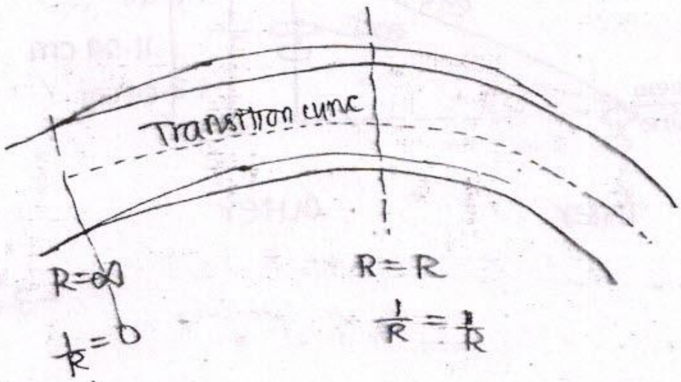
Transition curve :

When a straight portion is joined with a curved portion, a transition curve is provided at the junction to serve the following purpose:-



(i) To reduce the radius of curve in a gradual manner from $(R=\infty)$ at straight junction to $(R=R)$ at curved junction.

(ii) To avoid a sudden jerk to passengers due to centrifugal force.

(iii) To provide the SE (cant) in a gradual manner starting from zero at straight junction to full S.E (e) at curved junction.

Requirements of an ideal transition curve :

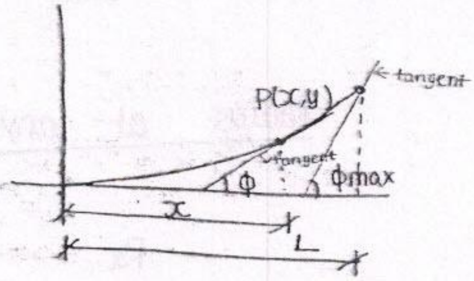
(i) The radius of curvature should be reduced in such a manner that the curve is tangential at the junction points.

At str. junction $R=\infty$

at curved junction $R=R$

Slope - at any point on the curve is the slope of tangent with X-axis at that point. This angle is also called spiral angle.

a) Spiral angle (ϕ):



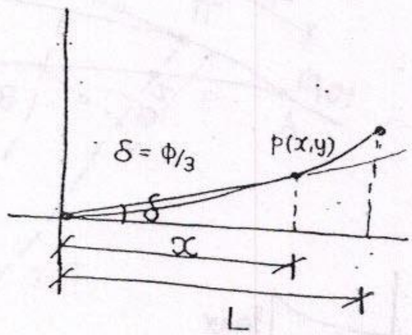
ϕ = slope at any point on the curve.

$$\boxed{\phi = \frac{x^2}{2RL}}$$

Max. spiral angle (ϕmax), at $x = L$

$$\phi max = \frac{x^2}{2RL} = \frac{L^2}{2RL} \Rightarrow \boxed{\phi max = \frac{L}{2R}}$$

b) Deflection angle (δ)



Slope of line joining any point on the curve with origin making with x-axis is called deflection angle.

$$\tan\delta = \delta = y/x$$

$$\delta = \frac{x^3}{6RL \times x} = \frac{x^2}{6RL} \; ; \; \boxed{\delta = \frac{\phi}{3}} \text{ at any point}$$

