time taken by wave to complete one second.

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Relation b/w λ, T & f

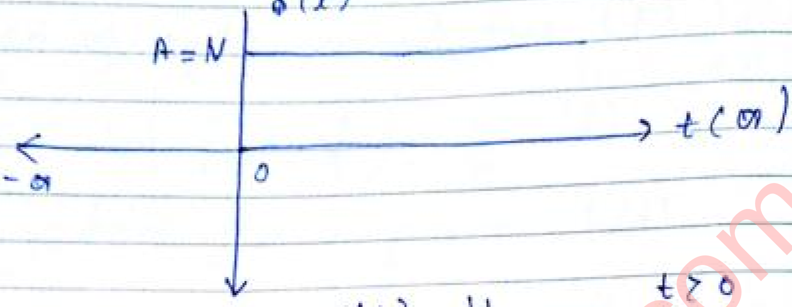
Phase It is amount of angle that is add or subtract in a signal. for eg

$$f(t) = \sin(\omega t + \phi)$$
$$\text{or } f(t) = \sin(\omega t - \phi)$$

some standard signal use in signal analysis

- unit step
- Impulse
- signum function
- Ram function

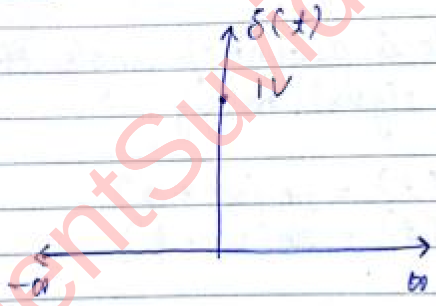
Unit Step Signal



+ve
-ve

$\delta(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Impulse

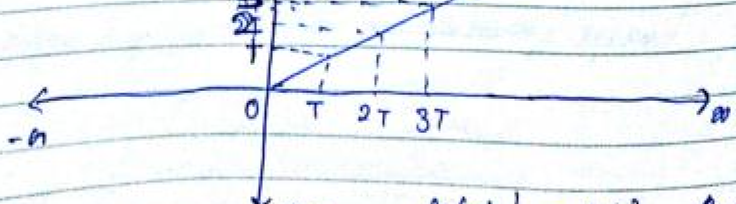


+ve
-ve

$\delta(t) = 1 \;, \; t = 0$

$= 0$ otherwise

Ram function

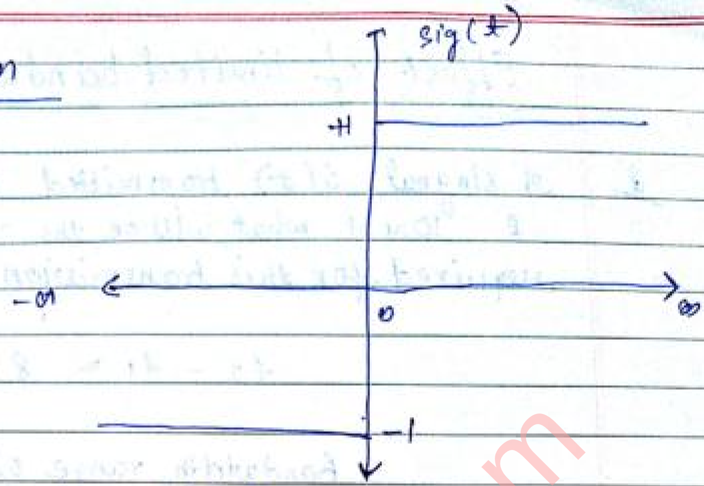


+ve

$r(t) = (t) \quad 0 \leq t$

otherwise

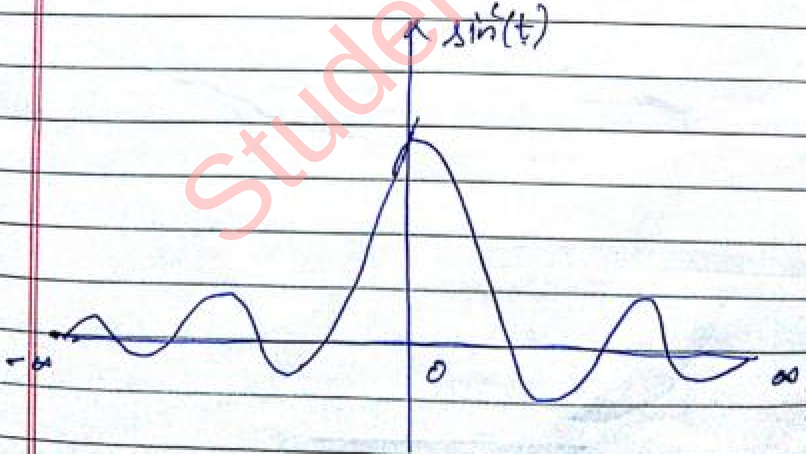
Signum function



$$sig(t) = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \\ 0, & \text{otherwise} \end{cases}$$

Sincy function

$$\text{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Effect of limited bandwidth :-

Q. A singnal $s(t)$ transmitted b/w the frequency 2MH & 10MH what will be the total bandwidth required for this transmission ?

$$f_2 - f_1 = 8 \text{ MH}$$

· Bandwidth range of frequency in which signal travels.

