UNIT: III

Terzaghi's Theory of one dimensional consolidation

The Theoretical concept of the consolidation process was developed by Terzaghi (1923)

Assumptions:

(i) Soil is homogeneous & fully saturated
(ii) Soil particles & water are incompressible.
(iii) The deformation of soil is due to change in volume.
(iv) Darcy's law for the velocity of flow of H_2O through soil is perfectly valid.
(v) coefficient of permeability is constant during consolidation.
(vi) Load applied in one direction & deformation occurs only in the direction of the load application.
(vii) coefficient of consolidation permeability is constant during the process of consolidation.
(viii) during the process of consolidation certain soil properties remain constant values for constant.

Terzaghi theory describes the time rate of consolidation of saturated clayey soils.

The differential equation for consolidation is

$$\frac{\partial u}{\partial t} = C_v \cdot \frac{\partial^2 u}{\partial z^2}$$

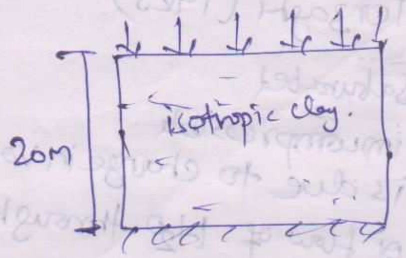
where C_v = coefficient of consolidation $C_v = \frac{K}{\gamma_w m_v}$

Problem:

A 20m thick isotropic clay stratum overlies an impervious rock. The coefficient of consolidation of soil is $5\times10^{-4}\ cm^2/sec$ (C_v).

Find the time required for 50% & 90% consolidation. Time factor for U=50% is 0.2 & for U=90% is 0.85 where U is of consolidation.

(ii) In order to accelerate the settlement rate, vertical & drains of 1m diameter were made at 5m centre to centre in the soil stratum throughout the area



$$t = \frac{T \cdot d^2}{C_v}$$

t = the time required time for the %. T = time factor

d ⇒ maximum drainage path

$$t_{90} = \frac{0.85 \times 20 \times 100 \times 20 \times 100}{5 \times 10^{-4}\ cm^2/sec}\ sec.$$

$$t_{90} = \frac{0.85 \times 20 \times 100 \times 20 \times 100 \times 10^4}{5 \times 365 \times 24 \times 60 \times 60}$$

$$t_{90} = 215.63\ \text{Years.}$$

$$t_{50} = \frac{0.2 \times 20 \times 100 \times 20 \times 100}{5 \times 10^{-4}}$$

$$= \frac{0.2 \times 20 \times 100 \times 20 \times 100 \times 10^4}{5 \times 365 \times 24 \times 60 \times 60}$$

$$t_{50} = 50.74\ \text{Years.}$$

A compressible layer is expected to have a total settlement e_f of 15 cm under a given loading. It settles by 3 cm e at the end of two t months after the application of load increment? How many months will be required to reach a settlement e of 7.5 cm? What is the settlement in 18 months? They had double drainage?

Degree of consolidation

$$U = \frac{e}{e_f} \times 100\%$$

$$t_1 = 60 \text{ days}; \quad U_1 = \frac{3\times 100}{15} = 20\%$$

$$t_2 = ? \quad U_2 = \frac{7.5}{15}\times 100 = 50\%$$

Time factors can be computed from the expression for $U < 60\%$

$$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2$$

For $U_1 = 20\%$ $\quad (T_v)_1 = \frac{\pi}{4}(0.2)^2 = 0.0314$

For $U_2 = 50\%$ $\quad (T_v)_2 = \frac{\pi}{4}(0.5)^2 = 0.1963$

$$\frac{t_2}{t_1} = \frac{(T_v)_2}{(T_v)_1} = \frac{0.1963}{0.0314}$$

$$\boxed{t_2 = \frac{0.1963 \times 60}{0.0314} = 375 \text{ days} = 12.5 \text{ months}}$$

$c_v \propto d^2/t$

$c_v = T d^2/t$

$c_v t = T d^2$

$\frac{t_1}{t_2} = \frac{T_1}{T_2}$

Again when $t = 18$ months

$$\frac{T_v}{T_{v1}} = \frac{t}{t_1}$$

$$T_v = \frac{t}{t_1}(T_v)_1 = 18/2 \times 0.0314$$

$$\boxed{T_{v1} = 0.2826}$$

$U=60\%$ $T_v = \pi/4 (0.6)^2 = 0.2826$,

approximate expression is valid.

$$U = \rho/e_f \times 100$$

$$60 = \rho/15 \times 100$$

$$\boxed{\rho = \frac{60 \times 15}{100} = 9 cm.}$$

The time of reach 40% consolidation on two way drained labouratory [illegible] thick Saturated clayey soil sample is 35 second.