Max. deflection angle: at $x = L$

$$\delta max = \frac{L^2}{6RL} = \frac{L}{6R} = \frac{\phi max}{3}$$

3. Curvature equation :

$$\frac{d^2y}{dx^2} = \frac{2x}{2RL} = \frac{x}{RL}$$

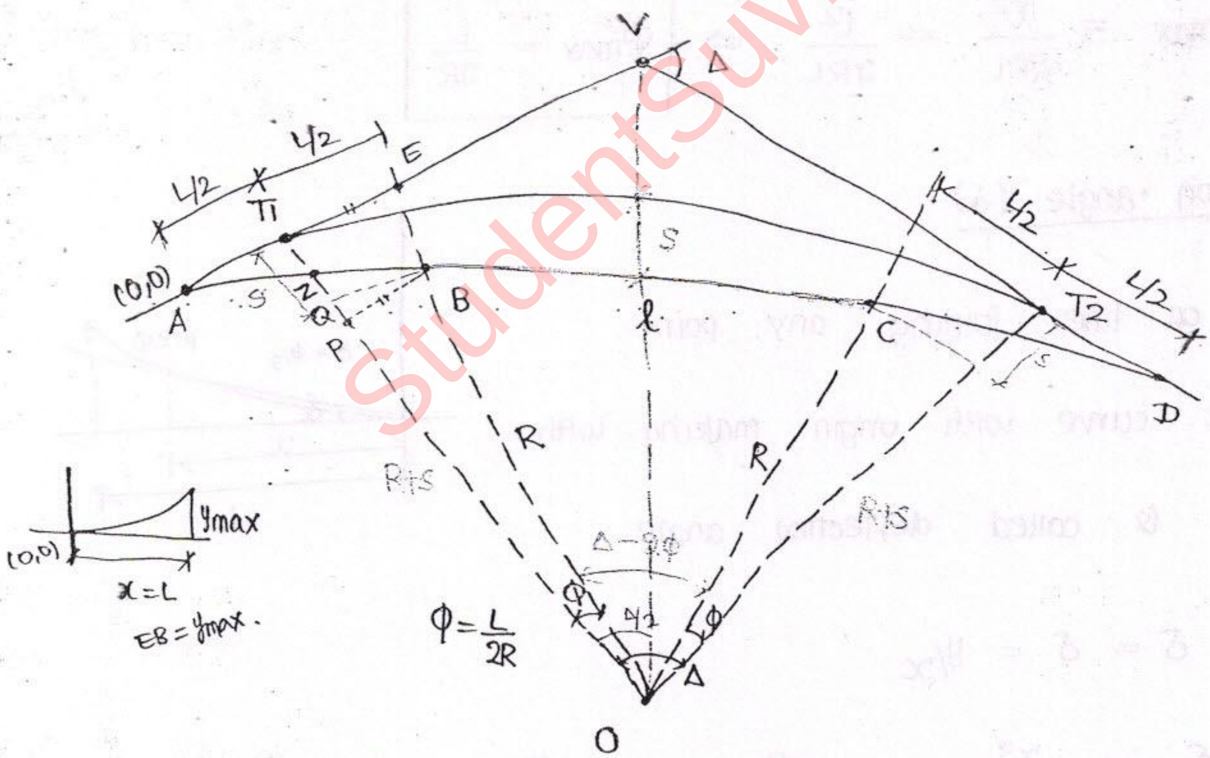
Radius at any point of curve :

$$R_x = \frac{1}{\text{curvature}} = \frac{RL}{x} \quad \ldots\ldots$$

At $x = L$

$$R_{min} = \frac{RL}{L} = R.$$

\#. Using transition curve on a railway track :



The total length of transition curve (L) is divided equally on the two side of tangent points T_1 and T_2 as shown. Due to providing transition curves at both ends of simple curve (say AB and CD), the simple curve is shifted by 's' distance called shift.

$$\angle T_1OB = \frac{L/2}{R} = \frac{L}{2R} = \phi \text{ (spiral angle)}.$$

$$\angle BOC = \Delta - 2\phi$$

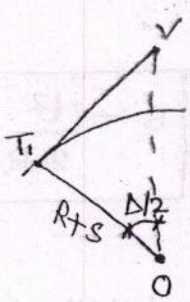
Total radius: $OT_1 = OT_2 = R+S$ where shift, $S = \frac{L^2}{24R}$

If chainage of V point known,

1. Total tangent length:

$$VA = VT_1 + T_1A$$

$$VA = (R+S)\tan\Delta/2 + L/2$$



2. Length of simple curve:

$$\underset{(l)}{BC} = \frac{2\pi R}{360}(\Delta - 2\phi)$$

chainage of A = Ch. V − VA

Chainage of B = Ch.A + AB $AB = L$.

chainage of C = chain B + BC $BC = l$.

chainage of D = chain C + CD $CD = L$

= ch. A + 2L + l.