SNR or $\frac{S}{N}$

S – Strength of Signal
N – strength of Noise

$$F[s(t)] = \int_{-\infty}^{\infty} e^{1-st} \cdot e^{-j\omega t} dt$$

$$\int_{-\infty}^{0} e^{t} \cdot e^{-j\omega t} dt + \int_{0}^{\infty} e^{-t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} e^{t(1-j\omega)} dt + \int_{0}^{\infty} e^{-t(1+j\omega)} dt$$

$$= \left[\frac{e^{t(1-j\omega)}}{(1-j\omega)} \right]_{-\infty}^{0} + \left[\frac{e^{-t(1+j\omega)}}{-(1+j\omega)} \right]_{0}^{\infty}$$

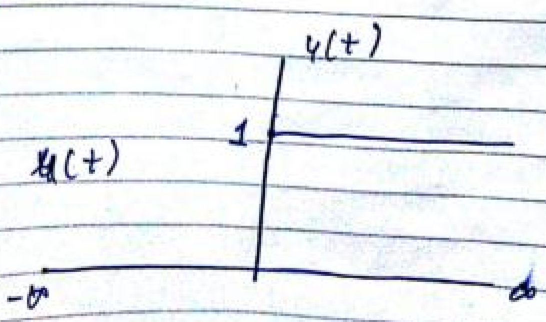
$$= \frac{1}{1-j\omega} \left[e^{0} - e^{-\infty} \right] + \left[\frac{1}{-(1+j\omega)} \right] \left[e^{-\infty} - e^{0} \right]$$

$$= \frac{2}{1-j^2\omega^2} = \frac{2}{1+\omega^2}$$

$j^2 = (\sqrt{-1})^2$

$j = -1$

Find the F(t) Transform of Unit Step Signal



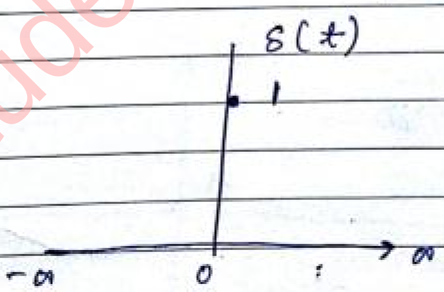
$$F[u(-t)] = u(t) = \begin{cases} t, & t \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

$$F[u(t)] = \int_{-\infty}^{\infty} u(t) e^{-j\omega t} dt$$

$$\int_{-\infty}^{0} 0 (\;)_T + \int_{0}^{\infty} (-e)^{j\omega}$$

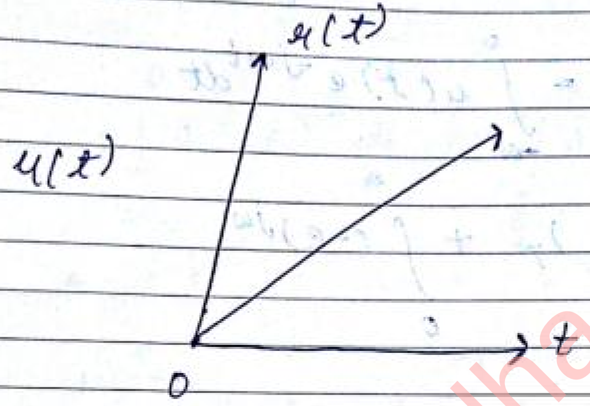
Find the F(t) Transform of ~~unit~~ δt Impulse signal