Determine the [illegible] required for 60% consolidation of the sa[illegible] [illegible] cm thick on the top of a rocky surface subjected to the same loading conditions as the labouratory samples.

consolidation is less than 60% in both the cases.

$$\text{Time factor } T_1 = \pi/4 \left(\frac{40}{100}\right)^2$$

$$= 0.785 \times 0.16 = 0.1256$$

$$T_2 = \pi/4 \left(60/100\right)^2 = 0.785 \times 0.36 = 0.2826.$$

For double drainage.

$T \quad C_v \propto d^2/t$

$C_v = T_v d^2/t$

$T_v = \frac{C_v \cdot t}{d^2}$

$T_1 = \frac{C_v t}{d^2}, \quad d = h/2$

$0.1256 = \frac{4 C_v \cdot t}{d^2}$

$0.1256 = \frac{4 C_v \cdot 35}{(0.01)^2} \longrightarrow (1)$

For single drainage $T_2 = \frac{C_v t}{d^2} \quad d = h$

$0.2826 = \frac{C_v \cdot t}{10^2} \longrightarrow (2)$

$\frac{(2)}{(1)} = \frac{0.2826}{0.1256} = \frac{C_v \cdot t}{100} \Big/ \frac{4 C_v \cdot 35}{(0.01)^2}$

$2.25 = \frac{C_v t}{100} \times \frac{0.01}{4 C_v \cdot 35}$

$\frac{0.2826}{0.1256} = \frac{C_v t \cdot 0.01}{100 \times 4 \times C_v \times 35}$

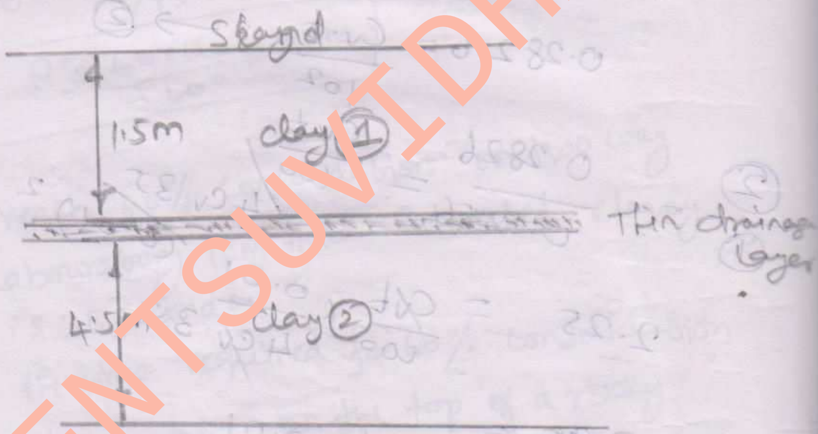
$t = \frac{0.2826 \times 4 \times 35 \times 100}{0.1256 \times (0.01)^2 \times 35}$

$t = \frac{0.2826 \times 4 \times 35 \times 100}{0.1256 \times (0.01)^2 \times 60 \times 60}$

$t = 87500$ hrs.

$= 3645.83$ days.

A 6m thick layer of clay is located two layers of free draining sand. Also there is a thin drainage layer within the clay at a depth of 1.5m from its top surface. The average value of C_v is found as $4.92 \times 10^{-2}\,mm^2/sec$. If a structure is constructed above the clay layer how many days would be required for it to attain half the ultimate settlement. Assume that the expression $T = \pi/4\, U^2$ is applicable for the entire range of consolidation.