Shift of curve :

$$T_1Q = S = T_1P - PQ$$

$$= EB - (OQ - OP)$$

$$= y_{max} - (R - R\cos\phi)$$

$$= \frac{x^3}{6RL} - R(1-\cos\phi)$$

For small angles $\sin\phi/2 = \phi/2$

$$= \frac{L^3}{6RL} - R \cdot 2\sin^2\phi/2 = \frac{L^2}{6R} - 2R\sin^2\phi/2$$

$$S = \frac{L^2}{6R} - 2R\left(\frac{\phi}{2}\right)^2 = \frac{L^2}{6R} - 2R\left(\frac{L}{2R.2}\right)^2$$

$$S = \frac{L^2}{6R} - \frac{L^2}{8R} = \frac{L^2}{24R}$$

$$\boxed{S = \frac{L^2}{24R}}$$ (shift)

#. Length of transition curve :

There are two approaches .

Try Both approaches, Higher value you set, use that approach.

1. First approach :

a) $L = 7.20\ e$

b) $L = 0.073\ e \cdot V_{max}$

c) $L = 0.073\ D \cdot V_{max}$

Where

e = cant in cm

D = cant deficiency in cm

V_{max} = speed of train in km

L = length of transition curve in meter.

2. Second approach :

a) Based on railway board formula :

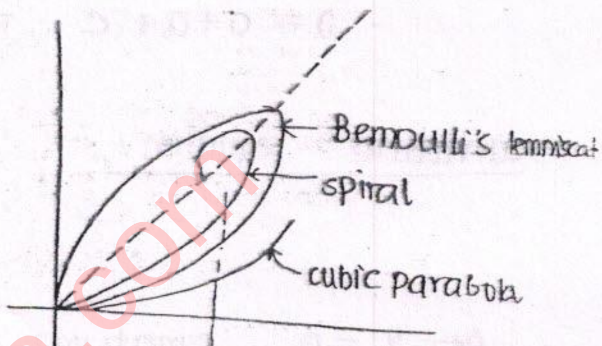
ii) The rate of change of curvature should be proportional to the rate of change of super elevation, so that full S.E can be provided within the length of transition curve.

Types of transition curve :

i) spiral → used for highway

ii) Bernoulli's lemniscate

iii) cubic parabola → used for railway transition curve.



Upto a certain degree of deflection angle, the shape of these three curves are almost identical.

Upto 4°, there is no difference.

Upto 9°, ~~there~~ curves are almost same.

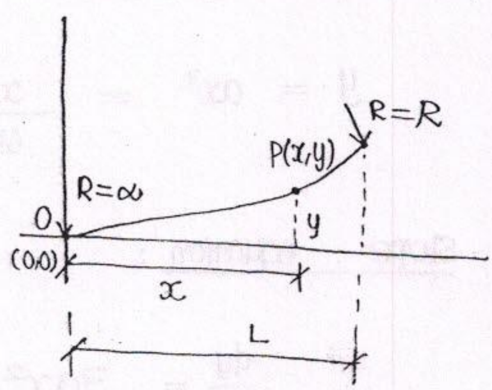
07.02.2014

Equation of cubic parabola :

General Equation :

$y = ax^3 + bx^2 + cx + d$

→ deflection Equation.



At point O, $x=0$, $y=0$

$0 = 0+0+0+d \Rightarrow \boxed{d=0}$

$$y = ax^3 + bx^2 + cx \rightarrow ①$$

Slope equation: $\frac{dy}{dx} = 3ax^2 + 2bx + c \rightarrow ②$

At $x = 0$, $\frac{dy}{dx} = 0$ (curve is tangential to x-axis at 0)

$$0 = 0 + 0 + c \Rightarrow \boxed{c = 0}$$

Curvature equation: $\frac{d^2y}{dx^2} = 6ax + 2b$

at $x = 0$, curvature $\frac{d^2y}{dx^2} = \frac{1}{R} = \frac{1}{\infty} = 0$

$$0 = 0 + 2b \Rightarrow \boxed{b = 0}$$

curvature at $x = L \Rightarrow \frac{d^2y}{dx^2} = 6ax$

$$\frac{1}{R} = 6aL \Rightarrow \boxed{a = \frac{1}{6RL}}$$

Final Equation - cubic parabola:

1. Deflection equation:

$$y = ax^3 = \frac{x^3}{6RL} \longrightarrow ①$$

2. Slope equation:

$$\frac{dy}{dx} = 3ax^2 = \frac{3x^2}{6RL} \Rightarrow \boxed{\frac{dy}{dx} = \frac{x^2}{2RL}} \longrightarrow ②$$

$$L = 4.4\sqrt{R}$$

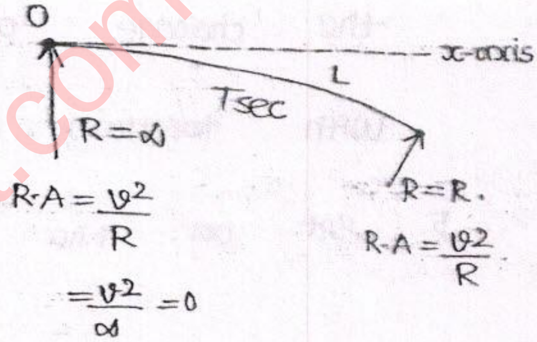
R = radius of curve in meter.

L = length of transition curve in meter.

b) <u>Based on rate of change of radial acceleration</u>:

C = rate of change of R.A.

= change of R.A from 0 to $\frac{v^2}{R}$ in T seconds.



$$C = \frac{v^2/R}{T}$$

Time taken by the vehicle to travel 'L' distance

$$T = \frac{L}{v} \quad \Rightarrow C = \frac{v^2/R}{L/v} = \frac{v^3}{LR}$$

<u>Length of transition curve</u> $\boxed{L = \frac{v^3}{CR}}$ $\longrightarrow$ ①

1 ft = 0.3048 m.
C = feet/s²/sec.
= 0.3048 m/s²/sec.

In railway, value of C = 0.3048 m/sec²/sec

have to use any one approach.

$$L = \frac{v^3}{0.3048 R} = 3.28\frac{v^3}{R}$$

Second approach is better.

$$\boxed{L = 3.28\frac{v^3}{R}}$$

v = speed in m/sec
= 0.278 V

R = radius in meter

L = Length of T.C in meter.

c) <u>Based on rate of change of S.E</u>:

$\boxed{L = 3.60\ e}$ ~~W~~

e = cant in cm.

Q1. Equilibrium cant is provided on B.G track for an average speed of 75 Kmph for a 4° curve.

1. Calculate actual cant provided for the curve.
2. Calculate max. speed that can be allowed on the track.
3. Calculate the length of transition curve required.
4. If chainage of intersection point is 4065 m, calculate the chainage of important points on the curve provided with transition curve for a deflection angle of 80°. for simple curve
5. Set out the transition curve at every 15 m distance.

Equilibrium speed = 75 Kmph = V.

Radius of curve $= \frac{1720}{4^\circ} = 430\,m$.

1. Actual cant to be provided :

$$e_{act} = \frac{GV^2}{127R} = \frac{1.676 \times 75^2}{127 \times 430} = 0.1726\,m$$

$$= 17.26\,cm$$

Max. cant that can be provided = 16.50 cm.

e_{act} (provided) = 16.50 cm.

2. Max. speed that can be allowed :

a) Safe speed by Martin's formula :

$$V_{max} = 4.35\sqrt{R-67} = 4.35\sqrt{430-67} = 82.87 \text{ kmph}$$

say 82 kmph.

b) Based on cant formula :

Among these min = V_{max}.

Theoretical cant = $e_{act} + D = 16.50 + 7.6 = 24.10$ cm.

$$V_{max} = \sqrt{\frac{127 \times R \times e_{th}}{G}} = \sqrt{\frac{127 \times 430 \times 0.2410}{1.676}} = 88.62 \text{ kmph}$$

Max. speed allowed on the track = 82 kmph.