$$F[\delta(t)] = ?$$



$$F(u(t)) = \int_{-\infty}^{\delta} \delta(t) e^{-j\omega t} dt$$

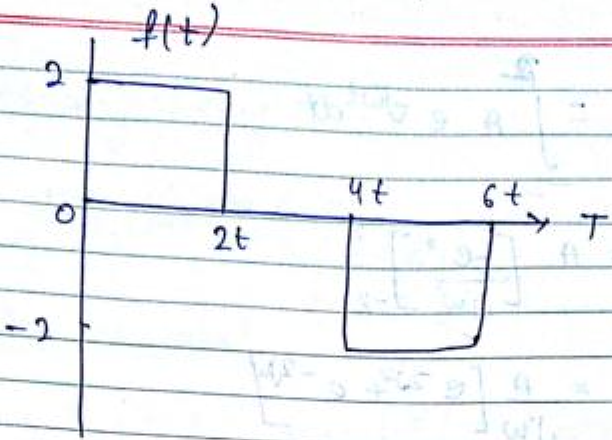
$F(t)$ Of RAM function



$$r(t) = t, \quad t \geq 0$$
$$\quad 0, \text{ otherwise}$$

$$\int_0^\infty t \cdot e^{-j\omega t}\, dt$$

Eg

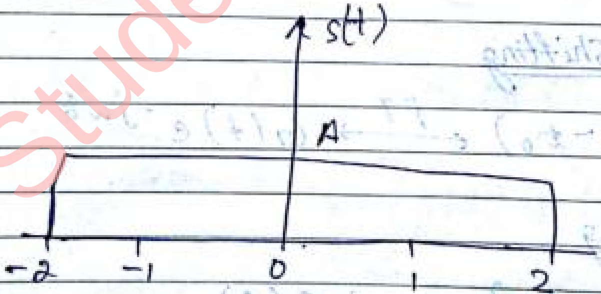


$$F(t) = \int_0^{2t} 2e^{-j\omega t}dt + \int_{2T}^{4T} (0)\,dt + \int_{4T}^{6T} -2e^{-j\omega t}dt$$

$$F(t) = \frac{2e^{-j\omega t}}{-j\omega t} + 0 + \frac{-2e^{-j\omega t}}{-j\omega t}$$

$$= \frac{-2e^{-j\omega t} + 2e^{-j\omega t}}{j\omega t} = 0$$

Eg



$$S(t) = \int_{-2}^{0} Ae^{-j\omega t}dt + \int_{0}^{2} Ae^{-j\omega t}dt$$

$$= \left[\frac{A e^{-j\omega t}}{-j\omega}\right]_{-2}^{0} + \left[\frac{A e^{-j\omega t}}{-j\omega t}\right]_{0}^{2} =$$

$$= \int_{-2}^{2} A \cdot e^{-j\omega t} dt$$

$$= A \left[\frac{-e^{t}}{j\omega} \right]_{-2}^{2}$$

$$= \frac{A}{j\omega} \left[e^{2j\omega} + e^{-2j\omega} \right]$$

Properties of F(t) Transform

① **linearity** If $\delta(t) \xrightarrow{F.T} \delta(f) \xrightarrow{F.T} \delta(f)$

$g(t) \xrightarrow{F.T} G(f)$

$$= F[a_1 g_1(t) + a_2 g_2(t)] \xleftrightarrow{F.T} a_1 G_1(f) + a_2 G_2(f)$$

② **Time Shifting**

$$g(t - t_0) \xleftrightarrow{FT} G(f) e^{-j\omega t}$$

③ **Scaling**

$$g(t) \longrightarrow G(f)$$

$$F[g(at)] \xleftrightarrow{FT} \frac{1}{|a|} G\left(\frac{f}{a}\right)$$

(4) Frequency Shifting

$$F(g(t)) \Leftrightarrow G(f)$$
$$g(t)e^{j\omega_0 t} \Leftrightarrow G(f-f_0)$$

(5) Time reversal

$$F[g(t)] \Leftrightarrow G(-f)$$

$$F[g(-t)] \longleftrightarrow G(-f)$$

(6) Time Differentiation

if $F[g(t)] \xleftrightarrow{F.T} G(f)$

(7) Integration

If $F[g(t)] \xleftrightarrow{F.T} G(f)$

then $F\left[\int_{-\infty}^{t} g(t)\,dt\right] \xleftrightarrow{F.T} \frac{1}{j\omega} G(f)$

(8) Convolution

$$F[g_1(t) * g_2(t)] \xrightarrow{F.T} F[G_1(t) * G_2(t)]$$

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(9) Duality

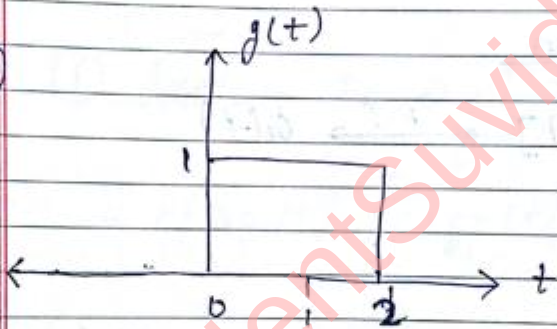
If $F(g(t) \xleftrightarrow{F.T} G(f)$'