$T_{v_1} = \pi/4\, U_1^2$ - - - (1) $d_1 = \frac{150}{2} = 75\,mm$

$T_{v_2} = \pi/4\, U_2^2$ - - - (2)

$T_{v_1} = \frac{C_v}{d_1^2} t = \frac{4.92 \times 10^{-4}}{10^{-6}(150/2)^2} . t$

$T_{v_1} = 8.74 \times 10^{-4} \times 10^{-4} t = 8.74 \times 10^{-8} t$

$T_{v_2} = \frac{C_v}{d_2^2} t = \frac{4.92 \times 10^{-4}}{(225)^2}$

$T_{v_2} = 9.71 \times 10^{-9} t$

$\pi/4\, U_1^2 = 8.747 \times 10^{-8} t$

$\pi/4\, U_2^2 = 9.7185 \times 10^{-9} t$.

$\frac{U_1^2}{U_2^2} = \frac{8.747 \times 10^{-8} \cdot t}{9.7185 \times 10^{-9}/t}$

$\frac{U_1^2}{U_2^2} = 9 \quad \therefore \frac{U_1}{U_2} = 3$

$\boxed{\Delta H = U \cdot H}$

$\Delta H_1 = U_1 H_1$

$\Delta H_1 = U_1 \times 1.5$

$\Delta H_2 = U_2 \times 4.5 = 4.5 U_2$

$\boxed{\Delta H = \Delta H_1 + \Delta H_2 = 1.5 U_1 + 4.5 U_2}$

substitute $U_1/U_2 = 3$ $\quad U_1 = 3U_2$

$1.5(3U_2) + 4.5U_2 = 3$

$1.5 \times (3 \times U_2) + 4.5U_2 = 3$

$U_2 = 4.5U_2 + 4.5U_2 = 3$

$9U_2 = 3$

$\boxed{U_2 = 1/3}$

$\pi/4\, U_2^2 = 9.7186 \times 10^{-9} t$

$\pi/4 (1/3)^2 = 9.7185 \times 10^{-9} t$

$\pi/4 \times 1/9 = 9.7185 \times 10^{-9} t$

$t = \frac{0.08722 \times 1}{10^{-9}} = 0.08722 \times 10^{9}$

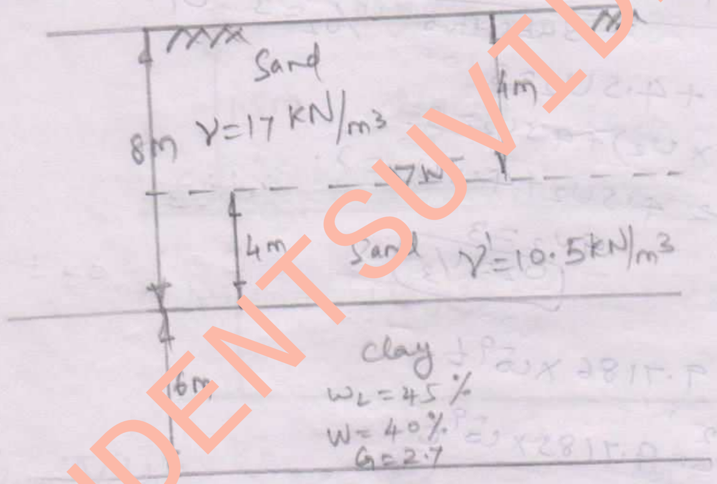
$= \frac{8.97 \times 10^{-3}}{10^{-9}}$

$= 8.97 \times 10^{6}$ Sec

$\boxed{t = 103.93 \text{ days}}$

A stratum of clay with an average liquid limit of 45% is 6m thick. the surface is located at a depth of 8m below the ground surface. The nature H_2O content of clay is 40% & the S.G is 2.7. Between ground surface & clay with the subsoil consists of fine sand.

The water is located at a depth of 4m below the ground surface. The average submerged unit wt of sand is 10.5 kN/m³ & unit wt of sand above the H_2O table is 17 kN/m³. The wt of building that will be constructed on the sand above clay increases the overburden pressure on the clay by 40 kN/m². Estimate the settlement of building.