3. Length of transition curve :

a) As per railway board :

$$L = 4.4\sqrt{R} = 4.4\sqrt{430} = 91.24 \text{ m}.$$

b) From rate of change of R.A :

$$L = \frac{v^3}{CR} = 3.28\frac{v^3}{R}$$

here $v = v_{max}$ (always) (in this formula)

$$= \frac{3.28 \quad (0.278 \times 82)^3}{430} = 90.36 \text{ m}$$

c) As per rate of change of S.E :

Among these, max = length T.C

$$L = 3.6 \times e$$

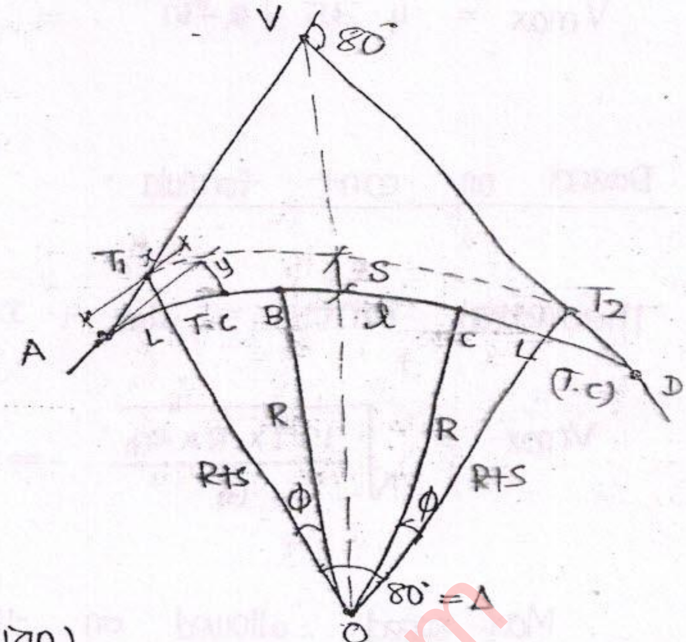
$$= 3.6 \times 16.50 = 59.4 \text{ m}.$$

Length of transition curve = 91.24 m

$$L = 91.24 \text{ m}.$$

(say 92 m)

4. chainage of imp points on curve :



$R = 430\,m$

$\Delta = 80^\circ$

spiral angle $\phi = \frac{L}{2R}$

$= \frac{92}{2 \times 430}$ (radian)

$\phi = 0.107$ (radian) $= 0.107 \times \frac{180}{\pi} = 6^\circ 7' 45.54''$

chainage $V = 4065\,m$.

Total tangent length $VA = VT_1 + T_1 A$

$= (R+S)\tan \Delta/2 + L/2$

$S = \frac{L^2}{24R}$

$= (430 + 0.82)\tan 40 + 92/2$

$= \frac{92^2}{24 \times 430} = 0.82\,m$

$= 407.50\,m$

Length of simple curve : $l = \frac{2\pi R}{360}(\Delta - 2\phi)$

$= \frac{2\pi \times 430}{360}(80 - 2 \times 6^\circ 7' 45.54')$

$= 508.39\,m$

chainage $V = 4065\,m$

$- VA = -407.5$

chainage $A = 3657.50\,m$

$+L = +92\,m$

$\text{chainage } B = 3749.50\ m$

$+l\,(BC) = +508.39\ m$

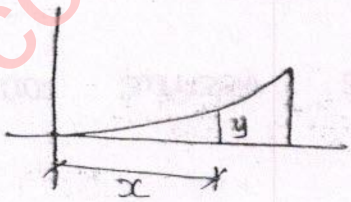
$\text{chainage } C = 4257.89\ m$

$+L = +92\ m$

$\text{chainage } D = 4349.89\ m.$

5. <u>setting out transition curve :</u>

Equation of transition curve (cubic parabola)



$$y = \frac{x^3}{6RL}$$

$$= \frac{x^3}{6 \times 430 \times 92} = \frac{x^3}{237360}\ m.$$

x	0	15 m	30 m	45 m	60 m	75 m	90 m	92 m
y	0	0.014m	0.114	0.384m	0.91	1.77	3.07	3.28

08.02.2014

#. <u>Max. speed as per length of transition curve :</u>

1. <u>for normal speed (speed < 100 kmph)</u>

$$V_{max} = \frac{134\ L}{e_{(mm)}}$$

$$(or) = \frac{134\ L}{D_{(mm)}}$$

whichever is less.