$F[G(t)] \longleftrightarrow g(-f)$

(1) $F[g(t)] = ?$

$F[G(t)]$

(a)



$$F(g(t)) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= \int_0^2 1 \cdot e^{-j\omega t} dt \qquad x = -j\omega t$$

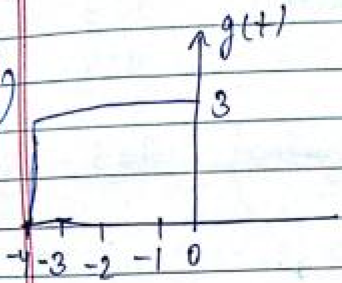
$$\cancel{= \int_0^2 e^{1-2t} e^{-j\omega t} dt}$$

$$\left.\frac{e^{ax}}{a}\right]_0^2$$

$$= \cancel{\int e^{1}}$$

$$\sin \omega_0 t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

(b)



(c) $g(t) = 2\sin \omega_0 t$

(d) $g(t)$ = signum function

(c) $$\int_{-\infty}^{\infty} 2\sin \omega_0 t \; e^{-j\omega t} \, dt$$

$$2\int_{-\infty}^{\infty} \sin \omega_0 t \cdot e^{-j\omega t} \, dt$$

$$\frac{e^{ax} - e^{-ax}}{2}$$

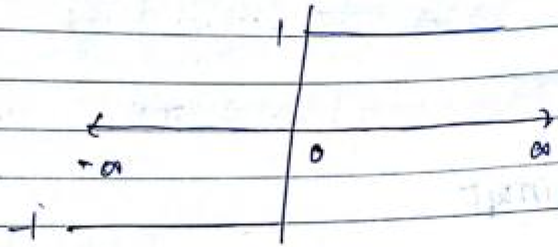
$$= \cancel{2}\int_{-\infty}^{\infty} \frac{e^{j\omega t} - e^{-j\omega t}}{\cancel{2}j} \cdot e^{-j\omega t} \, dt$$

$$= \int_{-\infty}^{\infty} \frac{e^{-j^2\omega^2 t^2} - e^{j^2\omega^2 t^2}}{j} \, dt$$

$$\frac{1}{j}\int_{-\infty}^{\infty} e^{-j^2\omega^2 t^2} - e^{j^2\omega^2 t^2} \, dt$$

$=$

(d) $g(t)$ = signum function



$$\int_{-\infty}^{0} -1\, e^{-j\omega t}\, dt + \int_{0}^{\infty} 1\, e^{-j\omega t}\, dt$$

$-$

ESD (Energy Spectral Density)

PSD

⑥ ESD (Energy per unit bandwidth)

$$ESD = \boxed{\frac{E}{B} = |G(f)|^2}$$ To be proved

$$g(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = g(t) * h(t) \quad \text{--- (1)}$$

F.T of (1)

$$Y(f) = G(f)\,H(f)$$

$$E = \int_{-\infty}^{\infty} |Y(f)|^2\,df$$

$$= \int_{-\infty}^{\infty} |G(f)\,H(f)|^2\,df$$

$$= \int_{-\infty}^{\infty} |G(f)|\,df$$

$$= |G(f)|^2 \int_{-f_M}^{f_M} df = |G(f)|^2\,B$$

$|G(f)|^2 [f_m - (-f_m)]$

$|G(f)|^2 [2f_m]$

$2f_m = B$

$$E = |G(f)|^2 \cdot B$$

$$\boxed{\frac{E}{B} = |G(f)|^2}$$ Proof

PSD (Power spectrum Density)

is defined as power ~~spec~~ per unit bandwidth

let us a signal $g(t) = \begin{cases} g(t) & -\frac{T}{2} \le t \le \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

~~Power~~

$$\underbrace{\int_{-\infty}^{-T/2} |g(t)|^2 dt}_{0} + \int_{-T/2}^{0} |g(t)|^2 dt + \int_{0}^{T/2} |g(t)|^2 dt + \underbrace{\int_{T/2}^{\infty} |g(t)|^2 dt}_{0}$$

$$\Rightarrow \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |g(t)|^2 dt.$$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |g(t)|^2 dt$$

$$\lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df$$

$$P = \lim_{T \to \infty} \frac{1}{T} |G(f)|^2 \underbrace{\int_{-\infty}^{\infty} df}_{B}$$

$$\frac{P}{B} = \lim_{T \to \infty} \frac{1}{T} |G(f)|^2$$

$$\boxed{PSD = \lim_{T \to \infty} \frac{1}{T} |G(f)|^2}$$