Pressure on the top of clay due to overburden $= \sigma$

$$\gamma \times h + \gamma' \times h = 4 \times 17 + 4 \times 10.5$$

$$\boxed{\sigma' = 110\ kN/m^3.}$$

Increase in pressure due to consolidation of building

$$= 40\ kN/m^2 = \Delta\sigma$$

$$\text{Settlement } S = \frac{C_c H}{\ldots} \log_{10} \frac{\sigma' + \Delta\sigma}{\sigma'}$$

$\Rightarrow 0.315 \times$

$$= \frac{C_c \times 6}{1+e_0} \log_{10} \frac{\sigma' + \Delta\sigma}{\sigma'}$$

$S_r e = w G$

$\boxed{e_0 = w_{sat}\, G}$

$C_c = 0.009(w_L - 10)$

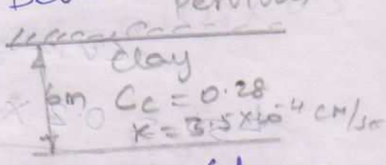
$C_c = 0.009(45-10) = 0.315$

Settlement $S = \frac{C_c H}{1+e_0} \log_{10} \frac{\sigma' + \Delta\sigma}{\sigma'}$

$$= \frac{0.315 \times 6}{1+1.08} \log_{10}\left(\frac{110+40}{110}\right)$$

$\boxed{S = 0.1224\,m}$

A saturated soil stratum 6m it lies above an impervious stratum & below a pervious stratum. It has a compression index of 0.28 & a coefficient of permeability of 3.5×10^{-4} cm/sec. Its void ratio at a stress of 150 kN/m² is 1.95. Determine



(i) the change in voids ratio

$$\Delta e = C_c \cdot \log_{10} \frac{\sigma_2}{\sigma_1}$$

$$= 0.28 \times \log_{10} \frac{210}{150} = 0.0409 \text{ (decrease)}$$

(ii) Settlement of soil stratum

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1+e_0}$$

$$\Delta H = \frac{\Delta e \times H_0}{1+e_0} = \frac{6 \times 0.0409}{1+1.95} = 0.0832\,m.$$

(iii) Time required for 50% consolidation

$$t_{50} = \frac{T_v \cdot d^2}{C_v} \quad d = 6\,m \text{ for single drainage}$$

& $\boxed{K = C_v \cdot m_v \cdot \gamma_w}$

A

$$C_v = \frac{k}{m_v \gamma_w}$$

$$m_v = -\frac{\Delta e}{1+e_0} \cdot \frac{1}{\Delta \sigma'}$$

$$= \frac{-\Delta e}{\Delta \sigma'} \cdot \frac{1}{1+e_0}$$

$$m_v = a_v \cdot \frac{1}{1+e_0}$$

$$C_v = \frac{(1+e_0)}{a_v \cdot \gamma_w}$$

$$C_v = \frac{k(1+e_0) \cdot \Delta\sigma'}{\Delta e \cdot \gamma_w}$$

$$= \frac{3.5\times10^{-6} \times (1+0.95)(200-150)}{0.0409 \times 9.8}$$

$$= 1544 \times 10^{-6}$$

$$C_v = 1.544 \times 10^{-3}\ m^2/sec.$$

$$t_{50} = \frac{0.2 \times (6)^2}{1.544\times10^{-3}}$$

$$= 4.663 \times 10^{3}$$

$$t_{50} = 4663\ sec.$$

A layer of clay 2m tk is subjected to a loading of 0.5kg/cm². one year after loading, the average consolidation is 50%. The layer has double drainage, (i) what is the coefficient of consolidation? (ii) if the coefficient of permeability is 3MM/year, what is the settlement after one years & how much time will the layer take to reach for 90% consolidation?

$H = 2m$, $d = 2/2 = 1m$

$U = 50\%$ $t = 1$ year. $\Delta\sigma = 0.5 kg/cm^2 = 0.5 \times 10^4$

(a) Determination of C_v : $U < 60\%$

$T_{v_1} = \pi/4 \times U_1^2 = \pi/4 (0.5)^2 = 0.1963$

$C_v = T_{v_1} \cdot d^2/t = 0.1963 \times 1^2/1 = 0.1963 \, m^2/year$

(b) Determination of settlement after 1 years.

$$m_v = k/c_v \cdot \gamma_w = \frac{3 \times 10^{-3} \, m/yr}{0.1963 m^2/year \times 1000 kg/m^3}$$

$= 15.283 \times 10^{-6} \, m^2/kg$

$\rho_f = m_v \cdot \Delta\sigma \cdot H_0 = 15.28 \times 10^{-6} \, m^2/kg \times 0.5 \times 10^4 \, kg/m^2 \times 2$

$= 0.1528 m$.