L = length of T.C in metre

e = cant in mm

D = deficiency in mm

V_{max} = kmph

2. For high speed (speed > 100 kmph)

$$V_{max} = \frac{198\,L}{e} \quad (\text{or}) \quad \frac{198\,L}{D} \quad (\text{whichever is less})$$

IES 2008. Q.4(b) Design the length of transition curve for a B.G track, having 4° curve and a cant of 12 cm. The max. design speed on the curve is 100 kmph. Also calculate the offsets at every 15 m interval and shift of circular curve. Assume cant deficiency = 7.60 cm

$$R = \frac{1720}{4^\circ} \Rightarrow R = 430\,m.$$

$$e_{th} = \frac{GV_{max}^2}{127\,R} = \frac{1.676 \times 100^2}{127 \times 430} = 0.3069\,m$$

$$= 30.69\,cm$$

$$e_{th} = e_{act} + D \Rightarrow e_{act} = 23.09\,cm$$

Max. cant that can be provided = 16.50 cm.

e_{act} (provided) = 16.50 cm.

Length of transition curve:

$$L = 4.4\sqrt{R} = 4.4\sqrt{430} = 91.24\,m.$$

Solution:

1. Max. speed allowed on the track:

a) max. design speed = 100 kmph.

b) Based on cant + cant Deficiency.

$e = 12\ cm$, $D = 7.60\ cm$

$e_{th} = 19.60\ cm = 0.1960\ cm$.

$$V_{max} = \sqrt{\frac{127 \times R \times e_{th}}{G}} = \sqrt{\frac{127 \times 430 \times 0.1960}{1.676}} = 79.9\ kmph$$

c) Martin's formula:

$$V_{max} = 4.35\sqrt{R-67} = 4.35\sqrt{430-67} = 82.88\ kmph.$$

The max. speed that can be allowed on the track is 79 kmph.

Shift: $s = \frac{L^2}{24R}$

$= \frac{92^2}{24 \times 430}$

$= 0.82 m.$

2. Length of transition curve:

a) $L = 4.4\sqrt{R} = 4.4\sqrt{430} = 91.24\ m.$

b) $L = \frac{V^3}{CR} = 3.28\ \frac{V^3}{R} = \frac{3.28 \times (0.278 \times 79)^3}{430}$

$= 80.80\ m.$

c) $L = 3.6 \times e = 3.6 \times 12\ cm$

$= 43.2\ m.$

So length of transition curve is $91.24\ m \simeq 92\ m$.

3. Offsets:

$y = \frac{x^3}{6RL} = \frac{x^3}{6 \times 430 \times 92} = \frac{x^3}{237360}$

x	0	15m	30m	45m	60m	75m	90m	92m
y	0	0.014	0.114	0.384	0.91	1.77	3.07	3.28

2. Calculate max. speed of train allowed on a track with $2°$ horizontal curve where S.E provided is 9.0 cm max. cant deficiency allowed is 10 cm. Max. sanction speed by railway board is 145 kmph. Length of transition curve is 125 m.