Settlement after one year $= \rho_f \cdot U_1 = 0.1528 \times 0.5$

$= 0.0764 \, m =$

(c) Time taken for 90% settlement

$U_2 > 90\% > 60$

$T_{v_2} = -0.9332 \log_{10}(1-0.9) - 0.0851$

Determination of coefficient of consolidation (c_v)

The recorded tks changes during one of the load stages in an odetometer test are used to evaluate the coefficient of consolidation (c_v). The process involves plotting tks changes against time (√ time (or) log time) & then fitting to this to the theoretical T_v. U_t curve.

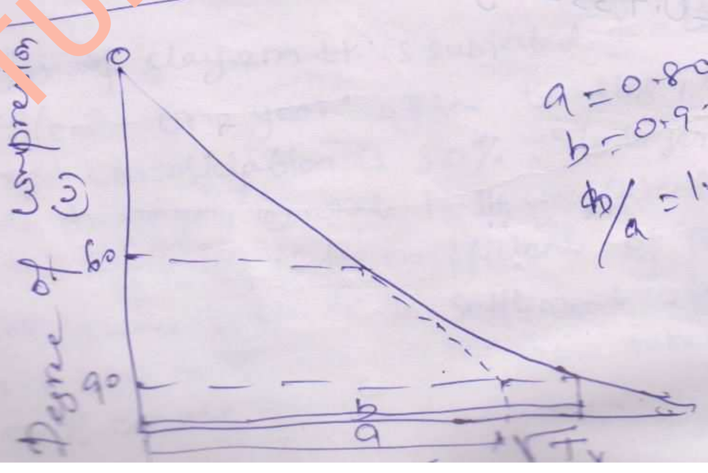
In this way known intercepts of T_v, U_t are located from which c_v may be located.

Two method's for fitting

① Square root of time fitting method.

② Logarithm of time fitting method

Square Root of time fitting method



Characteristic curve b/w $\sqrt{T_v}$ & U. The curve is straight upto $U = 60\%$ & when $U = 90\%$ then the distance to be equal to 1.15 times of $U = 60\%$

Taylor suggested that the characteristic of the theoretic curve to determination the 90% U point on a labour laboratory.

coefficient of consolidation

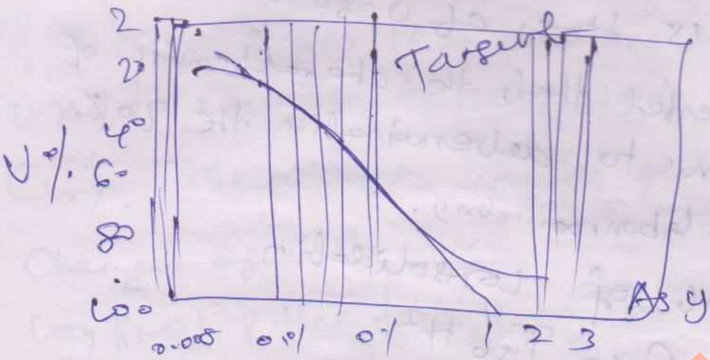
$$C_v = \frac{T_{90} H^2}{t_{90}}$$

$$\text{Primary compression ratio} = \frac{\text{Primary compression}}{\text{Total compression.}}$$

The log time fitting Method:

An Alternative method to Root time fitting method is Log time fitting method. this is widely useful when there is significant Secondary compression. Casagrande & RE Fadum (1939) devised this method consolidation curve is plotted on a semi logarithmic scale with time factor on logarithmic scale & degree of consolidation on arithmetic scale. Intersection of the tangent & asymptote is set the ordinate of 100%.

$$\boxed{C_v = \frac{T_{50} H^2}{t_{50}}}$$



Time factor T
(Log scale)

(Theoretical curve from Terzaghi's theory)

Degree of consolidation V Logarithm.

Laboratory curve

Time in minutes (log scale). Log of scale.

Sample thk / Dial gauge readings vs Laboratory curve.