Max. speed of train allowed :

a) Max. speed sanctioned by railway = 145 kmph.

b) As per cant and cant deficiency :

$$e_{th} = e_{act} + D = 9 + 10 = 19 \text{ cm.}$$

$$R = \frac{1720}{2°} = 860 \text{ m.}$$

$$V_{max} = \sqrt{\frac{127 \times R \times e_{th}}{G}} = \sqrt{\frac{127 \times 860 \times 0.19}{1.676}}$$

$$= 111.27 \text{ kmph.}$$

c) Martin formula :

$$V_{max} = 4.35\sqrt{R - 67} = 4.35\sqrt{860 - 67} = 122 \text{ kmph} > 100 \text{ kmph}$$

Let us use $V_{max} = \sqrt{R} \times 4.58 = 4.58\sqrt{880} = 134$ kmph.

d) Max. speed as per length of T.C :

(For high speed) > 100 kmph. $L = 125$ m.

$$V_{max} = \frac{198L}{e} = \frac{198 \times 125}{90} = 275 \text{ kmph.}$$

$$(\text{or}) = \frac{198 \times L}{D} = \frac{198 \times 125}{100} = 247.5 \text{ kmph.}$$

Max. speed allowed on the track = 111.27 kmph.

≃ (say) 111 kmph.

ES2007 Q.6C. Define cant and cant deficiency. Calculate length of T.C for B.G curved track having 5° deflection and a cant of 14 cm. Max. permissible speed on the track is 80 kmph.

Max. speed allowed on the track :

a) As per railway board = 80 kmph.

b) As per cant & cant deficiency : $R = \frac{1720}{5} = 344$ m

$e = 14$ cm, $D = 7.60$ cm, $e_{th} = 21.60$ cm.

$$V_{max} = \sqrt{\frac{127 \times R \times e_{th}}{G}} = \sqrt{\frac{127 \times 344 \times 0.216}{1.676}} = 75.04 \text{ kmph.}$$

c) Martin's formula :

$$V_{max} = 4.35\sqrt{R-67} = 4.35\sqrt{344-67} = 72.39 \text{ kmph.}$$

Max. speed allowed $= 72\,kmph$.

Length of transition curve :

a) $L = 4.4\sqrt{R} = 4.4\sqrt{344} = 81.6\ m$

b) $L = 3.28\dfrac{V^3}{R} = \dfrac{3.28\,(0.278 \times 72)^3}{344} = 76.46\ m.$

c) $L = 3.6 \times e = 3.6 \times 14 = 50.4\ cm.$

Length of transition curve $= 81.6\ m.$

Using first approach also, we can calculate length of T.C.

a) $7.2 \times e = 7.2 \times 14 = 100.8\ m.$

use both approaches. whatever max 'V' you get, use that approach

b) $L = 0.073 \times e \times V_{max} = 0.073 \times 14 \times 72 = 73.58\ m.$

c) $L = 0.073\,.\,D.\,V_{max} = 0.073 \times 7.6 \times 72 = 39.94\ m.$

We can use Length $= 100.8\ m.$

ES2003. Q.1.c) For a B.G track in transition zone, inorder to allow locomotives with max. permissible speed of 110 kmph. Calculate the following. 1. Radius of curve. 2. Degree of curvature 3. Super Elevation 4. Length of transition curve.

$V_{max} = 4.35\sqrt{R-67} = 110 = 4.35\sqrt{R-67}$

$R = 706.45\ m.$

For speed > 100 kmph , $V_{max} = 4.58\sqrt{R}$

$110 = 4.58\sqrt{R} \Rightarrow R = 576.84\,m.$

Degree $= \frac{1720}{R} \Rightarrow D^{\circ} = \frac{1720}{576.84}$ Railway use cant formula

degree = 2.98° (say 3°)

super elevation, $e_{th} = \frac{G V_{max}^2}{R \times 127}$

$= \frac{1.676 \times 110^2}{573 \times 127}$

$= 0.2786\,m$

$e_{th} = 27.86\,cm$

$e_{act} = 27.86 - 10\,cm = 17.86\,cm.$

~~Radius~~

* Radius of curvature $= \frac{1720}{3^{\circ}}$ $= 573\,m.$

* Deg. of curvature = 3°.

Max. 16.50 cm is allowed.

Let us increase the radius of curve. consider 2° degree curve

$R = \frac{1720}{2} = 860\,m$

degree of curvature = 2°.

cant value, $e_{th} = \frac{1.676 \times 110^2}{127 \times 860} = 0.1856\,m$

$= 18.56\,cm.$

$e_{act} = 18.56 - 10.0 = 8.56\,cm.$

Length of transition curve :

$L = 4.4\sqrt{R} = 4.4\sqrt{860} = 129.03\,m.$

b) $L = \frac{3.28 \times (0.278 \times 110)^3}{860} = 109.0\ m.$

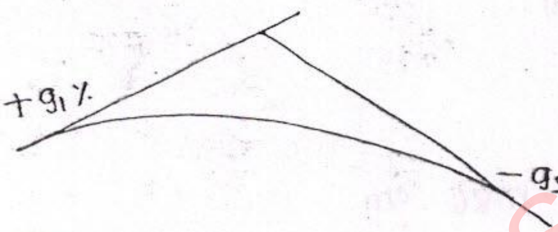
c) $L = 3.6 \times e = 3.6 \times 8.57 = 30.852\ m.$

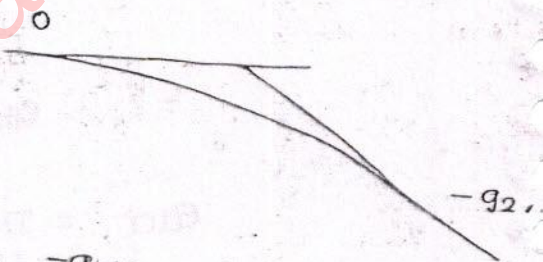
(Use) Length of transition curve = 129.03 m = 130 m.

Vertical curves :

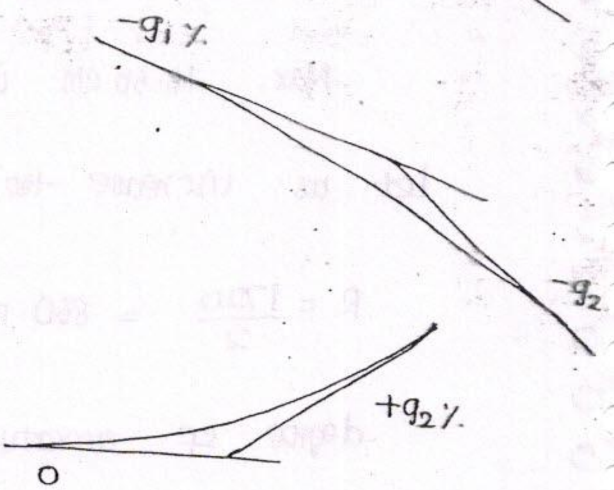
Type of vertical curve :

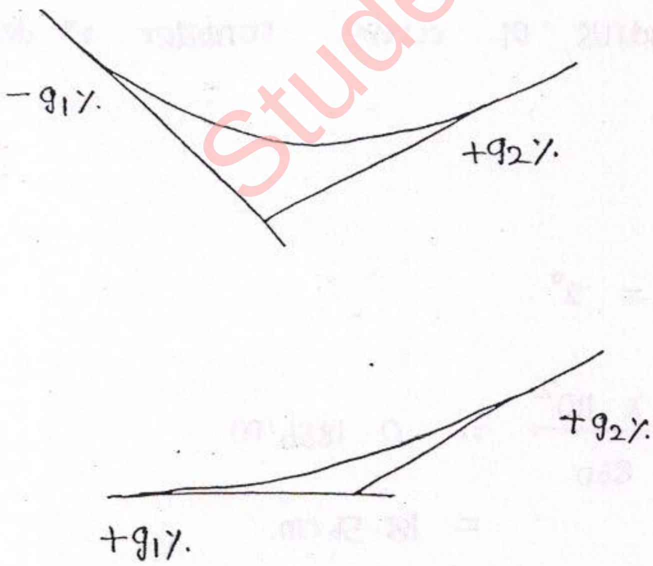
1. **Summit curve :**





2. **Valley curve :**





#. If first gradient $= \pm g_1\%$

second gradient $= \pm g_2\%$